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Mutual Coupling Calibration for L-Shaped Microstrip Antenna Array with Accurate 2-D Direction of Arrival Estimation

Jianhui Bao[1], Qiulin Huang[1, *], Xinhuai Wang[1], Peng Liu[2], and Xiaowei Shi[1]

Abstract—An improved mutual coupling calibration approach based on the element pattern reconstruction (EPR) method is proposed in this paper. Compared to previous calibration methods in which the calibration is carried out in the entire space, the space angle ranges to be calibrated in this method is partitioned according to the interested directions. Through the partition of the space angle region, the angle ranges to calibrate are narrowed, and thus more accurate calibration matrix can be obtained in corresponding angle regions. With the employment of the calibration matrix on 2-D DOA estimation, more effective mutual coupling calibration and more accurate DOA estimations are achieved by alternate iteration in each angle region. The validity of this method is verified by an L-shaped microstrip antenna array, and the performance of mutual coupling calibration is presented by the better accuracy in 2-D DOA estimations.

1. INTRODUCTION

It is well known that mutual coupling is one of the main error factors for the application of many super-resolution techniques in actual direction of arrival (DOA) estimation systems [1–4]. The actual manifold employed in these algorithms is not ideal point sources array manifold due to the antenna element performance and the mutual coupling in the array. Over the past years, many attempts have been made to reduce or to calibrate the mutual coupling effect, and many methods have been proposed. In 1983, open-circuit voltage method was proposed by Gupta and Ksienski [5]. The advantage of this method is that the mutual coupling analysis is characterized by mutual impedance matrix. And the definition of the mutual impedance is taken from the mutual impedance concept of circuit analysis such as Z parameters in network analysis. The definition is easily understood. However, this method treats the open-circuit voltages of the array as decoupled voltage, which is not exactly match the real situation of antenna array. The open-circuit scattering of the antenna exists, which is ignored in the open-circuit voltage method. Therefore, the calibration effect is weakened, and this method can only be used in the antenna array composed of simple wire elements. The receiving mutual impedance method takes the open-circuit scattering effect into account [6, 7]. This method has a good effect on wire elements, just as dipoles and monopole antenna array. The simple wire antenna element has a special character that the pattern is omnidirectional in horizontal plane when the antenna placed vertically. Therefore, the terminal voltage of the antenna response to various direction incident signals in horizontal plane is the same. Based on the special character of the simple wire antenna, the receiving mutual coupling impedance matrix is obtained by the experiment of which the incident signal is from the horizontal direction. Although it is effective for wire antenna element, this method is not effective generally for the microstrip antenna element because the pattern is non-omnidirectional and the response to the different incident signals directions are not consistent. The minimum-norm method also provides better

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* Corresponding author: Qiulin Huang (qlhuang1@mail.xidian.edu.cn).
[1] Science and Technology on Antenna and Microwave Laboratory, Xidian University, Xi'an, Shaanxi 710071, P. R. China. [2] Space Star Technology Co., Ltd and State Key Laboratory of Space-Ground Integrated Information Technology, Beijing 100086, P. R. China.

mutual coupling calibration than the open-circuit voltage method with consideration of the secondary scattering effect of an array [8]. Although more accurate mutual coupling calibrations are achieved, these methods have the same limitation as the open-circuit voltage method that there is requirement on the current distribution, which is more suitable for the wire antennas arrays. The joint estimation method can achieve the calibration matrix and DOAs of incident signals [9–12]. This method needs no priori information of incident signals, which is called blind estimation. However, it requires more calculation time and multiple iterations which are time consuming and impractical for real application.

The element pattern reconstruction (EPR) method is an alternative method on mutual coupling calibration, which overcomes the limitations of the previous methods on the current distribution and omnidirectional pattern. This method has a good effect not only on the calibration of omnidirectional antenna element array, but also on the non-omnidirectional antenna element array. It involves two parameters. One is the isolated element pattern when the element is in the isolated state. The other is the embedded element pattern when the element is arranged in the array where one element is excited and others are terminated with match loads. The calibration matrix is achieved by the far-field pattern transformation relation between the embedded element and isolated element. This method has been verified valid on the mutual coupling calibration of uniform line array and conformal antenna array for 1-D DOA estimation, 2-D DOA estimation, etc. [13–15]. While the sampled directions in the EPR method are generally chosen within 3 dB beam width, the calibration effect may not be obvious for the incident signals far from the 3 dB beam width. In addition, the mutual coupling calibration of the previous methods is made in entire space, which cannot adaptively calibrate mutual coupling according to the incident signals accurately and in real time.

For 2-D DOA estimation, L-shaped array has lower number of antenna elements and minimal estimation errors than other array shapes, such as cross, T-shaped and square types [16, 17]. However, there is little discussion about the applications of these mutual coupling calibration methods mentioned on L-shaped microstrip antenna array 2-D DOA estimation. Most research is based on the assumption that the elements of the L-shaped microstrip antenna array were ideal point sources in the literature. In [18], the experimental calibration method for 2-D DOA estimation with L-shaped patch array is proposed with reduced number of reference signals, while in this algorithms, the ideal point sources array manifold is employed in the eigenvalue decomposition process of the super resolution technique. Actually, the microstrip element in actual isolate situation is non-omnidirectional, which is not quite consistent with the ideal isolate point sources for most practical situations.

In this paper, an improved mutual coupling calibration method based on the EPR method is proposed. The space angle ranges to be calibrated in this method is partitioned according to the interested directions. More accurate calibration matrix is obtained through the partition of the space angle regions. More effective mutual coupling calibration and more accurate DOA estimations are obtained through the employment of the calibration matrix on 2-D DOA estimation by alternate iteration in each angle region. The numerical examples are shown using the five-element L-shaped microstrip antenna array in the following.

2. THEORY

To adapt various antenna shapes and achieve wider angle scope of DOA estimation, an improved mutual coupling calibration approach based on the EPR method is proposed in this paper. The EPR method is based on the fact that if the reconstructed pattern of the embedded element is consistent with that in the isolated state, then the corresponding calibrated received signal will be consistent with the received signal without mutual coupling effect. The new method achieves more accurate calibration matrix through the partition of the space angle regions. The angle ranges where the mutual coupling calibration is performed are corresponding to the directions of the incident signals, which realize that calibration metrics can be adaptively adjusted according to the incident signals. In the EPR method, there is an antenna array composed of N elements, and each element is terminated with the same load Z_L. In Eq. (1), $E_n(\theta,\varphi)$ and $E_n^i(\theta,\varphi)$ with $n=1,2,\ldots,$ and N represents the embedded element pattern and the isolated element pattern for each element, respectively. The transformation relation of the electric field main polarization component between two kinds of element pattern can be written as

$$\left[E_1^i(\theta,\varphi)\,E_2^i(\theta,\varphi)\ldots E_N^i(\theta,\varphi)\right]^T=\mathbf{C}\left[E_1(\theta,\varphi)\,E_2(\theta,\varphi)\ldots E_N(\theta,\varphi)\right]^T \tag{1}$$

where $\mathbf{C}$ is the calibration matrix for the total angle regions. In order to calculate the calibration matrix via Eq. (1), M directions are sampled, and then the pattern matrices in Eq. (1) will be transformed to Eq. (2),

$$\begin{bmatrix} E_1^i(\theta_1,\varphi_1)\ldots E_1^i(\theta_M,\varphi_M) \\ E_2^i(\theta_1,\varphi_1)\ldots E_2^i(\theta_M,\varphi_M) \\ \vdots \\ E_N^i(\theta_1,\varphi_1)\ldots E_N^i(\theta_M,\varphi_M) \end{bmatrix} = \mathbf{C} \begin{bmatrix} E_1(\theta_1,\varphi_1)\ldots E_1(\theta_M,\varphi_M) \\ E_2(\theta_1,\varphi_1)\ldots E_2(\theta_M,\varphi_M) \\ \vdots \\ E_N(\theta_1,\varphi_1)\ldots E_N(\theta_M,\varphi_M) \end{bmatrix} \tag{2}$$

where $\mathbf{C}$ is the least square solution, which satisfies $\min ||\mathbf{CE}-\mathbf{E}^i||$. And now, the calibration matrix for the angle range within the 3 dB beam width is obtained by the EPR method, and the least square solution can be given by

$$\mathbf{C} = \mathbf{E^i}\mathbf{E}^H\left(\mathbf{EE}^H\right)^{-1} \tag{3}$$

where $\mathbf{E}$ and $\mathbf{E}^i$ represent the pattern matrix for all embedded elements and for the isolated elements, respectively. The operator $||\cdot||$ denotes the F-norm of a matrix and superscript H the complex conjugate transpose.

In the improved method, the entire space is divided according to interesting directions, and the calibration is made in the corresponding regions. Then the calibration matrices can be employed on DOA estimation by alternate iteration. For the partition and employment on DOA estimation, there are three parameters for the determination of the angle regions, which are the rough DOAs of incident signals, DOA estimations empirical error and the maximum iteration error. The DOA estimation empirical error can be obtained through the experiment experience. Assume that the DOA estimations empirical error angle is $\delta(\theta,\varphi)_{error}$. Meanwhile, the incident signal angle achieved in the pth iteration is $\phi(\theta,\varphi)_k^p$; p is the iteration number; k represents the kth angle region. $\phi(\theta,\varphi)_{error}^{p+2}$ is defined as the maximum iteration error of incident signals angle between (p+1)th and pth iterations, just as shown in (4),

$$\phi(\theta,\varphi)_{error}^{p+2} = \max\left|\phi(\theta,\varphi)_k^{p+1} - \phi(\theta,\varphi)_k^p\right| \tag{4}$$

Therefore, the kth angle range $Region\,(k)^{p+2}$ which would be calibrated in (p+2)th iteration is just chosen as $[\phi(\theta,\varphi)_k^{p+1} - \delta(\theta,\varphi)_{error} - \phi(\theta,\varphi)_{error}^{p+2},\ \phi(\theta,\varphi)_k^{p+1} + \delta(\theta,\varphi)_{error} + \phi(\theta,\varphi)_{error}^{p+2}]$. The angle regions have been determined by the above steps. After the partition of the space angle region, the EPR method is employed in each region. There may be k angles regions for k incident signals, and each angle region has its own calibration matrix, which we call $\mathbf{C_{sub\_k}}$. According to the uniqueness of least-square solution for a certain area, there is the relationship in Eq. (5)

$$\min ||\mathbf{C_{sub\_k}}\mathbf{E} - \mathbf{E}^i|| \leq \min ||\mathbf{CE} - \mathbf{E}^i|| \tag{5}$$

$\mathbf{C_{sub\_k}}$ is the least square solution for the corresponding angle region, which is more accurate calibration matrix than $\mathbf{C}$.

By the partition of the space angle regions and the employment of the EPR method in these angle ranges, more accurate calibration matrices can be obtained. More accurate DOA estimation can be achieved by the alternate iteration of the improved method and DOA estimation algorithm in each angle region. The iteration is employed until the final iteration error of the calibration matrices, and the incident signal angles are both less than a threshold. The interaction can be bound to a threshold governed by the error of the phase between the former and the newly calculated incident signal angle. Assume that the received signal of the array is $\mathbf{X}$, and then the step of the improved mutual coupling calibration method with application to DOA estimation can be summarized as follows:

1) Initializes the calibrate matrix $\mathbf{C^p}$. $\mathbf{C^1}$= $\mathbf{I}$, $\mathbf{I}$ is the unit matrix and p the number of iterations with $p = 1,\ldots,P$.
2) Use iterations calibrate matrix C^p to achieve the calibrated signal $\mathbf{X_{new}}$with $\mathbf{X_{new}}$=$\mathbf{C^pX}$.
3) Calculate covariance matrix $\mathbf{R_{new}}$=$\mathbf{\frac{1}{T}X_{new}X_{new}^H}$, with T the number of samples.
4) Determine the angle regions. Using DOA estimation algorithm to estimate the incident signal angle $\phi(\theta,\varphi)_k^p$. Assume that there are K received signals and that the incident signal directions are $\left[\phi(\theta,\varphi)_1^p, \phi(\theta,\varphi)_2^p, \ldots, \phi(\theta,\varphi)_i^p, \ldots, \phi(\theta,\varphi)_K^p\right]$.

According to $\phi(\theta,\varphi)_i^p$ and $\phi(\theta,\varphi)_i^{p+1}$, the DOA estimations experience error $\delta(\theta,\varphi)_{error}$ and the maximum iteration error $\phi(\theta,\varphi)_{error}$, the space angle regions for the $(p+2)$th iteration which would be calibrated are determined. For the $(p+2)$th iteration, the space angle regions are [*Region* $(1)^{p+2}$, *Region* $(2)^{p+2}$, ..., *Region* $(k)^{p+2}$].

5) Compute the mutual coupling calibration matrix $\mathbf{C^p_{sub\_k}}$ using Eqs. (3) and (4) in each angle region with $k = 1, 2, \ldots, K$. Then the new calibration matrix $\mathbf{C^p_{sub\_k}}$ is employed on the DOA estimation to achieve more accurate incident signal directions $\phi(\theta,\varphi)_i^{p+1}$.

6) Convergence check. Compute difference between $\phi(\theta,\varphi)_i^{p+1}$ and $\phi(\theta,\varphi)_i^p$, and consider the lease-square solution that satisfies Eq. (5).

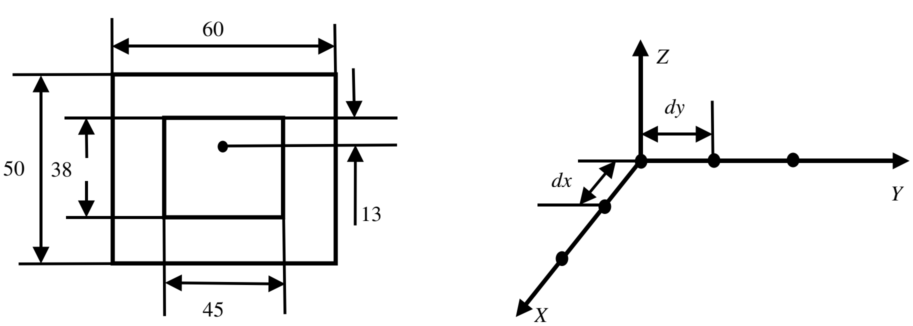
If $\left|\phi(\theta,\varphi)_i^{p+1} - \phi(\theta,\varphi)_i^p\right| < threshold$, and $\min||\mathbf{C_{sub\_k}E}-\mathbf{E}^i|| \leq \min||\mathbf{CE}-\mathbf{E}^i||$, the loop will break and end the algorithm.

The mutual coupling calibration of the improved method is carried out in the incident signal angle regions with moderately ignoring other angle regions. The total space is adaptively divided according to the rough DOA estimation. Through the partition, the angle range which will be calibrated is narrowed and more accurate calibration matrix can be obtained. The reconstruction of the embedded pattern to isolate element one is easier in a narrower angle range. Meanwhile, the angle range of DOA estimations is widened with the employment of the improved method on DOA estimation. In the following, the improved method is employed on the mutual coupling calibration of a five-element L-shaped microstrip antenna array, and the validity is verified by the better accuracy of 2-D DOA estimation.

3. NUMERICAL EXAMPLES

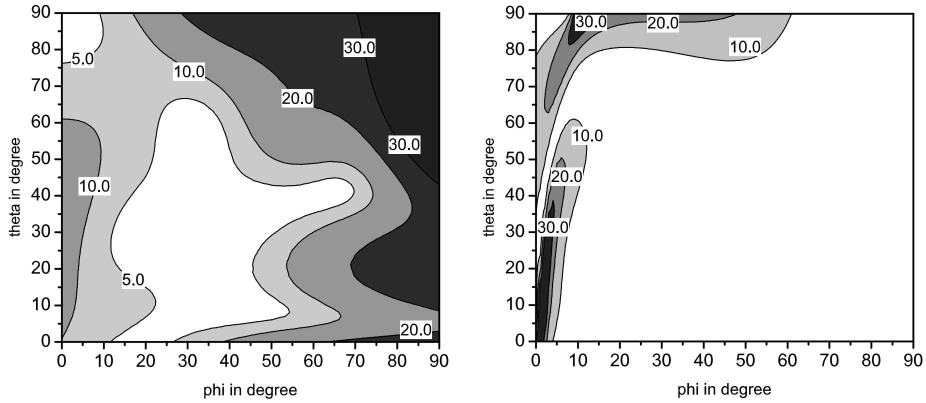
In this section, an L-shaped array with five linearly polarized microstrip elements has been designed. The element operates at 2.45 GHz with 3-dB beam width about 45°, and each element is terminated with the same load $Z_L = 50\,\Omega$. The structure of the microstrip element is shown in Fig. 1(a), and the element is printed on a substrate with relative dielectric constant of 2.2 and thickness of 3.0 mm. The L-shaped array is in the *xoy*-plane just as Fig. 1(b). The element spacings in x- and y-axis directions is dx and dy, respectively, with $dx = dy = 0.5\lambda$. The classic 2-D MUSIC algorithm is employed. The data sample is 2000, and the signal-to-noise ratio (SNR) of the incident signals involved is 20 dB. Meanwhile, the DOA estimations experience error $\delta(\theta,\varphi)_{error}$ is 5°, and we set the threshold of 0.01°. The EM simulation tool FEKO 6.0 is used to calculate the element patterns.

In order to illustrate the reconstructed effect of EPR method in certain area, the difference curves of the magnitude and phase between different states owning to the central element are shown in Fig. 2 and Fig. 3. The sampled directions are in the area where θ is [1°, 60°] and φ is [1°, 60°] for example. In the entire area, the calibration matrices are used to reconstruct the embedded element pattern. In



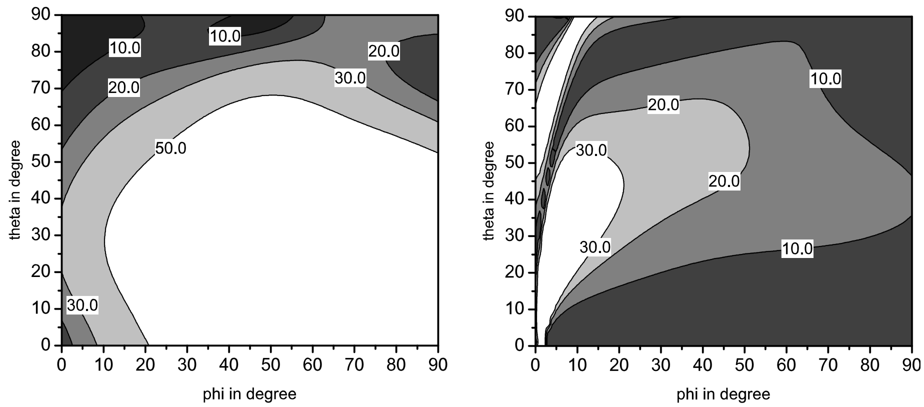
(a) (b)

Figure 1. Profile of the antenna element and array (Unit: mm). (a) The microstrip element. (b) The L-shaped array.



(a) (b)

Figure 2. The difference curves between the reconstructed embedded pattern and isolate state. (a) Magnitude difference curves. (b) Phase difference curves.



(a) (b)

Figure 3. The difference curves between without calibration and isolate state. (a) Magnitude difference curves. (b) Phase difference curves.

Fig. 2, it can be seen that the magnitude difference of the electric field between the reconstructed embedded pattern and the isolated one is less than 5 units in the area where θ covers [10°, 60°] and φ covers [10°, 60°], and the phase difference is less than 10 units for almost entire area. On the other hand, in Fig. 3, the magnitude difference of the electric field between the mutual coupling state and the isolated state is more than 50 units in most area, and the phase difference is from 10 to 30 units for the most area. It can be seen that the calibration matrices obtained by the EPR method is effective for the mutual coupling calibration of the area selected with some bias. In the actual situation, the entire angle regions are partitioned according to the incident signals, and the data are sampled in corresponding region. The angle regions are firstly got through the rough scan by DOA estimation algorithm. More accurate calibration matrices and smaller area which will be calibrated are achieved through the alternate iteration.

In DOA estimations, the calibration matrices are employed to calibrate the incident signals received by the L-shaped microstrip antenna array. To verify the validity of the improved method, two incident signals from different directions are employed at the same time. The signals are Signal 1 (10°, 20°) and Signal 2 (60°, 30°). One is within the 3-dB bandwidth, and the other is far from the 3-dB bandwidth of the L-shaped array. In contrast, there are three states listed, which are State1: without calibration where the array is in the mutual coupling state; State2: the EPR method whose mutual coupling of the array is calibrated by the EPR method; State3: the proposed method whose mutual coupling of the array is calibrated by the improved method proposed by this paper. Fig. 4 shows the contour charts and spatial spectra of the 2-D DOA estimations at various states.

As shown in Fig. 4(a), when the array is with mutual coupling, Signal 1 can be obtained with some bias, and the spatial spectra peak is only 12.3 dB. The bias between the state with mutual coupling and the isolated state is 1.2° in theta and 2.2° in phi. However, Signal 2 cannot be obtained by the array. It shows that the performance of the array is deteriorated by the mutual coupling between the elements

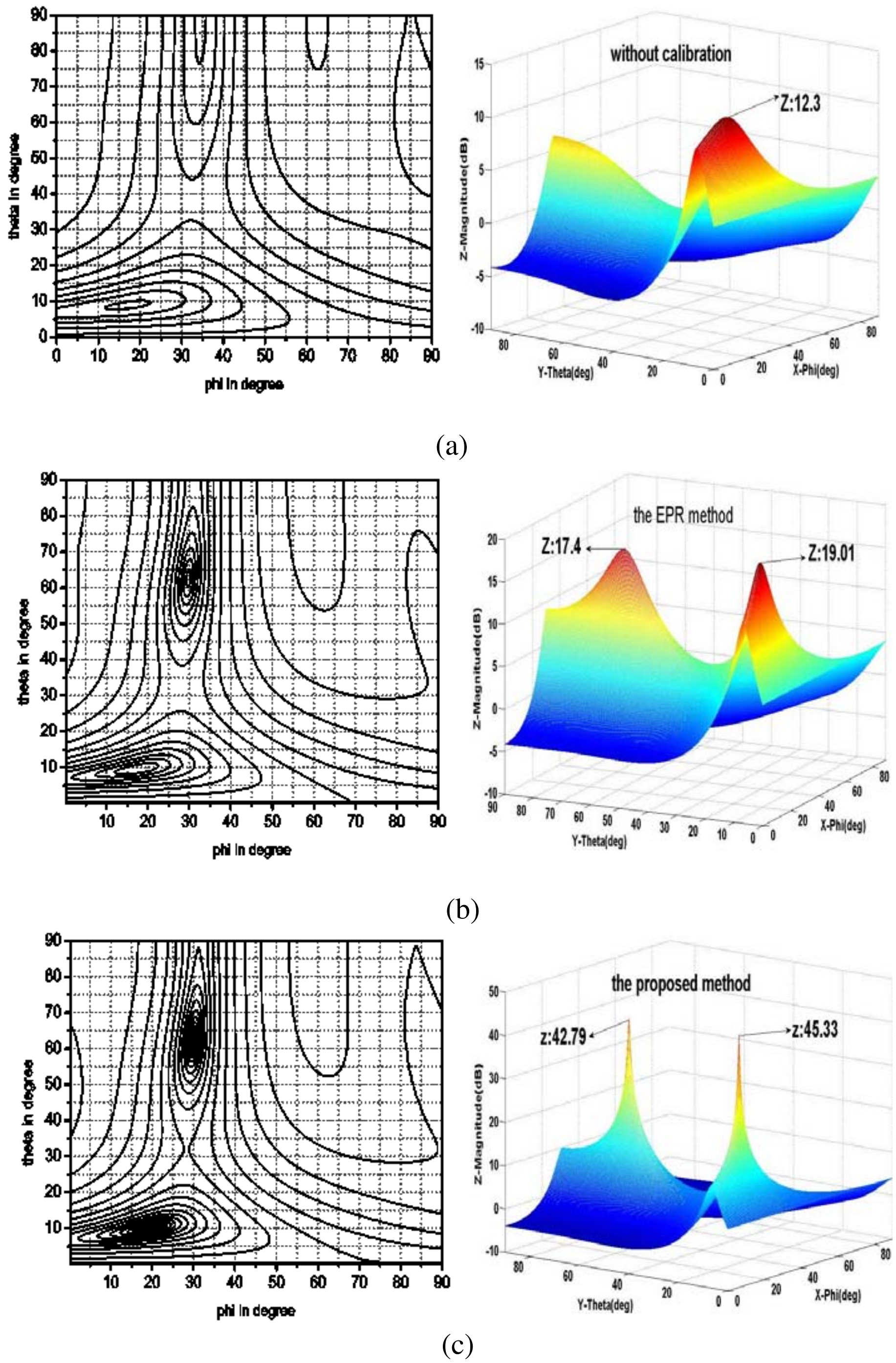


Figure 4. Contour charts and Spatial spectra of the 2D DOA estimation for various states. (a) Without calibration. (b) The EPR method. (c) The proposed method.

of the antenna array. In Fig. 4(b), the DOA estimation of the incident signals calibrated by the EPR method is completed with less bias than that of the array with mutual coupling state. The two signals are estimated with little bias from 0.4° to 2° between the state calibrated by the EPR method and the isolated state. Moreover, the spatial spectra peak is 19.01 dB which is better than the antenna in mutual coupling state. In Fig. 4(c), the two incident signals are achieved perfectly through the mutual coupling calibration and alternate iteration of the proposed method. The estimated incident signals calibrated by the proposed method are consistent with the isolated state where there is no mutual coupling effect, and the spatial spectra peak is 45.33 dB. By comparing and analyzing the results of various states in Fig. 4, the minimal bias and maximal spatial spectra peak for the estimation of the two incident signals are obtained by the proposed method. The improved method, characterized by the partition of the space angle region and alternate iteration, has a good effect on the DOA estimation.

For further verification, the DOA estimation bias and spatial spectra peak achieved by various calibration methods between the incident signals and the isolate antenna array are listed in Table 1, which shows the advantages of the proposed method over the EPR method. The average estimation bias of the proposed method is minimal compared with other states, whose bias is from 0° to 0.4°. It can be found that the proposed method is effective to estimate the incident signals which are far from the 3-dB bandwidth of the L-shaped array. It is also shown that the proposed method has a good effect on mutual coupling calibration with accurate 2-D DOA estimations. Additionally, because there are just 5 elements in the L-shaped array, the accurate DOA estimation can be obtained by 2 or 3 iterations for the two incident signals.

Table 1. Two-D DOA estimation bias and spatial spectra peak for the L-shaped array.

Incident signals (θ, φ)	DOA estimation bias (deg)			DOA estimation spatial spectra peak (dB)		
	Without calibration	EPR	Proposed method	Without calibration	EPR	Proposed method
(10°, 20°)	(1.2°, 2.2°)	(0.6°, 2.0°)	(0°, 0°)	12.3	19.01	45.33
(60°, 30°)	failure	(0.4°, 0.4°)	(0°, 0°)	failure	17.4	42.79

4. CONCLUSION

In this paper, an improved method is proposed to calibrate the mutual coupling effect with application to 2-D DOA estimation. The method achieves more accurate calibration matrices through the partition of the space angle area according to the incident signals. By the partition, the angle ranges to calibrate is narrowed. By the alternate iteration, the angle ranges of DOA estimations are widened. Meanwhile, the accuracy of the calibration matrix and 2-D DOA estimation is improved. The validity of the proposed method is verified by an L-shaped microstrip adaptive antenna array with five elements.

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