Progress In Electromagnetics Research Letters, Vol. 52, 63–69, 2015

Array Aperture Extension Algorithm for 2-D DOA Estimation with L-Shaped Array

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Abstract—In this paper, an array aperture extension algorithm is developed for two-dimensional (2-D) direction-of-arrival (DOA) estimation with L-shaped array. We enlarge the dimension of the covariance matrix by using the rotational invariance in conjunction with the property that the signal covariance matrix is real diagonal matrix. Estimation of DOAs is performed by processing this larger dimensional matrix. The simulation results indicate that our method can improve the DOA estimation accuracy.

1. INTRODUCTION

Estimation of two-dimensional (2-D) direction of arrival (DOA) of multiple incident signals using sensor array techniques has attracted considerable attention in many applications including radar, sonar, wireless communication, and seismic sensing [1], and lots of high-resolution algorithms were proposed in literatures [2–6] in the past decade. Most of the above-mentioned approaches directly deal with the covariance matrix of the received signals to estimate the DOAs of the signal sources. In this paper, we introduce a method to enlarge the dimension of the covariance matrix with L-shaped array. Our method extends the dimension of the covariance matrix by employing the rotational invariance and the property that the covariance matrix of the signal sources is real diagonal matrix. Then, the signal DOAs are found by processing the larger dimensional matrix. Computer simulations show that the proposed approach can obtain good DOA estimation performance.

Notations: The superscript $*$, T, H, and $\dagger$ denote the conjugate, the transpose, the conjugate transpose, and Moore-Penrose inverse respectively. $E\{\cdot\}$, $\mathbf{J}$, and $\mathbf{M}[i : j, :]$ stand for the statistical expectation, an exchange matrix with ones on its antidiagonal and zeros elsewhere, and a matrix consisting of the ith to jth rows of matrix $\mathbf{M}$ respectively.

2. DATA MODEL

Consider K narrowband far-field plane signals $\{s_k(t)\}_{k=1}^{K}$ from distinct directions impinging on an L-shaped array composed of two uniform linear arrays (ULAs) along x and z axes respectively. Each ULA consists of N isotropic sensors, and the inter-sensor spacing d along x and z axes is half-wavelength $\lambda/2$. Let α_k and β_k, $k = 1, 2, \ldots, K$, be the azimuth and elevation angles of the kth source. Note that the azimuth angle α_k is taken between the signal arrival direction and x axis, and the elevation angle β_k is taken between signal arrival direction and z axis, as shown in Figure 1.

The array manifold matrices can be given as

$$\mathbf{A}(\alpha) = [\mathbf{a}(\alpha_1), \mathbf{a}(\alpha_2), \ldots, \mathbf{a}(\alpha_K)] \tag{1}$$

$$\mathbf{A}(\beta) = [\mathbf{a}(\beta_1), \mathbf{a}(\beta_2), \ldots, \mathbf{a}(\beta_K)] \tag{2}$$

where $\mathbf{a}(\alpha_k) = [1, \xi_k, \ldots, \xi_k^{N-1}]^T$, $\xi_k = e^{j\pi\cos\alpha_k}$, $\mathbf{a}(\beta_k) = [1, \eta_k, \ldots, \eta_k^{N-1}]^T$, $\eta_k = e^{j\pi\cos\beta_k}$.

Received 15 January 2015, Accepted 6 March 2015, Scheduled 16 March 2015

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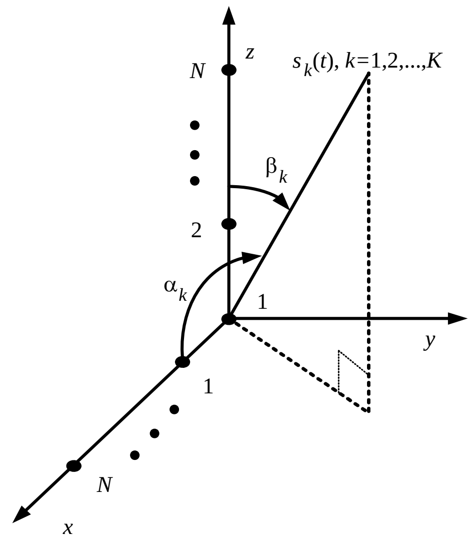

Figure 1. L-shaped array configuration for 2-D DOA estimation.

The observed vectors $\mathbf{x}(t)$ and $\mathbf{z}(t)$ can be written as

$$\mathbf{x}(t) = \mathbf{A}(\alpha)\mathbf{s}(t) + \mathbf{n}_x(t) \tag{3}$$

$$\mathbf{z}(t) = \mathbf{A}(\beta)\mathbf{s}(t) + \mathbf{n}_z(t) \tag{4}$$

where $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T$, $\mathbf{z}(t) = [z_1(t), z_2(t), \dots, z_N(t)]^T$, $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$, $\mathbf{n}_x(t) = [n_{x1}(t), n_{x2}(t), \dots, n_{xN}(t)]^T$, $\mathbf{n}_z(t) = [n_{z1}(t), n_{z2}(t), \dots, n_{zN}(t)]^T$. $\mathbf{s}(t)$ is a $K \times 1$ source vector, and $\mathbf{n}_x(t)$ and $\mathbf{n}_z(t)$ are additive noise in the x and z axes subarrays, respectively. Assume that the sources are uncorrelated with each other, and the noise is a white Gaussian random processes with zero-mean and variance σ^2, and is statistically independent of signal samples.

3. PROPOSED ALGORITHM

From the above assumption, we calculate the covariance matrix of the observations as

$$\mathbf{R}_{xz} = E\left\{\mathbf{x}(t)\mathbf{z}^H(t)\right\} = \mathbf{A}(\alpha)E\left\{\mathbf{s}(t)\mathbf{s}^H(t)\right\}\mathbf{A}^H(\beta) + E\left\{\mathbf{n}_x(t)\mathbf{n}_z^H(t)\right\} = \mathbf{A}(\alpha)\mathbf{R}_s\mathbf{A}^H(\beta) \tag{5}$$

where the diagonal matrix $\mathbf{R}_s = \operatorname{diag}\{p_1, p_2, \dots, p_K\}$ is the signal covariance matrix, and positive real number p_k stands for the power of the kth signal source. Note that $\mathbf{n}_x(t)$ and $\mathbf{n}_z(t)$ are spatially independent of each other, i.e., $E\{\mathbf{n}_x(t)\mathbf{n}_z^H(t)\} = \mathbf{0}$.

3.1. Covariance Matrix for Signal Subspace Identification

Performing the singular value decomposition (SVD) of $\mathbf{R}_{xz}$

$$\mathbf{R}_{xz} = \mathbf{A}(\alpha)\mathbf{R}_s\mathbf{A}^H(\beta) = \mathbf{U}\Sigma\mathbf{V}^H \tag{6}$$

where $\Sigma = \operatorname{diag}(d_1, d_2, \dots, d_K)$ with $d_1 \geq d_2 \geq \dots \geq d_K > 0$, $d_1, d_2, \dots, d_K$ stand for the K largest singular values of $\mathbf{R}_{xz}$, $\mathbf{U}$ and $\mathbf{V}$ stand for the left and right singular vectors of $\mathbf{R}_{xz}$ corresponding to K largest singular values, respectively.

It is well known that the K columns of $\mathbf{U}$ and $\mathbf{A}(\alpha)$ span the same range space. Therefore, there exists an invertible matrix $\mathbf{W}_1$ such that

$$\mathbf{U} = \mathbf{A}(\alpha)\mathbf{W}_1. \tag{7}$$

Divide $\mathbf{U}$ into two $(N-1)\times K$ matrices $\mathbf{U}_1$ and $\mathbf{U}_2$ such that $\mathbf{U}_1 = \mathbf{U}[1:N-1, :]$, $\mathbf{U}_2 = \mathbf{U}[2:N, :]$. Accordingly, $\mathbf{U}_1$ and $\mathbf{U}_2$ can be represented as

$$\mathbf{U}_1 = \mathbf{A}_1(\alpha)\mathbf{W}_1 \tag{8}$$

$$\mathbf{U}_2 = \mathbf{A}_2(\alpha)\mathbf{W}_1 \tag{9}$$

where $\mathbf{A}_1(\alpha) = \mathbf{A}(\alpha)[1:N-1, :]$, $\mathbf{A}_2(\alpha) = \mathbf{A}(\alpha)[2:N, :]$.

According to the rotational invariance, we have

$$\mathbf{A}_2(\alpha) = \mathbf{A}_1(\alpha)\mathbf{D}_\alpha \tag{10}$$

where $\mathbf{D}_\alpha$ is a diagonal matrix with $\mathbf{D}_\alpha = \mathrm{diag}(\xi_1, \xi_2, \ldots, \xi_K)$.

Combining (7), (8), (9) with (10), we get

$$\mathbf{U}\left(\mathbf{U}_2^\dagger\mathbf{U}_1\right)^N = \mathbf{A}(\alpha)\mathbf{W}_1\left(\mathbf{W}_1^{-1}\mathbf{A}_2^\dagger(\alpha)\mathbf{A}_1(\alpha)\mathbf{W}_1\right)^N = \mathbf{A}(\alpha)\mathbf{D}_\alpha^{-N}\mathbf{W}_1 = \mathbf{A}_{new1}(\alpha)\mathbf{W}_1 \tag{11}$$

where $\mathbf{A}_{new1}(\alpha) = [\mathbf{a}_{new1}(\alpha_1), \mathbf{a}_{new1}(\alpha_2), \ldots, \mathbf{a}_{new1}(\alpha_K)]$, $\mathbf{a}_{new1}(\alpha_k) = [\xi_k^{-N}, \xi_k^{-(N-1)}, \ldots, \xi_k^{-1}]^T$.

Substituting (7) into (6), we obtain

$$\mathbf{R}_s\mathbf{A}^H(\beta) = \mathbf{W}_1\Sigma\mathbf{V}^H. \tag{12}$$

We define the first matrix $\mathbf{R}_{12}$ as

$$\mathbf{R}_{12} \triangleq \mathbf{U}\left(\mathbf{U}_2^\dagger\mathbf{U}_1\right)^N\Sigma\mathbf{V}^H. \tag{13}$$

According to (11) and (12), $\mathbf{R}_{12}$ can be rewritten as

$$\mathbf{R}_{12} = \mathbf{A}_{new1}(\alpha)\mathbf{R}_s\mathbf{A}^H(\beta). \tag{14}$$

Likewise, the K columns of $\mathbf{V}$ and $\mathbf{A}(\beta)$ span the same range space. There exists an invertible matrix $\mathbf{W}_2$ satisfied the following equality

$$\mathbf{V} = \mathbf{A}(\beta)\mathbf{W}_2. \tag{15}$$

Divide $\mathbf{V}$ into two $(N-1)\times K$ matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ such that $\mathbf{V}_1 = \mathbf{V}[1:N-1, :]$, $\mathbf{V}_2 = \mathbf{V}[2:N, :]$. Accordingly, $\mathbf{V}_1$ and $\mathbf{V}_2$ can be represented as

$$\mathbf{V}_1 = \mathbf{A}_1(\beta)\mathbf{W}_2 \tag{16}$$
$$\mathbf{V}_2 = \mathbf{A}_2(\beta)\mathbf{W}_2 \tag{17}$$

where $\mathbf{A}_1(\beta) = \mathbf{A}(\beta)[1:N-1, :]$, $\mathbf{A}_2(\beta) = \mathbf{A}(\beta)[2:N, :]$.

By means of the rotational invariance, we obtain

$$\mathbf{A}_2(\beta) = \mathbf{A}_1(\beta)\mathbf{D}_\beta \tag{18}$$

where $\mathbf{D}_\beta$ is also a diagonal matrix with $\mathbf{D}_\beta = \mathrm{diag}(\eta_1, \eta_2, \ldots, \eta_K)$.

Combining (15), (16), (17) with (18), we have

$$\mathbf{V}\left(\mathbf{V}_2^\dagger\mathbf{V}_1\right)^N = \mathbf{A}(\beta)\mathbf{W}_2\left(\mathbf{W}_2^{-1}\mathbf{A}_2^\dagger(\beta)\mathbf{A}_1(\beta)\mathbf{W}_2\right)^N = \mathbf{A}(\beta)\mathbf{D}_\beta^{-N}\mathbf{W}_2 = \mathbf{A}_{new1}(\beta)\mathbf{W}_2 \tag{19}$$

where $\mathbf{A}_{new1}(\beta) = [\mathbf{a}_{new1}(\beta_1), \mathbf{a}_{new1}(\beta_2), \ldots, \mathbf{a}_{new1}(\beta_K)]$, $\mathbf{a}_{new1}(\alpha_k) = \left[\eta_k^{-N}, \eta_k^{-(N-1)}, \ldots, \eta_k^{-1}\right]^T$.

Substituting (15) into (6), we get

$$\mathbf{A}(\alpha)\mathbf{R}_s = \mathbf{U}\Sigma\mathbf{W}_2^H. \tag{20}$$

We define the second matrix $\mathbf{R}_{21}$ as

$$\mathbf{R}_{21} \triangleq \mathbf{U}\Sigma\left(\mathbf{V}\left(\mathbf{V}_2^\dagger\mathbf{V}_1\right)^N\right)^H. \tag{21}$$

From (19) and (20), $\mathbf{R}_{21}$ can be rewritten as

$$\mathbf{R}_{21} = \mathbf{A}(\alpha)\mathbf{R}_s\mathbf{A}_{new1}^H(\beta). \tag{22}$$

Next, we define the third matrix $\mathbf{R}_{11}$ as

$$\mathbf{R}_{11} \triangleq \mathbf{J}\left(\mathbf{U}\mathbf{U}_1^\dagger\mathbf{U}_2\Sigma\left(\mathbf{V}\mathbf{V}_1^\dagger\mathbf{V}_2\right)^H\right)^*\mathbf{J}. \tag{23}$$

According to (6), (7), (8), (9), (10), (15), (16), (17) and (18), $\mathbf{R}_{11}$ can be expressed as

$$\mathbf{R}_{11} = \mathbf{J}\Big(\mathbf{A}(\alpha)\mathbf{D}_\alpha \mathbf{W}_1 \Sigma (\mathbf{A}(\beta)\mathbf{D}_\beta \mathbf{W}_2)^H\Big)^* \mathbf{J} = \mathbf{A}_{new1}(\alpha)\mathbf{R}_s^* \mathbf{A}_{new1}^H(\beta). \tag{24}$$

Since $\mathbf{R}_s$ is real diagonal matrix, the equality $\mathbf{R}_s = \mathbf{R}_s^*$ holds. Thus, $\mathbf{R}_{11}$ can be represented as

$$\mathbf{R}_{11} = \mathbf{A}_{new1}(\alpha)\mathbf{R}_s \mathbf{A}_{new1}^H(\beta). \tag{25}$$

Combining $\mathbf{R}_{11}$, $\mathbf{R}_{12}$, $\mathbf{R}_{21}$ with $\mathbf{R}_{xz}$, we construct a new matrix $\mathbf{R}_{new}$

$$\mathbf{R}_{new} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{xz} \end{bmatrix} \tag{26}$$

According to (5), (14), (22) and (25), $\mathbf{R}_{new}$ can be expressed as

$$\mathbf{R}_{new} = \begin{bmatrix} \mathbf{A}_{new1}(\alpha) \\ \mathbf{A}(\alpha) \end{bmatrix} \mathbf{R}_s \begin{bmatrix} \mathbf{A}_{new1}(\beta) \\ \mathbf{A}(\beta) \end{bmatrix}^H = \mathbf{A}_{new}(\alpha)\mathbf{R}_s \mathbf{A}_{new}^H(\beta) \tag{27}$$

where $\mathbf{A}_{new}(\alpha) = [\mathbf{a}_{new}(\alpha_1), \mathbf{a}_{new}(\alpha_2), \ldots, \mathbf{a}_{new}(\alpha_K)]$, $\mathbf{a}_{new1}(\alpha_k) = [\xi_k^{-N}, \xi_k^{-(N-1)}, \ldots, \xi_k^{-1}, 1, \xi_k^1, \ldots, \xi_k^{(N-1)}]^T$, $\mathbf{A}_{new}(\beta) = [\mathbf{a}_{new}(\beta_1), \mathbf{a}_{new}(\beta_2), \ldots, \mathbf{a}_{new}(\beta_K)]$, $\mathbf{a}_{new}(\beta_k) = [\eta_k^{-N}, \eta_k^{-(N-1)}, \ldots, \eta_k^{-1}, 1, \eta_k^1, \ldots, \eta_k^{(N-1)}]^T$.

It is evident that $\mathbf{A}_{new}(\alpha)$ and $\mathbf{A}_{new}(\beta)$ correspond to an array manifold matrix of a ULA with $2N$ array elements and $\mathbf{R}_{new}$ corresponds to a covariance matrix of a cross-array with $2N + 2N$ array elements, which indicates that the number of array elements increases from $N + N$ to $2N + 2N$.

3.2. Estimation of the Azimuth α and the Elevation β

Performing the SVD of $\mathbf{R}_{new}$

$$\mathbf{R}_{new} = \mathbf{A}_{new}(\alpha)\mathbf{R}_s \mathbf{A}_{new}^H(\beta) = \mathbf{U}'\Sigma'\mathbf{V}'^H \tag{28}$$

where $\Sigma' = \text{diag}(d_1, d_2, \ldots, d_K)$ with $d_1 \geq d_2 \geq \ldots \geq d_K > 0$, $d_1, d_2, \ldots, d_K$ stand for the K largest singular values of $\mathbf{R}_{new}$, $\mathbf{U}'$ and $\mathbf{V}'$ stand for the left and right singular vectors of $\mathbf{R}_{new}$ corresponding to the K largest singular values, respectively.

Next, we define three matrices Σ, $\mathbf{U}$ and $\mathbf{V}$ as $\Sigma \triangleq \Sigma'^{1/2}$, $\mathbf{U} \triangleq \mathbf{U}'\Sigma$ and $\mathbf{V} \triangleq \mathbf{V}'\Sigma$. Accordingly, (28) could be rewritten as

$$\mathbf{A}_{new}(\alpha)\mathbf{R}_s \mathbf{A}_{new}^H(\beta) = \mathbf{U}\mathbf{V}^H. \tag{29}$$

We have known that $\mathbf{U}$ and $\mathbf{A}_{new}(\alpha)$ span the same range space, and $\mathbf{V}$ and $\mathbf{A}_{new}(\beta)$ span the same range space. Hence, the following two equalities hold

$$\mathbf{A}_{new}(\alpha) = \mathbf{U}\mathbf{P}_1 \tag{30}$$

$$\mathbf{A}_{new}(\beta) = \mathbf{V}\mathbf{P}_2 \tag{31}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are two invertible matrices.

Then, we can utilize the conventional ESPRIT procedure [7] to estimate the azimuth α_k and the elevation β_k. Divide $\mathbf{U}$ into two $(2N-1) \times K$ matrices $\mathbf{U}_1$ and $\mathbf{U}_2$ such that $\mathbf{U}_1 = \mathbf{U}[1 : 2N-1, :]$, $\mathbf{U}_2 = \mathbf{U}[2 : 2N, :]$.

Performing the eigenvalue decomposition (EVD) of $\mathbf{U}_1^\dagger \mathbf{U}_2$, the eigenvectors $\mathbf{P}_1{}'$ of $\mathbf{U}_1^\dagger \mathbf{U}_2$ must satisfy the following equality

$$\mathbf{P}_1{}' = \mathbf{P}_1 \mathbf{C}_1 \tag{32}$$

where $\mathbf{C}_1$ is a permutation matrix composed of a single nonzero constant along every row or column and zeros elsewhere. The eigenvalues λ_{α_k} of $\mathbf{U}_1^\dagger \mathbf{U}_2$ must be equal to $e^{j\pi \cos \alpha_k}$, $k = 1, 2, \ldots, K$. Accordingly, the azimuth α_k, $k = 1, 2, \ldots, K$, could be obtained by solving the following nonlinear equation

$$\alpha_k = \cos^{-1}\left[\frac{\arg(\lambda_{\alpha_k})}{\pi}\right] k = 1, 2, \ldots, K. \tag{33}$$

Similarly, performing the EVD of $\mathbf{V}_1^{\dagger}\mathbf{V}_2$, where $\mathbf{V}_1 = \mathbf{V}[1 : 2N-1, :]$, $\mathbf{V}_2 = \mathbf{V}[2 : 2N, :]$. The eigenvectors $\mathbf{P}_2{}'$ of $\mathbf{V}_1^{\dagger}\mathbf{V}_2$ must satisfy the following equality

$$\mathbf{P}_2{}' = \mathbf{P}_2\mathbf{C}_2 \tag{34}$$

where $\mathbf{C}_2$ is also a permutation matrix. The eigenvalues λ_{β_k} of $\mathbf{V}_1^{\dagger}\mathbf{V}_2$ must be equal to $e^{j\pi\cos\beta_k}$, $k = 1, 2, \ldots, K$. The elevation β_k, $k = 1, 2, \ldots, K$, could be obtained by solving the following nonlinear equation

$$\beta_k = \cos^{-1}\left[\frac{\arg(\lambda_{\beta_k})}{\pi}\right] \quad k = 1, 2, \ldots, K. \tag{35}$$

3.3. Pair Matching

Combining (29), (30) with (31), we have [8]

$$\mathbf{P}_1^H\mathbf{P}_2 = \mathbf{R}_s^{-1}. \tag{36}$$

Since $\mathbf{R}_s$ is a diagonal matrix, $\mathbf{R}_s^{-1}$ is also a diagonal matrix.

We define a matrix $\mathbf{P}$ as

$$\mathbf{P} \triangleq \mathbf{P}_1'^H\mathbf{P}_2{}'. \tag{37}$$

According to (32), (34) and (36), $\mathbf{P}$ can be represented as

$$\mathbf{P} = \mathbf{C}_1^H\mathbf{P}_1^H\mathbf{P}_2\mathbf{C}_2 = \mathbf{C}_1^H\mathbf{R}_s^{-1}\mathbf{C}_2. \tag{38}$$

Since $\mathbf{C}_1$ and $\mathbf{C}_2$ are two permutation matrix and $\mathbf{R}_s^{-1}$ is a diagonal matrix, $\mathbf{P}$ is also a permutation matrix. (38) implies that if the eigenvectors of $\mathbf{U}_1^{\dagger}\mathbf{U}_2$ and $\mathbf{V}_1^{\dagger}\mathbf{V}_2$ are unit vectors, only one element value is close to 1 for every row or column in $\mathbf{P}$. We can pair 2-D angles of the signals by utilizing this property. Let the eigenvectors of $\mathbf{U}_1^{\dagger}\mathbf{U}_2$ and $\mathbf{V}_1^{\dagger}\mathbf{V}_2$ be unit vectors, if $p_{i,j}$ is close to 1, the ith azimuth and jth elevation angles come from a couple of incident angles, where $p_{i,j}$ denotes the element value of the ith row jth column in $\mathbf{P}$.

4. COMPUTATIONAL COMPLEXITY

In this section, we analyse the computational complexity of the proposed method. From the derivation of the presented method, we can see that the computational complexity of our method mainly focuses on two SVD. One is to perform the SVD of a $N \times N$ matrix, and the computational cost is $24N^3 + 48N^3 + 54N^3 = 126N^3$ flops [10]. The other is to perform the SVD of a $2N \times 2N$ matrix, and the computational burden is $24(2N)^3 + 48(2N)^3 + 54(2N)^3 = 1008N^3$. Since our method need to deal with a larger dimensional matrix, the computational cost of our method increases somewhat. However, our method can enhance the DOA estimation accuracy, and the simulation would verify this conclusion in the next section.

5. SIMULATION RESULTS

In this section, we illustrate the performance of our method by simulations. We compare our method with CCM-ESPRIT in [2], JSVD in [3] and Cramer Rao bound (CRB) in [9]. An L-shaped array is employed with 8 sensors for each ULA. The elements of each antenna array are separated by a half-wavelength. For simplicity, we suppose that all signal sources are of equal power σ_s^2, and the input SNR is defined as $10\log_{10}(\sigma_s^2/\sigma_n^2)$. Define the root-mean-square-error (RMSE) of the DOA estimates from N Monte Carlo trials as

$$\mathrm{RMSE} = \sqrt{\frac{1}{NK}\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\left(\hat{\alpha}_k^{(n)} - \alpha_k\right)^2 + \left(\hat{\beta}_k^{(n)} - \beta_k\right)^2\right)} \tag{39}$$

where $\hat{\alpha}_k^{(n)}$ and $\hat{\beta}_k^{(n)}$ are the estimates of α_k and β_k for the nth Monte Carlo trial respectively, and K is the source number.

In the first simulation, we examine the estimation performance of three methods in terms of SNR. Three signal directions are set to $[\alpha_1, \alpha_2, \alpha_3] = [100°, 90°, 80°]$, and $[\beta_1, \beta_2, \beta_3] = [70°, 80°, 90°]$. The snapshot number is fixed at 500. The SNR varies from 0 dB to 15 dB.

In the second simulation, we research the RMSE of three methods with respect to the snapshot number. The incident directions of three sources are set to $[\alpha_1, \alpha_2, \alpha_3] = [120°, 110°, 100°]$ and $[\beta_1, \beta_2, \beta_3] = [100°, 110°, 90°]$. The SNR is fixed at 5 dB. The snapshot number ranges from 200 to 2000.

From Figure 2 and Figure 3, it can be observed that our method is superior to CCM-ESPRIT and JSVD in different SNR and the snapshot number. It is because that our method utilizes the information $\mathbf{R}_s = \mathbf{R}_s^*$ to estimate the DOAs of signal sources, while CCM-ESPRIT and JSVD do not utilize this information.

In the third simulation, we investigate the estimation accuracy of the azimuth angles of three methods for different SNR. Three signal directions are set to $[\alpha_1, \alpha_2, \alpha_3] = [60°, 70°, 80°]$, and $[\beta_1, \beta_2, \beta_3] = [110°, 120°, 130°]$. The snapshot number is fixed at 500. The SNR varies from 0 dB to 15 dB.

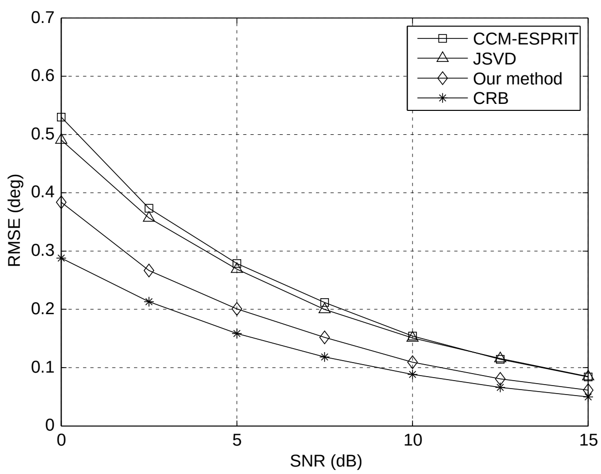

Figure 2. RMSE versus the SNR for 500 snapshots.

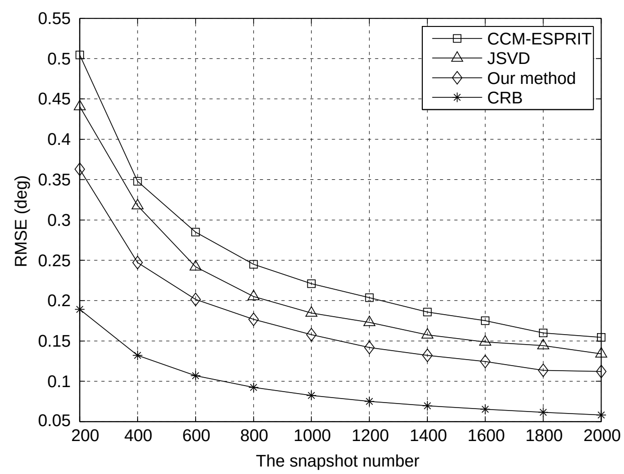

Figure 3. RMSE versus the snapshot number, SNR = 5 dB.

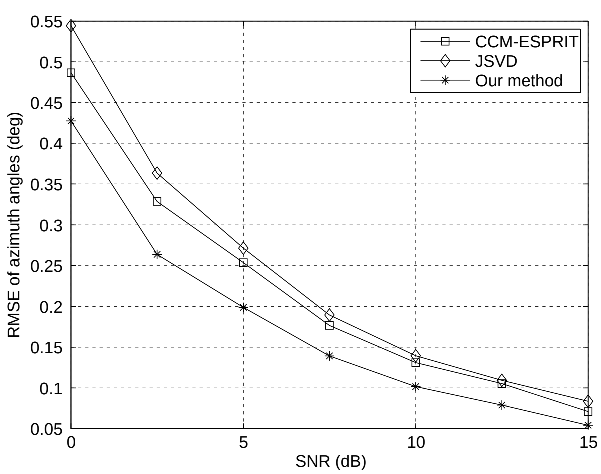

Figure 4. RMSE of azimuth angles versus the SNR for 500 snapshots.

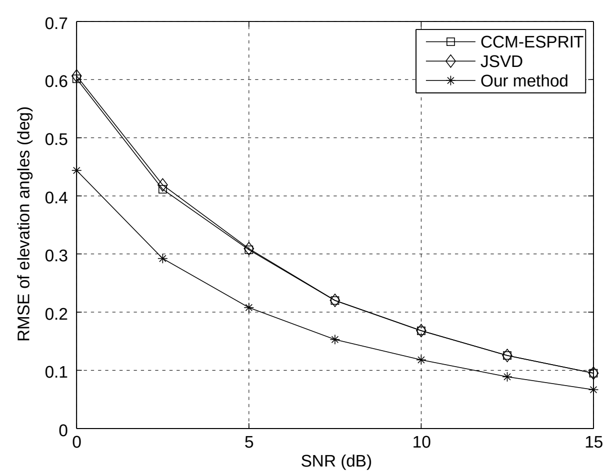

Figure 5. RMSE of elevation angles versus the SNR for 500 snapshots.

In the fourth simulation, we test the estimation accuracy of the elevation angles of three methods for different SNR. The simulation condition is the same as the third one.

From Figure 4 and Figure 5, we can see that the DOA estimation accuracy of azimuth angles and azimuth angles is improved respectively by using our array aperture extension algorithm. The reason for this is that our method exploits more information to detect the DOAs of the signal sources.

6. CONCLUSION

This paper proposes an array aperture extension algorithm for 2-D DOA estimation with L-shaped array. Our method exploits rotational invariance, as well as the property that the signal covariance matrix is real diagonal matrix to construct a larger dimensional covariance matrix corresponding to a cross-shaped array with more array sensors. It is equal to enlarging the array aperture. Thus, our method achieves higher DOA estimation accuracy. Simulation results confirm the validity of the proposed method.

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