

Analysis of Complex Discontinuities in Circular Waveguides Using Hybrid Finite Element Method and Multimodal Variational Method

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Abstract—We propose a hybrid multimodal variational method (MVM) and finite element method (FEM) to the analysis of complex 2D discontinuities in circular waveguides. The finite element method characterizes waves in arbitrarily shaped discontinuities, and the total response of the circuit is obtained by applying the multimodal variational method. The proposed hybrid method is successfully applied to the full-wave analysis of discontinuities with great practical interest (i.e., circular, cross-shaped, off-centered, ridged, multi-aperture irises, etc.), thus improving CPU time and memory storage against several full-wave finite element method based computer aided design (CAD) tools (i.e., HFSS High Frequency Structural Simulator).

1. INTRODUCTION

Numerous electromagnetic CAD tools for devices in millimeter waves and microwave have been developed in the last three decades (i.e., modal, space meshing techniques). The modal methods, such as Mode Matching (MM) technique [1], Generalized Scattering Matrix (GSM) analysis [2] and Multimodal Variational Method (MVM) [3], use a modal development of electromagnetic fields on the basis of eigenmodes of constituent waveguides. Based on the mode-matching technique, the authors of [4] proposed a procedure that makes use of the admittance matrix characterization of waveguide discontinuities. The number of modes used in the input and the output of the circuits determine the size of the scattering matrix [1, 2]. Conversely, in our multimodal variational formulation [3], the size of the scattering matrix depends only on the number of accessible modes, which is independent from the total number of modes used. Consequently, compared to other modal methods such as MM and GSM, the size of the scattering matrix is reduced, which minimizes the memory storage and the computational time without modifying the response of the circuit.

However, when arbitrarily-shaped cross section discontinuities are considered, the modal basis is no longer systematic. The use of space meshing techniques, i.e., FEM [5], produces accurate results. Nevertheless, these techniques require important CPU time and memory storage. To alleviate these weaknesses, several hybrid modal and space meshing methods have been proposed [5–11]. In [9–11], we presented hybrid FEM-MVM tools to the studying of complex discontinuities in unfilled and filled rectangular waveguides.

The circular waveguide components find numerous applications in telecommunication's systems. In this paper, we extend our hybrid FEM and MVM tool to the accurate and fast analysis of complex discontinuities in circular waveguides. The analysis program is developed in Matlab on a PC (Intel 3 GHz and 4-GB RAM).

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2. THEORY

We consider a complex 2D discontinuity between two circular waveguides as shown in Fig. 1 where S represents the aperture of the discontinuity. In the case of simple discontinuities between waveguides, the MVM provides a full-wave characterization of the complete structure [3]. Its principle is to minimize the following variational and stationary function [3]:

$$f(E_t) = \langle E_t | \hat{Y} E_t \rangle \quad (1)$$

$\langle \cdot | \cdot \rangle$ denotes the inner product and \hat{Y} the self-adjoint admittance matrix operator of the discontinuity [3].

When considering simple S aperture (i.e., rectangle, circle, ellipse, etc.) the tangential electrical field E_t within the discontinuity can be expanded over the modal eigenfunction basis that satisfies the boundary conditions. When considering a complex S aperture as in Fig. 1, the determination of the modal eigenfunction basis is no longer analytical. To overcome this limitation, we express the tangential electrical field E_t as in the FEM edge formulation [5]. The complex S aperture is divided into rectangular elements (in cylindrical coordinate system). Fig. 2 shows the shape of one element (e). In local numbering, the tangential electric field within the rectangular element (e) can be expressed as follows [5]:

$$E_t^e(x, y) = \sum_{m=1}^4 E_m^e N_m^e(x, y) \quad x, y \in e \quad (2)$$

N_m^e are the interpolation functions, and E_m^e denote the tangential electric fields along the m th edge. We selected meshes conform to the coordinate lines of cylindrical coordinate system, which allows for separation of variables. However, for more complex apertures, triangle elements can be adopted. In global numbering, the tangential electric field within the aperture S can be expressed as:

$$E_t(x, y) = \sum_{p=1}^M B_p g_p(x, y) \quad (3)$$

M denotes the number of edges in the S aperture and B_p the unknown tangential electric field along the p th edge

$$B_p = E_m^e \quad (4)$$

and

$$g_p(x, y) = N_m^e(x, y) \quad x, y \in e \quad (5)$$

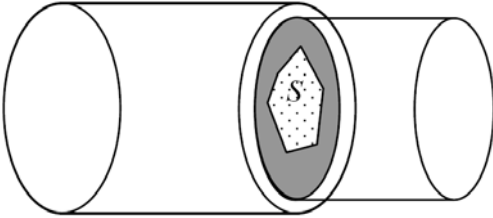


Figure 1. Complex discontinuity between circular waveguides.

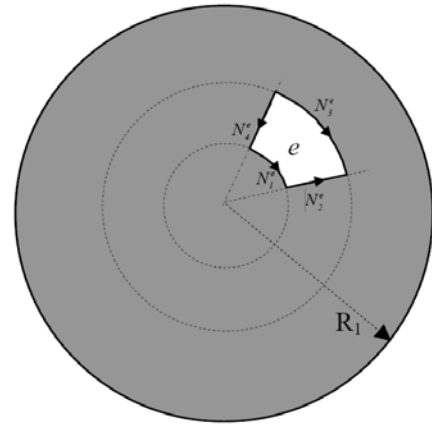


Figure 2. Rectangular element (e) in cylindrical coordinate system.

In this paper, we choose first order interpolation functions. By combining (1) and (3), we obtain the following function:

$$f(E_t) = \left\langle \sum_{p=1}^M B_p g_p(x, y) \left| \hat{Y} \sum_{q=1}^M B_q g_q(x, y) \right. \right\rangle \quad (6)$$

The scattering matrix of the circuit is obtained by minimizing the expression (6) with respect to B_p . The size of the basis functions depends only on the number of edges M in the discontinuity which is smaller than the number of modes used.

3. RESULTS

In our study, we analyse various forms of irises that have important practical interests in circular waveguide technology. Theoretically, the scattering matrix is of infinite dimension corresponding to the infinite number of modes. For numerical computations and after making a convergence study of the S-parameters, the number of modes is truncated to a finite size. To choose the number of modes used, we found in practice that the optimal way is firstly to arrange modes of the entire circuit by ascendant cut-off frequencies. Then, we take all modes having cut-off frequencies below a selected cut-off frequency F . The performances of the proposed hybrid method are compared to those obtained using HFSS. We found that both numerical CAD tools provided similar accuracies for all studied discontinuities which show the versatility of the proposed hybrid method. However, to converge, as presented in our earlier publications for complex rectangular waveguide discontinuities [9–11], we find that HFSS takes about 5 hours while our method takes a few minutes which represents an overall reduction in CPU time more than 99% in all studied structures.

3.1. Circular Iris

We consider a centred circular iris in a circular waveguide. Fig. 3 shows the geometries and meshing of the aperture. We selected a circular iris because the modal methods can obtain precise responses that can be used as references to compare the performances of the studied numerical methods. When applying a modal analysis [1–3], for dominant TE_{11} mode excitation, it is sufficient to consider only TE_{1m} and TM_{1m} modes, where m is an integer. However, when applying our hybrid method we have to consider all modes.

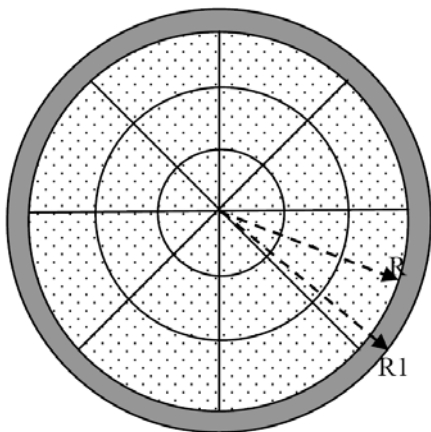


Figure 3. Geometries and meshing of the circular aperture (24 elements). $R = 12$ mm, $R_1 = 13.4$ mm.

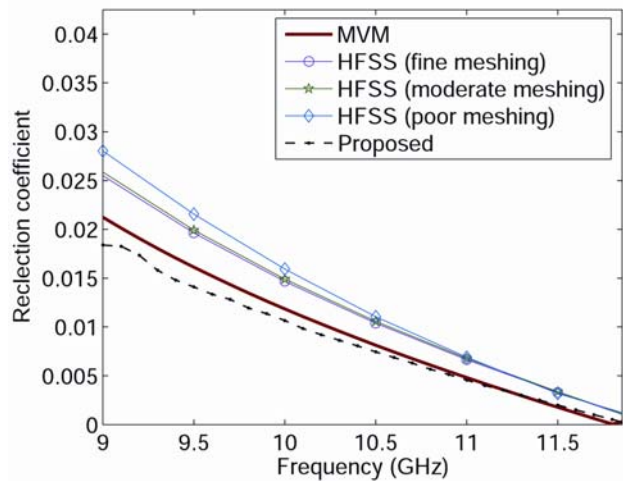


Figure 4. Reflection coefficient of the circular aperture ($F = 150$ GHz).

Figure 4 displays the reflection coefficient of the circular iris using the suggested method and HFSS, respectively. For the classic MVM, we employed a sufficiently high number of modes (i.e., $F = 150$ GHz) to obtain a precise response of the circuit which can be used as reference. We observe that the proposed hybrid method, using 24 elements and $F = 150$ GHz, is successfully applied to the design of the discontinuity

3.2. Ridged Iris

The next example deals with the analysis of a circular ridged waveguide which is typically used in circular waveguide dual-mode CWDM filters to tune and couple each pair of degenerated modes [12]. To reduce the manual tuning, an efficient software tool for the full wave electromagnetic simulation of the complex discontinuity is required.

We consider multi-ridged irises in a circular waveguide. Figs. 5 and 7 show the geometries and the meshing of the studied apertures. Figs. 6 and 8 display the reflection coefficients for both cases using the suggested method and HFSS. We observe that both responses agree.

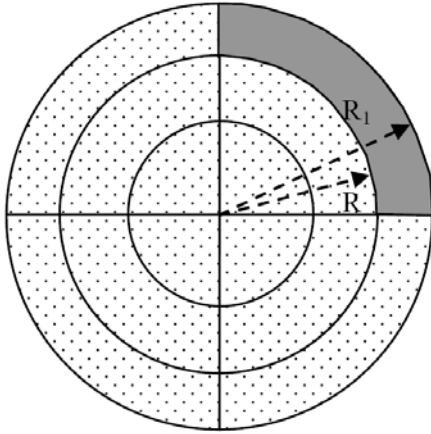


Figure 5. Geometries and meshing of the ridged aperture (11 elements), $R = 10$ mm.

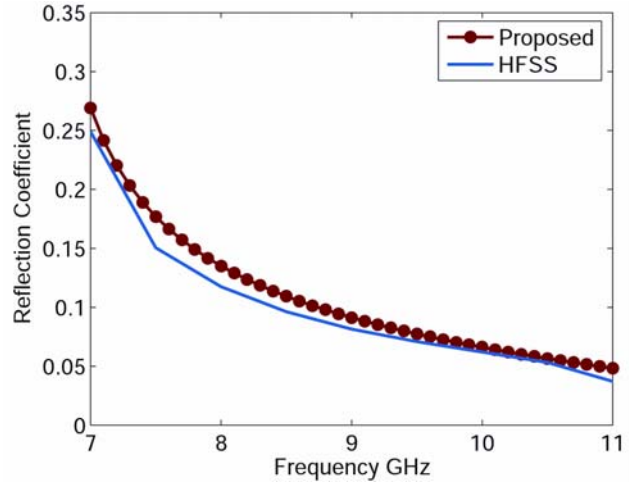


Figure 6. Reflection coefficient of the ridged aperture ($F = 150$ GHz).

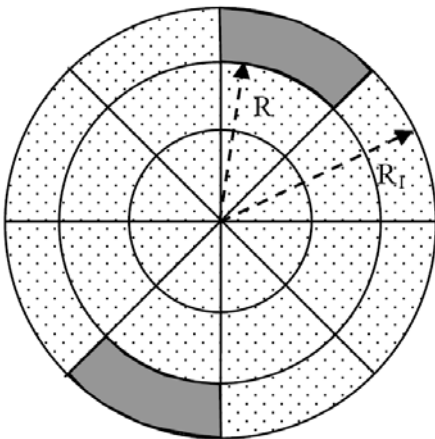


Figure 7. Geometries and meshing of the two ridged aperture (22 elements), $R = 11$ mm.

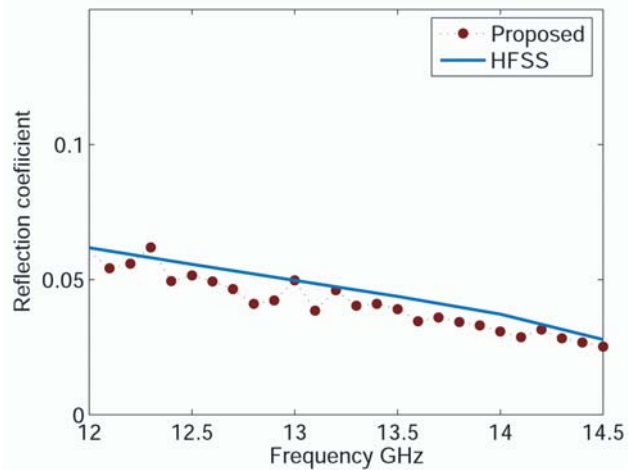


Figure 8. Reflection coefficient of the two ridged aperture ($F = 170$ GHz).

3.3. Multiaperture Iris

Multi-aperture resonant irises are commonly utilized for design of various kinds of filters such as bandpass and band rejection filters. They can be used as a tool to introduce additional attenuation poles to the bandpass filter response [13].

Figure 9 represents an example of a multi-aperture iris in circular waveguide which has arc-shaped opening. This kind of irises is widely studied in [13]. The performances of the proposed hybrid method are compared to those obtained using HFSS (see. Fig. 10). We observe that both results are comparable.

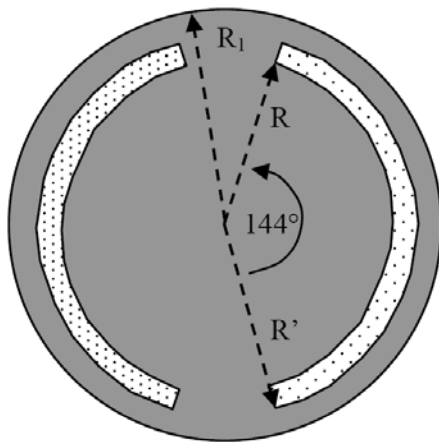


Figure 9. Geometries and meshing of the two aperture iris (2 elements), $R = 10$ mm, $R' = 12$ mm.

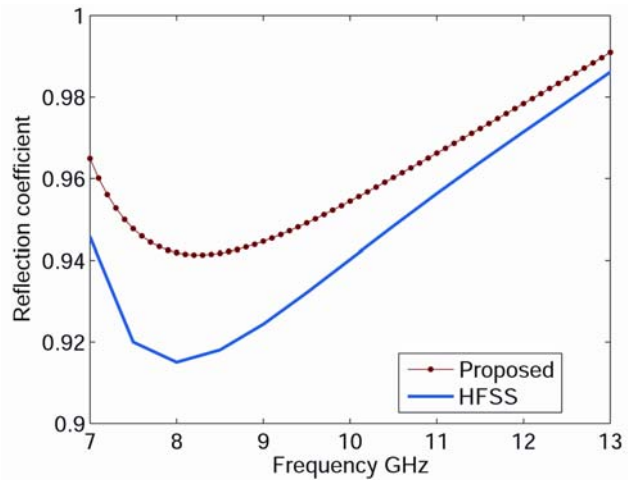


Figure 10. Reflection coefficient of the two aperture iris ($F = 150$ GHz).

3.4. Cross Shaped Iris

The cross-shaped irises are generally used for the inter-resonator couplings in dual mode filters. Fig. 11 shows the geometries and meshing of the considered cross-shaped iris. Fig. 12 displays the reflection coefficients using the suggested method and HFSS. We observe that the two responses agree.

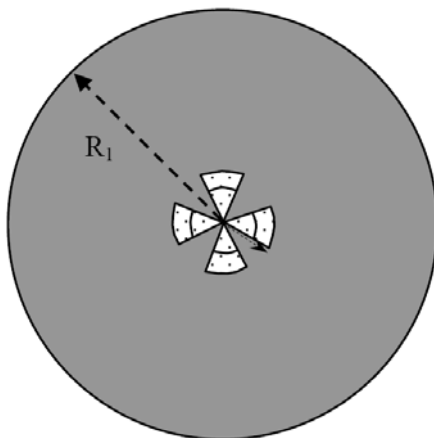


Figure 11. Geometries and meshing of the cross-shaped aperture (8 elements), $R = 4$ mm.

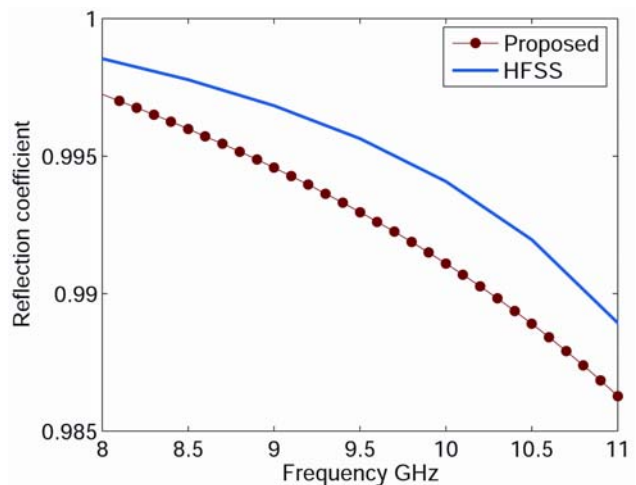


Figure 12. Reflection coefficient of the cross-shaped aperture ($F = 250$ GHz).

3.5. Off-Centred Iris

The off-centred irises offer more design flexibilities than concentric ones because they provide larger susceptance range [14, 15]. Our present effort is directed toward the analysis of complex off-centred irises in a circular waveguide. Fig. 13 shows geometries and the meshing of an off-centred cross-shaped iris. As observed in Fig. 14, the precisions of the proposed hybrid method are comparable to those obtained using HFSS.

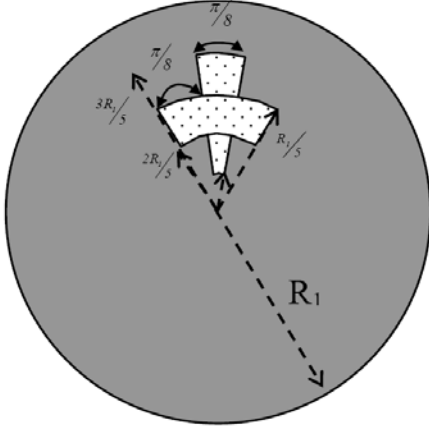


Figure 13. Geometries and meshing of the off-centred cross-shaped aperture (3 elements).

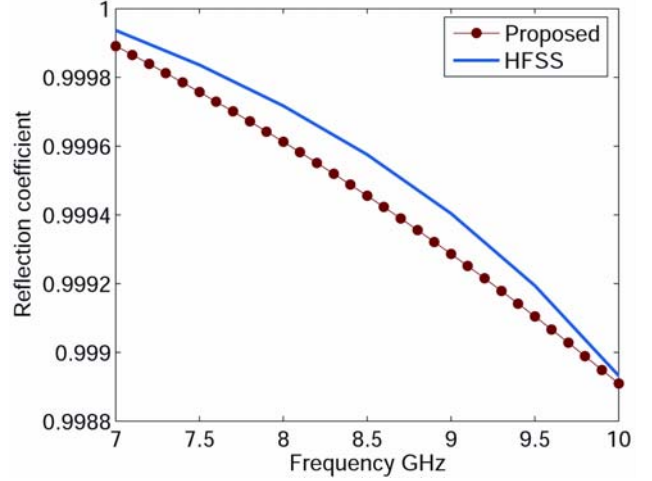


Figure 14. Reflection coefficient of the off-centred cross-shaped aperture ($F = 350$ GHz).

4. CONCLUSION

We proposed a fast and accurate hybrid FEM-MVM tool to the complete CAD of 2D complex discontinuities in circular waveguides. The introduced hybrid FEM-MVM has been accurately applied to the analysis of several application examples of highly practical interest such as cross-shaped, multi-aperture, off-centered and ridged irises thus improving the computing time against FEM based CAD tools. Our hybrid method can be applied to the fast and accurate analysis of more structures containing complex discontinuities in circular waveguides.

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