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Homogenization of Periodic Objects Embedded in Layered Media

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Abstract—An effective medium modeling technique is proposed to homogenize the periodic objects embedded in layered media. The homogenization is based on the same scattering coefficients. An integral equation based approach is adopted to solve the scattering problem of original structures. Our modeling results are compared with Maxwell-Garnett mixing formula and published results. Good agreements have been observed. Periodic metal patches embedding in layered dielectric structure is fabricated and measured to validate the modeling technique. The difference between experiment results and proposed modeling results is less than 3%.

1. INTRODUCTION

Periodic structures have been investigated intensively over the decades. By embedding periodic objects in layered media, many practical applications have been proposed. Such applications include bandgap material, frequency selective surface, metamaterial, optical waveguides, and novel antenna substrate [1, 2]. The properties of the composite materials, such as effective permittivity, permeability or conductivity, can be controlled by changing the constitutive parameters, the volume and geometric shape of the inclusions.

There are different approaches to homogenize the composite materials. Analytical formulas such as Maxwell-Garnett mixing formula [3] is derived based on spherical inclusions and applied to the case when the size of inclusions is small comparing with the period. In recent years, numerical techniques such as method of moments (MoM), finite difference method (FDM), and finite element method (FEM) have been adopted to solve the problem [4–7]. The MoM methods usually have better accuracy. In [6], an integral equation approach combining equivalence principle and connection scheme (EPACS) is presented. This approach avoids evaluating the sophisticated multilayered periodic Green's function so that it is very efficient. In [7], we present an effective medium modeling approach based on the same scattering coefficients. After solving the scattering problem using EPACS, a retrieving procedure is then applied to get the effective parameters for the homogenized material from the scattering parameter. This method is applicable to all kinds of material, as long as the structures are electrical small. In this letter, we focus on the homogenization of periodic structures; explore the effect of different inclusion shape and filling rate in periodic structures. Both simulation and experiment results are presented.

This paper is organized as follows. In Section 2, the modeling methodology is presented together with numerical examples. Section 3 provides experiment validation. The conclusions are made in Section 4.

2. MODELING METHODOLOGY AND NUMERICAL RESULTS

The structure of interest is as illustrated in Fig. 1. The structure is assumed to be infinitely large in the horizontal plane (XY plane) while the inclusions are doubly periodical in x and y directions. The

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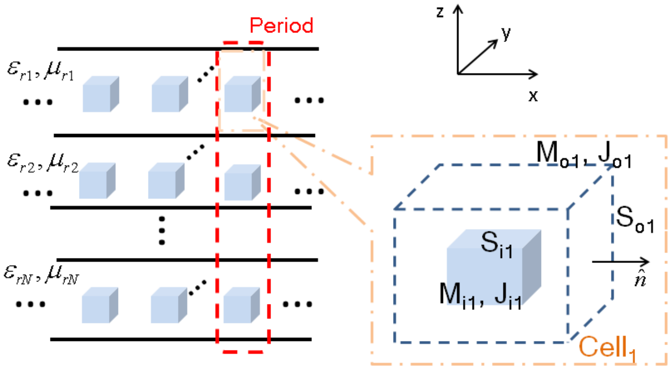

Figure 1. Periodic objects embedded in layered media.

parameters of each dielectric layer, such as the permittivity, permeability, and thickness can be different. One period is divided into N cells based on number of dielectric layers. With the help of equivalence principle, our computation domain is constrained in just one cell. To make a more general case, the inclusions are assumed to be dielectric objects which have their own permittivity and permeability. This enables us to model real metal with finite conductivity. Thus we have equivalent source on both the surface of the cell S_o and the surface of the inclusion S_i as well.

The electric field integral equation (EFIE) and magnetic field integral equation (MFIE) are applied on the outer surface of cell1 S_{o1} [6, 7] and for the surface of inclusion, the PMCHWT formulation is applied [7]. Then the integral equations are discretized. The unknowns on the surface of inclusions and the four sides of outer surface (the surface of dielectric box) are eliminated by applying the periodic boundary condition. After that, the connection scheme is adopted to enforce the tangential continuity of neighboring cells so that the relation between the top layer and bottom layer is determined [6, 7]. Solving the matrix equation for the equivalent source and the reflection/transmission coefficients are calculated in terms of the equivalent sources. After the scattering coefficients are calculated, a retrieval procedure is applied to retrieve the effective constitutive parameters of the effective homogenized material [8, 9].

To numerically validate our proposed approach, periodical dielectric spheres in free space case is simulated first. The relative permittivity of the sphere is 40. The period is a cube with side length set to be 0.1 mm and the solution frequency is set to be 2 GHz. The radius of the sphere is adjusted to fulfill different filling rate (FR), which is defined as the ratio of inclusion volume over cell volume. When the inclusion is spherical shape and the FR is small, the MG formula works as good approximation. We change the size of the sphere while keeping the same period dimension so that we get permittivity at different FRs. The results are compared with both MG formula [3] and results from [10]. These three results are illustrated in Fig. 2. As we can see, when the filling rate is below 35%, all these three results have little difference. When the filling rate goes up, the MG formula results show larger difference while the other two stay close. The reason is that when the filling rate is high, the interactions between inclusions become strong but the MG formula cannot catch the interaction. Wu and Whites [10] extracted the permittivity based on electric flux and averaged electric field while in our proposed method, the effective permittivity is extracted from the scattering parameter. Thus two approaches give slightly different results.

In the EPACS approach, the inclusions are described using surface mesh. This enables us to simulate various geometries but not limited to ellipsoid. Moreover, EPACS accepts complex permittivity input so that we can model different types of inclusions: lossless dielectric, lossy dielectric, metal with finite conductivity or perfect electric conductor (PEC). The following simulation case is periodic PEC cubes in lossless dielectric slab with a relative permittivity of 2. We also change the volume of the cube inclusion so that we get results at different FRs. The extracted permittivity are shown in Fig. 3 and compared with MG formula. Though the two results still agree with each other when filling rate is low, but the difference become obvious at even lower FR comparing to spherical inclusion case. This is due to the MG formula is derived based on sphere inclusion rather than other shapes.

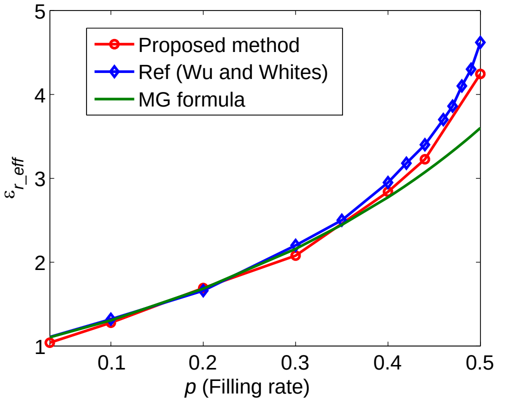

Figure 2. Effective permittivity of periodic dielectric sphere as a function of filling rate. Dielectric spheres are placed in a period of 0.1 mm by 0.1 mm by 0.1 mm cube. The frequency in our modeling is 2 GHz. The results are compared with MG formula and those from [10].

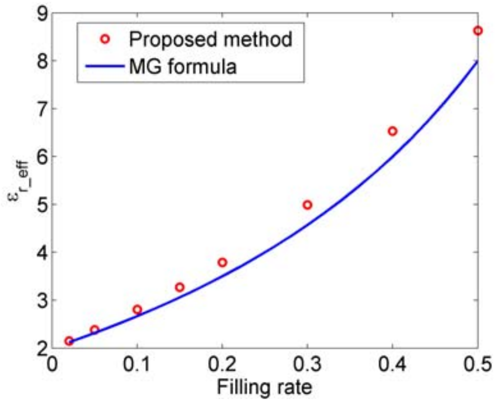

Figure 3. Effective permittivity of periodic metal cubes as a function of filling rate. The period and solution frequency are configured the same way as case shown in Fig. 2.

3. EXPERIMENT VALIDATION

Two layers of periodic metal patches embedded in dielectric material was fabricated and evaluated for the relative permittivity of this structure. We used standard 4 metal layer printed circuit board (PCB). The inner two metal layers are designed to have periodic metal patches. Meanwhile the top metal and the bottom metal form microstrip (MS) pairs which are used to determine the substrate permittivity. With this configuration, the substrate of the MS pairs becomes layered media embedded with periodic objects. We obtained the effective relative permittivity of the homogenized substrate by analyzing the propagation constant of MS pairs.

The photo of the fabricated board and structure illustration are as shown in Fig. 4. By changing the size of the metal patches in inner layers, we have implemented three different filling rates. One configuration is to place the metal patch of 0.127 mm by 0.127 mm by 0.035 mm in a period of 0.254 mm by 0.254 mm by 0.711 mm, which gives us a filling rate of 1.25%. Another configuration is to make the metal patch size to be 0.635 mm by 0.635 mm by 0.035 mm while the period to be 0.762 mm by 0.762 mm by 0.711 mm, which gives a filling rate of 3.47%. There are three sets of microstrip line pairs, corresponding the filling rate in the substrate to be 0 (no motel inclusion), 1.25% and 3.47%, respectively.

The effective permittivity of substrate is obtained as follows. After the S parameters of one microstrip line pair are measured, the equivalent propagation constant γ is calculated using by [11, 12]:

$$\begin{aligned} \gamma &= \ln\left[\left(A \pm \sqrt{A^2-4}\right)\Big/2\right]\Big/(l_1-l_2) \\ A &= \left(T_{11(1)}T_{22(2)} + T_{11(2)}T_{22(1)}\right) - \left(T_{21(1)}T_{12(2)} + T_{21(2)}T_{12(1)}\right) \\ T_{11(i)} &= -\det\left(S_{(i)}\right)\big/S_{21(i)}, \quad T_{12(i)} = S_{11(i)}\big/S_{21(i)}, \\ T_{21(i)} &= -S_{22(i)}\big/S_{21(i)}, \quad T_{22(i)} = 1\big/S_{21(i)}. \end{aligned}$$

where the subscript i indicates different MS line, l the length of the MS line, and $\det(S_{(i)})$ the determinant of S parameter of MS line i.

The effective permittivity is then derived from the equivalent propagation constant using equations [13]:

$$\begin{aligned} \varepsilon_{req} &= -\gamma^2\big/k_0^2 \\ \varepsilon_{r\_eff} &= \frac{1 + (2\varepsilon_{req} - 1)\sqrt{1 + 12h/w_{eff}}}{1 + \sqrt{1 + 12h/w_{eff}}} \end{aligned}$$

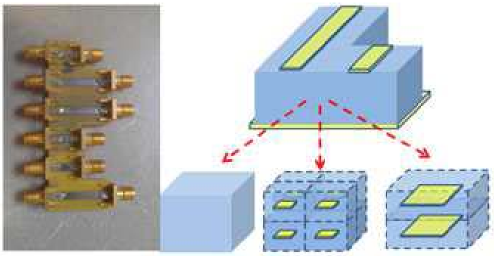

Figure 4. Photo of fabricated PCB and illustration for periodic metal patches. Top and bottom metal layers form MS pairs while two inner metal layers are configured as periodic metal patches with different filling rate.

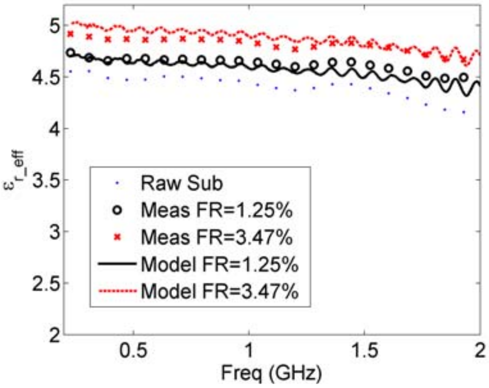

Figure 5. Effective permittivity of periodic metal patches embedded in layered media, measurement results and modeling results for two different filling rate and frequency range of 200 MHz to 2 GHz.

where:

$$w_{\mathit{eff}} = w + \frac{1.25t}{\pi}\left[1 + \ln\left(2h/t\right)\right],$$

k_0 is the wavenumber in free space, h the thickness of the substrate, w the width of the microstrip line, and t the thickness of the metal. The effective permittivity of substrate is the permittivity for the homogenized substrate. The experiment results are then compared with our proposed modeling results.

The measurement results are shown in Fig. 5. The measurement is performed in frequency band 200 MHz to 2 GHz. Measured results are plotted in discrete circles and crosses, for filling rate of 1.25% and 3.47%, respectively. The results for raw substrate are also plotted in discrete dots, for reference. Modeling results are also provided in Fig. 5 using solid and dashed line for filling rate of 1.25% and 3.47%, respectively. From Fig. 5, both experiment results and modeling results suggest that the effective permittivity rises when periodic metals are embedded. The results also suggest that higher filling rate gives higher effective permittivity. The difference between experiment and modeling results is less than 3%. This good correlation between the experiment and modeling validates our proposed homogenized approach.

4. CONCLUSION

In this letter, we propose an effective medium modeling approach to homogenize layered media with periodic inclusions. The approach is based on the same scattering coefficients. The scattering problem of original structure is solved by an integral equation based approach named EPACS. The effective constitutive parameters are retrieved from the scattering coefficients. Numerical results for different cases are provided and compared with MG formula and results from literatures. Comparing with MG formula, our proposed method works fine for higher inclusion filling rate case and also for various inclusion shapes besides spheres. Structures of periodic metal patches embedded in layered dielectric material are fabricated using PCB technology. The designed structures are measured and experiment data are analyzed and compared with result of proposed modeling approach. The difference between experiment results and modeling results is less than 3%.

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