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# Physical and Computational Aspects of Antenna Near Fields: The Scalar Theory

**Said M. Mikki* and Yahia M. M. Antar**

**Abstract**—The main goal of the present paper is to analyze the structure of the near field radiated by scalar point sources. The motivations for this study are the strong connection with interaction problem and the need for some insights to be utilized in the later, much more involved study of the full-wave vectorial case. We first suggest that the radial direction is the most convenient at the current time for observing the structure of the near field and proceed to derive the radial Green's function of the problem in a simple analytical closed form. The obtained expressions are then studied and their physical features are illuminated, especially in connection with the engineering radiation problem. The overall understanding of the near field problem obtained here will help in guiding the devolvement for the more complicated sources sometimes encountered in applications and theory.

## 1. INTRODUCTION

There has been a growing interest throughout the last few years in the topic of antenna near fields, motivated by both theoretical and applied concerns, where there appears to be a convergence toward more compact systems working at various spatial scales, for example nanoscale applications, metamaterials, miniaturized microwave and millimeter technology, etc. [1–3]. On another hand, the design and devolvement of large and complex antenna arrays force us to decrease the spacing between the elements, resulting in strong mutual coupling, a phenomenon that involves the near field of the radiating sources. The topic of electromagnetic energy has also been studied extensively in the last few decades, especially in connection with reactive energy, e.g., see the comprehensive paper [4]. More recently, the authors have undergone a general study of several fundamental aspects of the problem of the near fields and the connection with energy and mutual coupling, see [5–12, 15, 16]. In particular, the core of their approach can be found in [8, 13, 14], where it was proposed that one of the most important aspects of the antenna field is its dynamic tendency to propagate or not propagate differently in different directions in space. The analysis was based on the classic plane wave spectrum (Weyl) expansion (see [20] for background material) combined with a dynamic rotation of the local coordinate system in order to study the physical structure of the radiation problem. The analysis revealed the complexity of energy aspects in antennas, especially those related to localized and stored energy. However, it is possible using this approach to quantify many important features in the localization of the radiation field by computing the total nonpropagating part and comparing it with the total field. Due to the complexity of the analysis, no numerical study was presented in [8] for simple and exact special cases. We attempt here to focus on such smaller problems but within the perspective of the general insights obtained previously in the researches conducted by the authors.

The topic of the present paper is an investigation of the fundamental physical aspects of the near field produced by radiators comprised of scalar sources. The generalization to the full-wave vectorial

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case is more involved but relies on the insights developed after the study of the scalar case presented here. The overall structure of this paper is the following. In Section 2, we first provide a general motivation for the study of the scalar theory. In Section 3, we formulate the problem starting from the vantage point of [8], which is reviewed here for self completeness. In particular, we show how the traditional Weyl expansion is combined with our dynamic approach to shed some light on the subtle manner in which the radiated field tend to propagate in different radial directions. In Section 4, we show that the radial field generated by a point scalar source, i.e., the radial Green's function, can be evaluated in simple analytical form when the origin of the coordinate system is located at the source itself. In Section 5, we provide in-depth analysis of the analytical results just obtained with comparison to numerical evaluation of the spectral integrals. Several interesting features are observed, for example the total vanishing of the propagating parts at certain spheres even in the far zone. The important topic of the interaction energy between the propagating and nonpropagating parts is also discussed. In Section 6, we give a brief indication of how to deal with the case of multiple sources, for example antenna arrays or continuous source distributions. Finally, we end with conclusions. The theory proposed in the present paper is applied extensively† to develop new numerical analysis tools suitable for antenna design and development.

## 2. MOTIVATION FOR THE STUDY OF STUDY OF SCALAR NEAR FIELD THEORY

Although the electromagnetic problem is strictly speaking never scalar, the scalar case provides a very attractive viewpoint to the topic at large and presents some very compelling advantages, which we briefly review here. To begin with, the vector Hemholtz equation, which governs the propagation of electromagnetic fields in the space surrounding the source, is obtained from the scalar Hemholtz equation, which governs the scalar problem, by updating the scalar field of the latter to the three Cartesian components of the former [17, 18]. Therefore, although the vector problem is considerably more complex, it is actually based on a the scalar case. Understanding the simpler theory provides therefore good foundations for working with the full-wave vectorial case. This approach has been favourite for many throughout the history of electromagnetic theory.

Another important advantage for studying the scalar theory is provided by the fact that many fundamental *vectorial* electromagnetic problems can be reformulated in a form that resembles the scalar case. That does not mean that a full-wave problem can be made equivalent to a scalar one, which is contradiction in terms, but that certain complex aspects in the vectorial case can be put in the easier form of the scalar one. We will give here a general example that illustrates this idea.

Consider a generic interaction problem between two current distributions $\mathbf{J}_1(\mathbf{r})$ and $\mathbf{J}_2(\mathbf{r})$ radiating in free spaces within volume supports $V_1$ and $V_2$. Suppose we are interested in studying the phenomenon of energy exchange between the two sources, for example, the problem of mutual coupling in antenna arrays. Any such study will sooner or later involve the consideration of basic integrals of the following form

$$I = \int_{V_1} d^3 r \int_{V_2} d^3 r' \, \mathbf{J}_1(\mathbf{r}) \cdot \left( \bar{\mathbf{I}} + \frac{1}{k^2} \nabla\nabla\cdot \right) g(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_2(\mathbf{r}'), \tag{1}$$

where $g(\mathbf{r}, \mathbf{r}')$ is the free-space Green's function (3). Physically, the direct meaning of the expression (1) reduces to understanding energy exchange between the antennas represented by $\mathbf{J}_1(\mathbf{r})$ and $\mathbf{J}_2(\mathbf{r})$ as mediated by the "channel" $\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = (\bar{\mathbf{I}} + 1/k^2 \nabla\nabla\cdot) g(\mathbf{r}, \mathbf{r}')$, i.e., the full-wave vectorial Green's function. However, it is possible to apply integration by parts in order to convert (1) into an expression containing only integrals of the form

$$I' = \int_{V_1} d^3 r \int_{V_2} d^3 r' \mathbf{J}_1(\mathbf{r}) \cdot g(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_2(\mathbf{r}'), \, I'' = \int_{V_1} d^3 r \int_{V_2} d^3 r' \nabla \cdot \mathbf{J}_1(\mathbf{r}) \, g(\mathbf{r}, \mathbf{r}') \, \nabla' \cdot \mathbf{J}_2(\mathbf{r}'). \tag{2}$$

Technically speaking, the trick is to move the differential operators in (1) from the free-space Green's function to the source. This is, for example, done routinely in method of moment formulations, for instance see [17].

† Mikki, S. M. and Y. M. M. Antar, "A new technique for the analysis of energy coupling and exchange in general antenna systems," Submitted to *IEEE Trans. Antennas Propagat.*

From the physical point of view, the problem has not changed. However, from the computational aspect, the integrals (2) are much easier to work with. Indeed, for examining the problem of the near field, as will be done below, the analysis of the free-space Green's function $g(\mathbf{r}, \mathbf{r}')$ into propagating and nonpropagating modes using (2) is much easier to perform than the original (1). *Therefore, understanding the scalar near field problem can provide considerable help in tackling the interaction problem in general electromagnetic systems both computationally and theoretically.* For example, it will be shown in Section 4 that the radial total propagating and nonpropagating parts can be put in simple analytical forms and that there is here no need to perform numerical integration.

Finally, a strong motivation for the study of the scalar case is the relation with the far-field problem in the full-wave vectorial case. Indeed, the scalar Green's function (3) is nothing but the spherical wave representing the far field of any antenna [18]. While we are mainly interested here in studying the near field behaviour of this wave, we will have occasions to say something interesting about its behaviour in the far zone. On the other hand, the spherical wave (3) contributes to the near field of any antenna and becomes important even before reaching the far zone, for example in the intermediate field zone [7]. Therefore, the insight developed here for the scalar can be considered a direct contribution to studying one aspect of the full-vectorial near field of any antenna.

A direct utilization of the proposal motivated by the expression (2) can be found[‡], where extensive numerical results and more examples beyond the present paper can be found there.

## 3. DEVELOPMENT OF THE RADIAL LOCALIZED NEAR-FIELD GREEN'S FUNCTION

The scalar problem is the one connected with establishing the scalar field $\psi(\mathbf{r})$ produced by a scalar source density $\rho(\mathbf{r})$ defined on a compact support $V$. The fields can be rigourously defined everywhere, including the source region, but in the present we are concerned mainly with the problem in the exterior region $\mathbb{R}^3 - V$. Therefore, we work with infinite, homogenous, and isotropic space containing only a single source enclosed within the (possibly multi-connected) region $V$. In the time harmonic case, the wave equation reduces to the scalar Helmholtz equation $\nabla^2\psi(\mathbf{r}) + k^2\psi(\mathbf{r}) = 0$. The scalar Greens function of this equation is given by the well-known expression [19]

$$g\left(\mathbf{r}, \mathbf{r}'\right) = \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \tag{3}$$

The importance of this Green's function arises from the fact that the radiated fields can now be expressed bt the following intuitive form

$$\psi\left(\mathbf{r}\right) = \int_V d^3r\, g\left(\mathbf{r}, \mathbf{r}'\right) \rho\left(\mathbf{r}\right). \tag{4}$$

We would like to further decompose the Green's function into two parts, one pure propagating and the other evanescent. This task can be accomplished by using the Weyl expansion [2, 17]

$$\frac{e^{ikr}}{r} = \frac{ik}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dpdq \frac{1}{m} e^{ik(px+qy+m|z|)}, \tag{5}$$

where[§]

$$m(p, q) = \begin{cases} \sqrt{1-p^2-q^2}, & p^2+q^2 \leq 1 \\ i\sqrt{p^2+q^2-1}, & p^2+q^2 > 1 \end{cases}. \tag{6}$$

The Weyl expansion shows that the total scalar Greens function can be divided into the sum of two parts, one as pure propagating waves and the other as evanescent, hence nonpropagating part. Explicitly, we write [2]

$$g\left(\mathbf{r}, \mathbf{r}'\right) = g_{\mathrm{ev}}\left(\mathbf{r}, \mathbf{r}'\right) + g_{\mathrm{pr}}\left(\mathbf{r}, \mathbf{r}'\right), \tag{7}$$

‡ Mikki, S. M. and Y. M. M. Antar, "A new technique for the analysis of energy coupling and exchange in general antenna systems," Submitted to *IEEE Trans. Antennas Propagat.*

§ Throughout this paper, the explicit dependance of $m$ on $p$ and $q$ will be suppressed for simplicity.

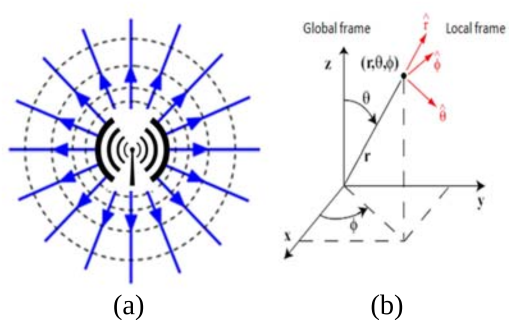


**Figure 1.** Radial near field. A generic antenna is shown in (a) with the radial direction field along which a decomposition of the total field into propagating and nonpropagating modes is enacted. (b) We show the notation of the coordinate systems used. The global coordinate system is the Cartesian $xyz$ system. The local frame is given the notation of the spherical coordinates and the use of double primes is avoided for convenience in the notation.

where the propagating and nonpropagating (evanescent) parts are given, respectively, by the expressions

$$g_{\rm ev}\left(\mathbf{r},\mathbf{r}'\right)=\frac{ik}{8\pi^2}\int_{p^2+q^2>1}dpdq\frac{1}{m}e^{ik[p(x-x')+q(y-y')]}e^{im|z-z'|}, \tag{8}$$

$$g_{\rm pr}\left(\mathbf{r},\mathbf{r}'\right)=\frac{ik}{8\pi^2}\int_{p^2+q^2<1}dpdq\frac{1}{m}e^{ik[p(x-x')+q(y-y')]}e^{im|z-z'|}. \tag{9}$$

By transforming the double integrals into cylindrical coordinates, making use of the integral representation of the Bessel function, we write the Weyl expansion in the following form [2, 17]

$$g_{\rm ev}\left(\mathbf{r},\mathbf{r}'\right)=\frac{k}{4\pi}\int_0^\infty duJ_0\left(k\rho_s\sqrt{1+u^2}\right)e^{-k|z-z'|u}, \tag{10}$$

$$g_{\rm pr}\left(\mathbf{r},\mathbf{r}'\right)=\frac{ik}{4\pi}\int_0^1 duJ_0\left(k\rho_s\sqrt{1-u^2}\right)e^{ik|z-z'|u}, \tag{11}$$

where $\rho_s=\sqrt{(x-x')^2+(y-y')^2}$.

We now introduce the radial localized near field of the radiation problem first proposed in [8]. The main idea is to distinguish between three coordinate systems in the radiation problem:

(i) Global coordinate system.
(ii) Source coordinate system.
(iii) Local coordinate system.

The global and source frames are the conventional ones used in electromagnetic theory, and are usually denoted by the unprimed and primed notation, e.g., $\mathbf{r}$ and $\mathbf{r}'$ respectively. In [8], it was found after a critical examination of the traditional approach (carried out in [7]) that in order to adequately describe the engineering aspects of the radiation problem in the near field, there is a need to introduce a *third* coordinate system, the *local* frame, which was denoted there by double prime notation $\mathbf{r}''$. The physico-engineering significance of this new frame is quite straightforward and can be explained briefly as follows.

Consider a generic antenna system as shown in the left of Figure 1. As we now know, the field in the near zone can no longer be considered a pure propagating wave. Instead, it has the ambiguous character of being composed of both propagating and nonpropagating parts.[‖] A Fourier analysis (i.e., via the Weyl expansion) of the spatial field is then required in order to decompose radiation into propagating and nonpropagating modes along any arbitrary chosen direction in space. This analysis, therefore, requires the specification of a direction at each point in the exterior region of the antenna problem along which

[‖] In literature, this division is usually termed "static" and "radiated" fields. The difficulties of this approach, which is based on a hasty generalization of phenomena applicable only to small dipoles antennas, were criticized in [7].

the above mentioned decomposition into propgating/nonpropagating parts will be effectuated. The *radial* local frame is one natural choice that was pointed out in the general analysis of [8]. As can be seen from Figure 1, it consists of choosing the direction field of the Fourier decomposition to coincide with the familiar radial field, i.e., at each location $\mathbf{r}$ in the exterior region, we attach the direction $\hat{r}$ and analyze the total field into propagating and nonpropagating modes along this vector. In this way, instead of the total field (which does not say anything about the inner dynamic structure of the near field), we obtain *two* different fields, the propagating radial near field, and the nonpropagating radial near field. The sum of the two will obviously give back the total field; however, we now obtain through this decomposition a firm grasp of how the original field structure splits dynamically as we move the local observation frame around the source.

The engineering motivations of this process are evident: we need to know how the field behaves around specific parts of the physical structure supporting the current distribution. By fixing the global frame on this rigid physical structure, and then rotating the local frame *with respect to the global frame*, we obtain in essence the relational dynamics of the problem in the sense that the tendency of the field to split into propagating and nonpropagating modes is described in reference to the physical body of the antenna itself. The three coordinate systems are then tightly connected with each other in a process that eventually leads to the explication of a physically real phenomena: the dynamical production of the propagating field with respect to the geometry of the source. It is the opinion of the authors that the conventional approach to energy and propagation in antenna systems using the methods of input impedance, reactive energy, and radiation density, are not adequate to deal with the problem in the way described above.

In Figure 1, we use the unprimed notation $xyz$ to refer to the global frame. The local frame will be referred to by the usual spherical coordinates $r$, $\theta$, and $\phi$. However, *no double prime is used here.* The reader should be aware that the angles $\theta$ and $\phi$ refer to the *direction field* along which the dynamic decomposition into propgating/nonpropgating modes will be performed. No semantic confusion between the typical spherical angles used to define position in $\mathbf{r}$ and these two angles above should be made, although their numerical values happen to coincide only in this case of radial localized near field.¶

## 4. DERIVATION OF THE RADIAL LOCALIZED NEAR-FIELD GREEN'S FUNCTION FOR THE SCALAR PROBLEM

To proceed further, we need to write down the local frame coordinates explicitly in terms of the global frame. To do this, the following rotation matrix is employed for the derivation of the matrix elements (13).

$$\bar{\mathbf{R}}(\theta,\varphi) \equiv \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix}, \tag{12}$$

where the elements are given by

$$\begin{aligned} &R_{11} = \sin^2\phi + \cos^2\phi\cos\theta, \\ &R_{12} = -\sin\phi\cos\phi\,(1-\cos\theta)\,, \\ &R_{13} = -\cos\phi\sin\theta, \quad R_{21} = -\sin\phi\cos\phi\,(1-\cos\theta)\,, \\ &R_{22} = \cos^2\phi + \sin^2\phi\cos\theta, \quad R_{23} = -\sin\phi\sin\theta, \\ &R_{31} = \cos\phi\sin\theta, \quad R_{32} = \sin\phi\sin\theta, \quad R_{33} = \cos\theta. \end{aligned} \tag{13}$$

Using this matrix, we can express the local frame coordinates in terms of the global frame's using the following relations

$$\mathbf{r}'' = \bar{\mathbf{R}}(\theta,\phi)\cdot\mathbf{r}, \quad \mathbf{r}'_s = \bar{\mathbf{R}}(\theta,\phi)\cdot\mathbf{r}'. \tag{14}$$

It should be immediately stated that this rotation matrix will also rotate the $x''y''$-plane around the $z''$-axis with some angle. However, it was shown in [8] that the total propagating and nonpropagating modes are independent of rotation of the local frame around its $z'$-axis.

¶ The more general decomposition theorem, of which the radial localized field is only a special case, was discussed extensively in [8].

Since the scalar Green's function (3) is invariant with respect to rotation of the coordinate system, we replace $\mathbf{r}$ in (5) by $\mathbf{r}''$ using (14). This leads to

$$\frac{\exp(ikr)}{r} = \frac{ik}{2\pi}\int_{-\infty}^{\infty} dpdq \frac{1}{m} e^{i\mathbf{K}\cdot(\bar{\mathbf{R}}\cdot\mathbf{r})}, \tag{15}$$

where the spectral variable (wavevector) is given by

$$\mathbf{K} = \hat{x}kp + \hat{y}kq + \hat{z}\operatorname{sgn}(z - z')\,km. \tag{16}$$

Here, sgn stands for the signum function. From the expressions of the rotation matrix elements (13)), we easily find that

$$\frac{\exp(ikr)}{r} = \frac{ik}{2\pi}\int_{-\infty}^{\infty} dpdq \frac{1}{m} e^{ikmr}. \tag{17}$$

We next further simplify this integral by converting the double integral into a single integral and then removing the singularity of the integrand. To achieve this, we work in the polar coordinates of the spectral domain, i.e., we employ $p = v\cos\varphi$ and $q = v\sin\varphi$. The integral (17) becomes then

$$\frac{\exp(ikr)}{r} = \frac{ik}{2\pi}\int_{0}^{2\pi}\int_{0}^{\infty} dvd\varphi \frac{v}{\sqrt{1-v^2}} e^{ik\sqrt{1-v^2}r}. \tag{18}$$

Since the integrand is independent of $\varphi$, we evaluate the outer integral and then separate the Green's function into two parts as in (7). The outcome is

$$g_{\text{ev}}^{\text{rad}} = ik\int_{1}^{\infty} dv \frac{v}{\sqrt{1-v^2}} e^{ik\sqrt{1-v^2}r}, \tag{19}$$

$$g_{\text{pr}}^{\text{rad}} = ik\int_{0}^{1} dv \frac{v}{\sqrt{1-v^2}} e^{ik\sqrt{1-v^2}r}. \tag{20}$$

Finally, perform the transformations of variable $u = -i\sqrt{1-v^2}$ and $u = \sqrt{1-v^2}$ in (19) and (20), respectively. We obtain

$$g_{\text{ev}}^{\text{rad}} = k\int_{0}^{\infty} du\, e^{-kur} = \frac{1}{r}, \tag{21}$$

$$g_{\text{pr}}^{\text{rad}} = ik\int_{0}^{1} du\, e^{ikur} = \frac{e^{ikr}-1}{r}. \tag{22}$$

These are the main results of the scalar theory of the radial propagating-evanescent Green's functions. In the remaining parts of this section, we explore some of their salient consequences.

## 5. DISCUSSION OF THE RESULTS AND THEIR PHYSICAL CONSEQUENCES

Before discussing the nature of the analytical expressions of the radial localized Green's functions derived above, we first present some numerical studies justifying the physical importance of the radial direction in describing the near field of the source. We consider a simple example comprised of a single point source located at the origin and ask how the radiate field tend to propagate at the point (0, $d$, 0). Figure 2 presents the propagating scalar Green's function numerically when the rotation matrix has $\theta = \pi/2$. The variation with respect to $\phi$ are recorded at position (0, 10$\lambda$, 0), hence in the far field of the source. It is interesting that at exactly the radial direction, in this case $\phi = \pi/2$, the total propagating part is zero! On the other hand, for directions around the radial orientation the propagating field has significant value. At a slightly different position (0, 10.25$\lambda$, 0), Figure 3 shows that, contrary to the previous case, the propagating part has its most significant value concentrated at the radial direction. (The same results are also obtained for the position (0, 9.75$\lambda$, 0).) This phenomena, the vanishing of the radial propagating part at some locations in space, will be explained below based on the analytical expressions (19) and (20). For the time being, we notice that most of the time the field tends to propagate maximally in the radial direction. For example, consider Figure 4 where this time we observe the spectral composition of the radiated field in the near-field zone. It is clear that energy can be found propagating in all directions, but the highest concentration of energy is located in the radial direction. *Our conclusion is that even though at certain discrete locations the radiated field has zero total propagating part, almost everywhere in space energy tends to flow maximally in the radial direction.* This provides some further justification for the theoretical and practical importance of the radial Green's function besides their obvious intuitive motivation.

Let us now examine carefully the structure of the radial Green's functions (19) and (20). Our purpose is to gain some insight into the physical settings of the problem by exploiting the analytical simplicity of the derived expressions.

**Spherical symmetry**. Both scalar radial Green's functions are independent of $\theta$ and $\phi$. This is expected since a point source in the scalar problem has no structure. Therefore, the problem is spherically symmetric. However, note that this applies only to the *radial* decomposition chosen in the present paper.

**Independence of the wavenumber**. A striking feature in the expression (21) is the fact that the dependence of the total radial evanescent field on $k$ cancels out in the final formula. In particular, the localized near field is independent of frequency in the scalar problem. This shows that all scalar radiation problems possess a single universal structure of the nonpropagating field. This is connected with the next observation.

**Connection between the evanescent field and electrostatics**. The evanescent radial Green's function is nothing but (numerically speaking) the electrostatic Green's function. This suggests that

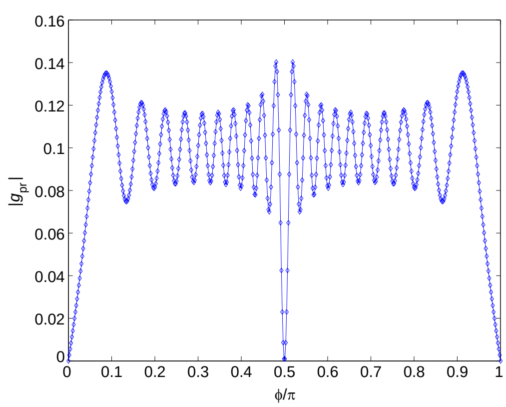


**Figure 2.** Spectral content of the propagation scalar Green's function observed at $(0,\ 10\lambda,\ 0)$ with $\theta = \pi/2$.

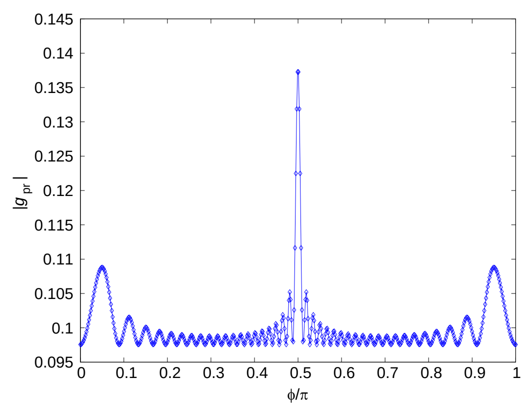


**Figure 3.** Spectral content of the propagation scalar Green's function observed at $(0,\ 10.25\lambda,\ 0)$ with $\theta = \pi/2$.

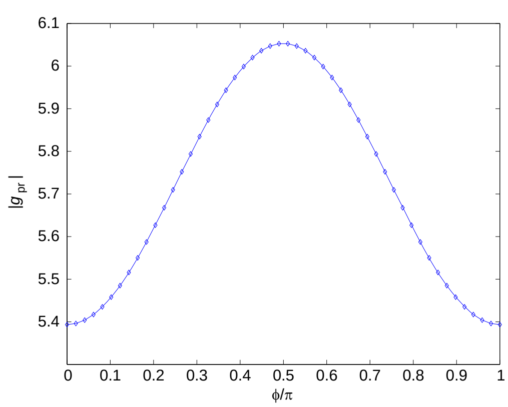


**Figure 4.** Spectral content of the propagation scalar Green's function observed at $(0,\ 0.15\lambda, 0)$ with $\theta = \pi/2$.

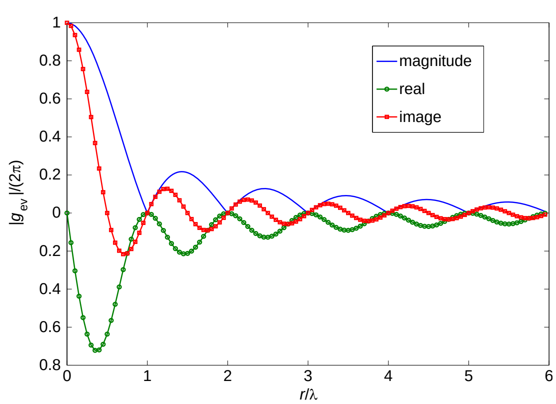


**Figure 5.** Radial propagating scalar Green's function.

there is deep connection between the electromagnetic near field theory and potential theory.

**Singularity of the radial propagating-evanescent Green's functions**. The evanescent Green's function is singular at $r = 0$. This is, however, not the case with the propagating Green's function, which takes a nonzero value at the origin. Indeed, by L'hospital rule we have

$$\lim_{r\to 0} g_{\rm pr}^{\rm rad}(r) = \lim_{r\to 0} \frac{e^{ikr}-1}{r} = ik \lim_{r\to 0} \frac{e^{ikr}}{1} = ik. \tag{23}$$

This shows that *the radiation field is already propagating right at the source location itself*. In other words, the field is created already in a propagating state! This result is surprising since one would expect intuitively that if the propagating part is finite at the source, it would approach the value zero there. Instead, the calculation (23) proves that the propagating field behaves differently in the neighborhood of the source.

Figure 5 shows the radial propagating scalar Green's functions. It is clear that the propagating part attains its greatest magnitude at $r = 0$, implying that not only the propagating field is nonzero at the source location, but actually it is maximal there.

**Rate of decay**. Another very interesting observation is that the rate of decay of both the propagating and evanescent scalar Green's functions is the same. Indeed, since $\exp(ikr) - 1$ is bounded, the function $g_{\rm rv}(r)$ is asymptotically like $1/r$. Therefore, both the propagating and evanescent fields behave in the far field as $1/r$. *The evanescent field contribute to the far field by the same level of magnitude contributed by the pure propagating part.*

**Vanishing of the total propagating part**. From (22) we see that the propagating field can vanish at any sphere

$$r = 2n\lambda, \quad n \in \mathbb{N}. \tag{24}$$

This means that at point on any sphere with radius equals exactly an even multiple of $\lambda$, the totality of radial propagating modes all cancel each other. *On such sphere, the radiation field is completely nonpropagating (evanescent.) along the radial directions.*

**Energy ratio**. Consider now the energy density of the propagating and evanescent parts of the radiation field. We define the energy ratio $\Gamma$ as

$$\Gamma := \frac{|g_{\rm pr}(r)|^2}{|g_{\rm ev}(r)|^2} = \left|e^{ikr} - 1\right|^2. \tag{25}$$

Figure 6 illustrates this energy ratio. It is clear that at the source the ratio is zero although the propagating part is not zero as we found above. This is possible because the evanescent field energy is infinite at the source. The fact that the limit of the ratio is zero restores some of the intuitive picture expected in radiation problem. *That is, although the individual total propagating field is nonzero at the source, its relative strength compared with the total evanescent field is zero.*

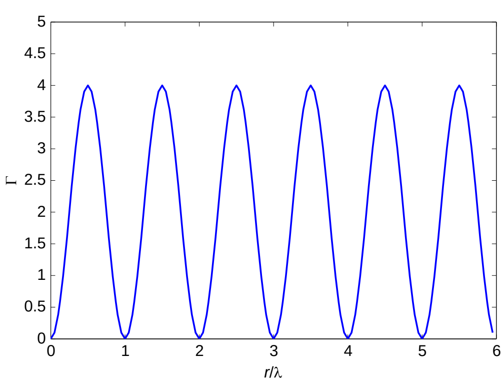


**Figure 6.** The energy ratio between the radial propagating and evanescent energy densities for the scalar problem.

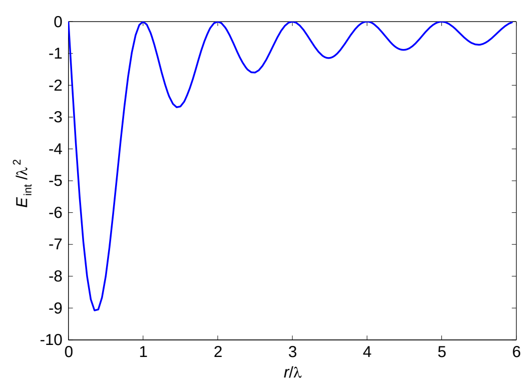


**Figure 7.** The interaction energy between the radial propagating and evanescent energy densities for the scalar problem.

The truly interesting feature of the energy ratio $\Gamma$ is that the vanishing of the total propagating part extends to the far field zone as was already observed numerically in Figure 2. Indeed, any even multiple of $\lambda$ satisfies the condition (24) and hence even deep in the far zone, one still encounters spheres where there is no total propagating mode. This is somehow contrasting with the naive common sense picture we usually draw about radiation problems where it is expected that the field is "totally propagating" in the far zone. Notice that the total field on the spheres $r = n\lambda$ is not zero. Actually, the radiation field of the scalar problem is never zero. Only the total pure propagating part is zero. This is again connected with the fact established above that in the far zone, both the propagating and evanescent modes are at the same level of magnitude.

**Interaction energy**. Consider now the total energy density in terms of the propagaing and evanscent parts. Simple calculations shows that

$$|g(r)|^2 = \underbrace{\frac{1}{r^2}}_{\substack{\text{self}\\ \text{evanscent}\\ \text{energy}}} + \underbrace{\frac{2}{r^2}\mathrm{Re}\left\{e^{ikr}-1\right\}}_{\text{interaction energy}} + \underbrace{\frac{\left|e^{ikr}-1\right|^2}{r^2}}_{\substack{\text{self propgating}\\ \text{energy}}}. \tag{26}$$

Therefore, the interaction energy between the propagating and evanescent parts is given by the simple expression

$$\mathcal{E}_{\text{int}} := \frac{2}{r^2}\mathrm{Re}\left\{e^{ikr}-1\right\} = \frac{2}{r^2}(\cos kr - 1). \tag{27}$$

Figure 7 shows the variation of the interaction energy (27) with distance. It is immediately observed that $\mathcal{E}_{\text{int}} \leq 0$ for all $r$. This is not surprising, since the self energy density of the radial evanescent part is actually equal to the total field energy, necessitating that when the propagating and evanescent parts mutually interact with each other, they both loss energy.

Notice that $\mathcal{E}_{\text{int}} = 0$ for all $r = 2n\lambda$, where $n$ is a natural number. This is in harmony with the fact that at these spheres, the propagating part is zero so the interaction energy is trivially also zero. However, at the spheres $r = (2n+1)\lambda$, the interaction energy reaches its smallest value at $\mathcal{E}_{\text{int}} = -2/r^2$. In this case, the total propagating field energy density is equal to twice the total energy $1/r^2$.

Such observations suggest that care should be taken when attempting to interpret the evanescent part of the total field. It is usually assumed that the energy "stored" in the field is located in the nonpropagating part of this field. However, the simple calculation above points to opposite conclusions, for total energy seems to be distributed not only among the propagating and evanescent modes, but also in the interaction zone between the two. Now, although the evanescent modes for arbitrary (nonpoint) source distribution will tend to be localized around the antenna, the propagating modes obviously exist everywhere. The fact that part of the total energy is always allocated to the *interaction* between purely propagating field and a localized one seems to imply that part of the *stored* energy may be related somehow to propagating waves escaping to infinity! This is another evidence indicating the difficulty in dealing with stored energy using a time-harmonic theory.⋆

**Total radial evanescent energy**. From the energy density expressions we can compute the total energy in any finite volume by simple integration

$$W_{\text{ev, pr}} = \int_V d^3r\, \mathcal{E}_{\text{ev, pr}}(r), \tag{28}$$

Consider a spherical volume bounded between the radii $r = a$ and $r = b$. The total electric radial evanescent energy in this region can be readily calculated as

$$W_{\text{ev}} = 4\pi(b-a). \tag{29}$$

We note that $W_{\text{ev}} = 4\pi b$ at $a = 0$, which is finite, although the evanescent field itself is infinite at the source. However, as $b \to \infty$, the total energy diverges. Therefore, *the total radial evanescent energy is infinite, even though the individual modes are pure evanescent and decays exponentially at infinity.*

⋆ In [8], the authors suggested that stored energy cannot be properly understood without a transient (time-dependent) analysis of the antenna system.

The total radial propagating energy is computed in the following form

$$W_{\mathrm{pr}} = 8\pi \left[ b - a - \frac{1}{k} (\sin kb - \sin ka) \right]. \tag{30}$$

The limit does not exist in an infinite domain as should be the case with pure propagating modes (i.e., total propagation energy is always infinite in any radiation problem.) The total interaction energy is simply the negative of the total propagating energy (30).

## 6. THE CASE OF MULTIPLE SCALAR SOURCES

Knowledge of the scalar radial Green's functions in the closed analytical form (19) and (20) derived in Section 4 permits us to compute the near field decomposition due to an arbitrary source. However, there is a subtlety in the multiple point source case that need to be clarified first. We noted in Section 4 that the derivation of the closed form expressions above is possible only when the origin of the coordinate system is chosen to coincide with the point source location. Obviously, when there are two or more point sources, this is not possible simultaneously. Therefore, the physical situation appear a bit more complicated in this case.

Before giving the mathematical expressions of the near field decompositions, we first consider the problem as illustrated graphically in Figure 8. We have three point sources $\rho_1(\mathbf{r})$, $\rho_2(\mathbf{r})$, and $\rho_3(\mathbf{r})$ and it is desired to know how the near field behaves at point $P$. When the method of Section 4 is used to compute the the propagating and evanescent parts of the field due to each source taken individually, the radial contributions of the sources $\rho_1(\mathbf{r})$, $\rho_2(\mathbf{r})$, and $\rho_3(\mathbf{r})$ will appear as $\hat{k}_1$, $\hat{k}_2$, and $\hat{k}_3$, respectively. By invoking the physically intuitive assumption that the net near field tendency to propagate or not propagate is the superposition of the separate sources taken individually, we may assume that the total propgating/nonpropgating parts of the near field at point $P$ will be described with respect to the vector $\hat{k}_P$, which is the unit vector parallel to $\hat{k}_1 + \hat{k}_2 + \hat{k}_3$. This is to be understood, however, only as an *average* direction of energy flow.

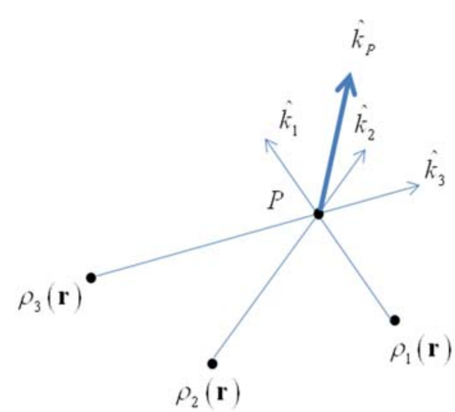


**Figure 8.** A diagram of three sources producing a near field at point $P$. Each point source will contribute a total propagating field along the direction specified by the line joining it with the observation point. A rough estimate of the direction of the resultant sum is the vector $\hat{k}_P$, which can be considered an average direction.

If we have a continuous source in a volume $V$, the expressions of the total propagating and nonpropagating parts follow from the expressions (19) and (20)

$$\psi_{\mathrm{ev}}(\mathbf{r}) = \int_V d^3r' \frac{\rho(\mathbf{r}')}{r}, \tag{31}$$

$$\psi_{\mathrm{pr}}(\mathbf{r}) = \int_V d^3r' \frac{\rho(\mathbf{r}') \left(e^{ikr} - 1\right)}{r}. \tag{32}$$

The unit vector of "most likely propagation" is estimated as

$$\hat{k}_P = \frac{\displaystyle\int_V d^3r' \left(\mathbf{r} - \mathbf{r}'\right)}{\left\| \displaystyle\int_V d^3r' \left(\mathbf{r} - \mathbf{r}'\right) \right\|}. \tag{33}$$

Therefore, the direction along which the near field tends to "maximally" propagate (or localize) depends on the geometric configuration of the relative positions of the radiating point sources with respect to each others. Evidently,, *each source current distribution will produce a near field localization pattern that reflects the shape of the radiator.*

Finally, we mention that it is possible to improve the estimation (33) by inserting a "weight" at each propgation/nonpropagation direction proportional to the magnitude of the total propagating part at the point under consideration. In this case, we can replace (33) by

$$\hat{k}_P = \frac{\displaystyle\int_V d^3r' \left|\psi_{\rm pr}\left(\mathbf{r}\right)\right| \left(\mathbf{r} - \mathbf{r}'\right)}{\left\| \displaystyle\int_V d^3r' \left|\psi_{\rm pr}\left(\mathbf{r}\right)\right| \left(\mathbf{r} - \mathbf{r}'\right) \right\|}. \tag{34}$$

The use of the absolute value of $\psi_{\rm pr}(\mathbf{r})$ in (34) is intentional and aims at eliminating any cancellation from the total sum. The reason is that from the energy viewpoint, a negative or positive total propagating parts are the same. A negative amplitude of a plane wave does not imply that the direction of propagation is reversed. Consequently, in computing the average direction of energy flow, it is more appropriate to use only positive values.

## 7. CONCLUSION

The paper presented a study of the near field structure of the basic problem of scalar point sources radiating in free space. The topic was approached from the new perspective of the dynamic structure of the near field based on careful spectral analysis of radiation using the plane wave spectrum combined with a rotating coordinate frame of reference. The analysis focused on the radial flow of energy given its intuitive appeal. We motivated this choice by some numerical examples indicating that most of the energy tend to move maximally in the radial direction. The radial Green's function of the scalar problem was then derived in simple analytical closed form for both the propagating and nonpropagating parts. We studied in depth the physical structure of the radiation problem revealed by these expressions and documented some interesting observations. Finally, a quick formulation for the array or multiple source problem was outlined, which may be used in the future for near-field shaping and focusing applications.

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