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Electrodynamic Characteristics of a Radial Impedance Vibrator on a Perfect Conduction Sphere

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Abstract—A problem of the spherical antenna consisting of a thin radial monopole located on a perfectly conducting sphere is solved. The antenna is excited at the base by a voltage δ-generator. An approximate analytical solution of the integral equation for the current on a thin impedance vibrator was found by the method of successive iterations. The solution is physically correct for arbitrary dimensions of the spherical antenna and for any value of surface impedance distributed along the monopole. The validity of the problem formulation is provided by using the Green's function for the Hertz vector potential in unbounded space outside the perfectly conducting sphere and by writing the initial integral equation for the current on the monopole. Influence of the monopole dimensions and surface impedance upon the radiation characteristics and the input impedance of the spherical antenna is studied by numerical evaluations using zero order approximation. The input impedance of the monopole was determined by the method of induced electromotive forces (EMF) using the current distribution function thus obtained.

1. INTRODUCTION

The electrodynamic theory of rectilinear impedance vibrators in an infinite medium is considered in [1] and in the references therein. The theory for the vibrators inside the rectangular waveguide was also presented in [1]. Almost all electrodynamic problems mentioned in these references were solved using Cartesian coordinates. A system of spherical coordinates $\{\rho; \theta; \varphi\}$ was used in [1–7] where radially oriented radiators near conducting spheres were investigated. The need to study characteristics of spherical antenna is defined by practical interest, since dipole radiators are widely used on mobile objects, including aircrafts [8]. It should be noted that impedance vibrators as radiators were studied only in [1, 7] while a Hertz dipole was considered in [2], and perfectly conducting vibrators were investigated in [3–6].

The results presented in [1, 7] are aimed at the construction of approximate analytical solution for the current on the radial impedance vibrator located near (or on) a conducting sphere by the method of successive iterations. To achieve this, a traditional approach based on the use of the Green's function constructed in [6] for the space outside a perfectly conducting sphere was used. The Green's function for the Hertz vector potential of electric type $\vec{\Pi}(\rho, \theta, \varphi)$ determined in [6] from the Helmholtz equation, based on the full wave equation in spherical coordinates. The authors supposed that such form of the Green's function would allow writing the integral equation for the current on the vibrator in terms of a differential operator graddiv $\vec{\Pi}(\rho, \theta, \varphi)$ applied to the true Hertz potential. However, as further studies shown [9] the solution for the current obtained in [1, 7] is physically valid only for large spheres, when

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relation of radius to the wavelength is much greater than unity. Of course, such condition excludes the possibility of analysis of resonant spherical scatterers, the most interesting for practical applications.

Thus, the problem of analytical solution of the integral equation for the current in the radial impedance monopoles, physically correct for spherical antenna of arbitrary dimensions, remains actual today. No less important is the study of influence of the surface impedance upon the radiation characteristics and input impedance of the spherical antennas that are not yet covered extensively in the literature. These issues are the subject of this paper.

2. PROBLEM FORMULATION AND INITIAL INTEGRAL EQUATIONS

Consider a perfectly conducting sphere and thin cylindrical impedance vibrator. The sphere radius is $\tilde{R}$, the vibrator radius and length are r and L, respectively, such that inequalities $(r/L) \ll 1$ and $(r/\lambda) \ll 1$ hold. The vibrator's axis coincides with the direction ρ', $\theta' = \theta_0$, $\varphi' = \varphi_0$ (Fig. 1).

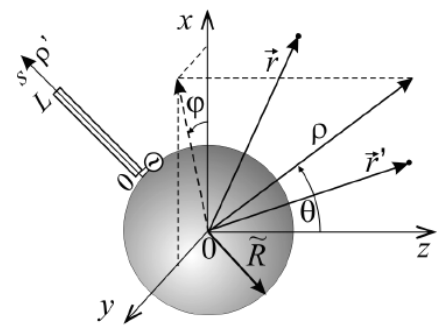


Figure 1. The spherical antenna consisting of the impedance monopole and sphere.

According to the thin wire model, the field of the surface current is equivalent to the field of linear current $J(\rho')$ flowing along the longitudinal axis of the vibrator. Then, the electric vector Hertz potential will have only a radial component

$$\Pi_\rho(\vec{r}) = \frac{1}{i\omega\varepsilon_1} \int_{\tilde{R}}^{\tilde{R}+L} J(\rho') \, G^e_{\rho\rho'}(\rho, \theta, \varphi; \rho', \theta_0, \varphi_0) \, d\rho', \quad (1)$$

where $\vec{r}$ is a radius-vector of the observation point, ε_1 the dielectric constant of the medium, ω the angular frequency (time dependence is in the form $e^{i\omega t}$), and $G^e_{\rho\rho'}(\rho, \theta, \varphi; \rho', \theta', \varphi')$ the electric Green's function for the space outside the perfectly conducting sphere [6]

$$G^e_{\rho\rho'}(\rho, \theta, \varphi; \rho', \theta', \varphi') = -\sum_{n=0}^{\infty} \frac{n+1/2}{2\pi} H_n(\rho, \rho') \, P_n\left[\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi - \varphi')\right]. \quad (2)$$

Here

$$H_n(\rho, \rho') = \begin{cases} k_1 h_n^{(2)}(k_1\rho') \left[h_n^{(2)}(k_1\rho) Q_n\left[y_n\left(k_1\tilde{R}\right)\right] - y_n(k_1\rho)\right], & \tilde{R} \le \rho < \rho', \\ k_1 h_n^{(2)}(k_1\rho) \left[h_n^{(2)}(k_1\rho') Q_n\left[y_n\left(k_1\tilde{R}\right)\right] - y_n(k_1\rho')\right], & \rho > \rho', \end{cases}$$

$$Q_n\left[y_n\left(k_1\tilde{R}\right)\right] = \frac{k_1\tilde{R} y_{n-1}\left(k_1\tilde{R}\right) - n y_n\left(k_1\tilde{R}\right)}{k_1\tilde{R} h_{n-1}^{(2)}\left(k_1\tilde{R}\right) - n h_n^{(2)}\left(k_1\tilde{R}\right)},$$

$P_n[\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi - \varphi')]$ are Legendre polinomials, $h_n^{(2)}(k_1\rho) = j_n(k_1\rho) - i y_n(k_1\rho)$ the spherical Hankel functions of the second kind, and $j_n(k_1\rho)$ and $y_n(k_1\rho)$ the spherical Bessel and Neumann functions, respectively [10].

For the constant internal impedance per unit length, $z_i = \text{const}$ [Ohm/m] on the vibrator generatrix, which can be approximate by a radial ray segment in the direction $\theta = \theta_0$ and $\varphi = \varphi_0 + r/(\tilde{R} + L/2)$, the initial integral equation can be written as [11]

$$\frac{d^2\left[k_1\rho\Pi_\rho(\rho)\right]}{d\rho^2} + k_1^2\left[k_1\rho\Pi_\rho(\rho)\right] = -E_{0\rho}(\rho) + z_i J(\rho), \tag{3}$$

where $E_{0\rho}(\rho)$ is the radial component of the extraneous excitation field, $k_1 = k\sqrt{\varepsilon_1\mu_1}$, $k = 2\pi/\lambda$, λ the wavelength in free space, and μ_1 the permeability of the medium. Equation (3), in contrast to that in [1, 7], is written in terms of the differential operator rotrot$\,[\vec{\rho}^0, \vec{\tilde{\Pi}}(\rho, \theta, \varphi)]$ for the Hertz pseudovector defined as $\vec{\tilde{\Pi}}(\rho, \theta, \varphi) = k_1\rho\Pi_\rho\vec{\rho}^0 + \vec{\theta}^0\Pi_\theta + \vec{\varphi}^0\Pi_\varphi$ ($\vec{\rho}^0, \vec{\theta}^0, \vec{\varphi}^0$ are unit vectors) through true Hertz vector of the electric type, whose components are expressed as in (1). Using well-known relation, one can easily see that the self-consistency condition for the electromagnetic field is taken into account separately for spherical waves of E-type. Using the expressions (1) and the notation $G^e_{\rho\rho'}(\rho, \rho') = G(\rho, \theta_0, \varphi_0 + r/(\tilde{R} + L/2); \rho', \theta_0, \varphi_0)$, Equation (3) can be written as

$$\left[\frac{d^2}{d\rho^2} + k_1^2\right]\left(k_1\rho\int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho')G(\rho, \rho')d\rho'\right) = -i\omega\varepsilon_1 E_{0\rho}(\rho) + i\omega\varepsilon_1 z_i J(\rho). \tag{4}$$

The structure of Equation (4) coincides with that of widely used Pocklington equation in the theory of thin wire antennas [12, 13].

3. EQUATION SOLUTION FOR THE CURRENT BY THE METHOD OF CONSISTENT ITERATIONS

The singularity of the quasi-stationary type in the kernel of the integral Equation (4) can be isolated as in [12, 13], using the following identity transformations

$$\int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho')G(\rho, \rho')d\rho' = J(\rho)\Omega(\rho) + \int\limits_{\tilde{R}}^{\tilde{R}+L}\left[J(\rho')G(\rho, \rho') - \frac{J(\rho)}{R(\rho, \rho')}\right]d\rho', \tag{5}$$

where $R(\rho, \rho') = \sqrt{(\rho - \rho')^2 + r^2}$, $\Omega(\rho) = \int\limits_{\tilde{R}}^{\tilde{R}+L}\frac{d\rho'}{\sqrt{(\rho-\rho')^2+r^2}} = \ln\left[\frac{\sqrt{(L-\rho)^2+r^2}+(L-\rho)}{\sqrt{\rho^2+r^2}-\rho}\right]$, and the mean integral value is equal $\bar{\Omega}(\rho) = 2\ln(L/r) - 0.614$. Let us define a functional

$$F\left[k_1\rho, J(\rho)\right] = \left[\frac{d^2}{d\rho^2} + k_1^2\right]\left(k_1\rho\int\limits_{\tilde{R}}^{\tilde{R}+L}\left[J(\rho')G(\rho, \rho') - \frac{J(\rho)}{R(\rho, \rho')}\right]d\rho'\right), \tag{6}$$

and a small parameter $\alpha = -1/\bar{\Omega}(\rho) \approx \frac{1}{2\ln(r/L)}$. Then Equation (4) can be represented as

$$\left[\frac{d^2}{d\rho^2} + k_1^2\right]\left[k_1\rho J(\rho)\right] = i\omega\varepsilon_1\alpha E_{0\rho}(\rho) - i\omega\varepsilon_1\alpha z_i J(\rho) - \alpha F\left[k_1\rho, J(\rho)\right]. \tag{7}$$

Using notation $\tilde{\tilde{k}}_1 = k_1\sqrt{1 + i\alpha\omega\varepsilon_1 z_i/k_1} = k_1\sqrt{1 + i2\alpha\bar{Z}_S/(\mu_1 kr)}$, where $\bar{Z}_S = \bar{R}_S + i\bar{X}_S$ is the normalized (on 120π Ohm) surface impedance, and the equation for the current on the impedance vibrator can be written as

$$\left[\frac{d^2}{d\rho^2} + \tilde{\tilde{k}}_1^2\right]\left[k_1\rho J(\rho)\right] = \alpha\left\{i\omega\varepsilon_1 E_{0\rho}(\rho) + \tilde{F}\left[k_1\rho, J(\rho)\right]\right\}, \tag{8}$$

where $\tilde{F}[k_1\rho, J(\rho)] = i\omega\varepsilon_1 z_i(\rho-1)J(\rho) - F[k_1\rho, J(\rho)]$. Inverting the operator on the left side of Equation (8), we obtain the general solution for arbitrary extraneous sources $E_{0\rho}(\rho)$:

$$\begin{aligned} k_1\rho J(\rho) &= C_1 \sin\left(\tilde{\tilde{k}}_1\rho\right) + C_2 \cos\left(\tilde{\tilde{k}}_1\rho\right) \\ &\quad - \frac{i\omega\varepsilon_1\alpha}{2\tilde{\tilde{k}}_1}\left\{\int_{\tilde{R}}^{\rho} E_{0\rho}(\rho')\sin\left[\tilde{\tilde{k}}_1(\rho-\rho')\right]d\rho' - \int_{\rho}^{\tilde{R}+L} E_{0\rho}(\rho')\sin\left[\tilde{\tilde{k}}_1(\rho-\rho')\right]d\rho'\right\} \\ &\quad + \frac{\alpha}{2\tilde{\tilde{k}}_1}\left\{\int_{\tilde{R}}^{\rho} \tilde{F}\left[k_1\rho, J(\rho)\right]\sin\left[\tilde{\tilde{k}}_1(\rho-\rho')\right]d\rho' - \int_{\rho}^{\tilde{R}+L} \tilde{F}\left[k_1\rho, J(\rho)\right]\sin\left[\tilde{\tilde{k}}_1(\rho-\rho')\right]d\rho'\right\}, \end{aligned} \tag{9}$$

where C_1 and C_2 are arbitrary constants determined from the boundary conditions at the ends of the monopole, and $E_{0\rho}(\rho)$ is exciting field. In accordance with the method of successive iterations [12], as a zero approximation for the vibrator current $J_0(\rho)$, we choose from solution (9) the following expression:

$$\begin{aligned} k_1\rho J_0(\rho) &= C_1 \sin\left(\tilde{\tilde{k}}_1\rho\right) + C_2 \cos\left(\tilde{\tilde{k}}_1\rho\right) \\ &\quad - \frac{i\omega\varepsilon_1\alpha}{2\tilde{\tilde{k}}_1}\left\{\int_{\tilde{R}}^{\rho} E_{0\rho}(\rho')\sin\left[\tilde{\tilde{k}}_1(\rho-\rho')\right]d\rho' - \int_{\rho}^{\tilde{R}+L} E_{0\rho}(\rho')\sin\left[\tilde{\tilde{k}}_1(\rho-\rho')\right]d\rho'\right\}. \end{aligned} \tag{10}$$

Consider excitation of vibrators by a point voltage δ-generator $E_{0\rho}(\rho) = V_0\delta(\rho-\tilde{R})$, located at the base of the monopole. Here V_0 is amplitude of the extraneous field, and δ is the delta function. Then, the zero approximation, according to (10), for the of the monopole current can be written as

$$k_1\rho J_0(\rho) = C_1 \sin\left(\tilde{\tilde{k}}_1\rho\right) + C_2 \cos\left(\tilde{\tilde{k}}_1\rho\right) - \frac{i\omega\varepsilon_1\alpha V_0}{\tilde{\tilde{k}}_1}\sin\tilde{\tilde{k}}_1\left(\rho-\tilde{R}\right) \tag{11}$$

or

$$J_0(\rho) = \tilde{C}_1 j_0\left(\tilde{\tilde{k}}_1\rho\right) + \tilde{C}_2 y_0\left(\tilde{\tilde{k}}_1\rho\right) - \frac{i\omega\varepsilon_1\alpha V_0}{k_1}\frac{\sin\tilde{\tilde{k}}_1(\rho-\tilde{R})}{\tilde{\tilde{k}}_1\rho}, \tag{12}$$

where $j_0(\tilde{\tilde{k}}_1\rho)$ and $y_0(\tilde{\tilde{k}}_1\rho)$ are the zero order spherical functions equal to

$$j_0\left(\tilde{\tilde{k}}_1\rho\right) = \frac{\sin\left(\tilde{\tilde{k}}_1\rho\right)}{\tilde{\tilde{k}}_1\rho}, \quad y_0\left(\tilde{\tilde{k}}_1\rho\right) = -\frac{\cos\left(\tilde{\tilde{k}}_1\rho\right)}{\tilde{\tilde{k}}_1\rho}. \tag{13}$$

Using the boundary condition $J_0(\tilde{R}+L) = 0$ we can find C_2

$$C_2 = C_1 tg\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] + \frac{\alpha i\omega\varepsilon_1 V_0}{k_1}\cdot\frac{\sin\left(\tilde{\tilde{k}}_1 L\right)}{\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]}. \tag{14}$$

Squiggles over the constants are omitted. After substitution (14) into (12) we obtain

$$\begin{aligned} J_0(\rho) &= C_1\left[j_0\left(\tilde{\tilde{k}}_1\rho\right) + tg\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] y_0\left(\tilde{\tilde{k}}_1\rho\right)\right] \\ &\quad + \frac{\alpha i\omega\varepsilon_1 V_0}{k_1}\cdot\left\{\frac{\sin\left(\tilde{\tilde{k}}_1 L\right)}{\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]} y_0\left(\tilde{\tilde{k}}_1\rho\right) - \frac{\sin\tilde{\tilde{k}}_1\left(\rho-\tilde{R}\right)}{\tilde{\tilde{k}}_1\rho}\right\}. \end{aligned} \tag{15}$$

The boundary condition $J_0(\tilde{R}+L) = 0$ is satisfied in expression (15) for arbitrary values of C_1, which can be found from the condition of the monopole excitation at the point of its contact with the conducting sphere. At this point, by virtue of the continuity, the equality $\operatorname{div}[k_1\rho J_0(\rho)]|_{\rho=\tilde{R}} = 0$ should hold. The product $k_1 \rho J_0(\rho) = J_{act}(\rho)$ should be considered as the current active value. Then after identical transformations we obtain

$$C_1 = -\frac{\alpha i\omega\varepsilon_1 V_0}{k_1} \frac{\tilde{\tilde{k}}_1 \tilde{R}\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] + \sin\left(\tilde{\tilde{k}}_1 L\right)\left[2\cos\left(\tilde{\tilde{k}}_1\tilde{R}\right) - \tilde{\tilde{k}}_1\tilde{R}\sin\left(\tilde{\tilde{k}}_1\tilde{R}\right)\right]}{2\sin\left(\tilde{\tilde{k}}_1 L\right) - \tilde{\tilde{k}}_1\tilde{R}\cos\left(\tilde{\tilde{k}}_1 L\right)}. \tag{16}$$

Thus, the expression for the monopole current can be represented in the form convenient for numerical calculations

$$J_0(\rho) = -\frac{\alpha i\omega\varepsilon_1 V_0}{k_1}\left\{C_j j_0\left(\tilde{\tilde{k}}_1\rho\right) + C_y y_0\left(\tilde{\tilde{k}}_1\rho\right) + \frac{\sin\left[\tilde{\tilde{k}}_1\left(\tilde{R}-\rho\right)\right]}{\tilde{\tilde{k}}_1\rho}\right\}, \tag{17}$$

where

$$C_j = \frac{\tilde{\tilde{k}}_1 \tilde{R}\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] + \sin\left(\tilde{\tilde{k}}_1 L\right)\left[2\cos\left(\tilde{\tilde{k}}_1\tilde{R}\right) - \tilde{\tilde{k}}_1\tilde{R}\sin\left(\tilde{\tilde{k}}_1\tilde{R}\right)\right]}{2\sin\left(\tilde{\tilde{k}}_1 L\right) - \tilde{\tilde{k}}_1\tilde{R}\cos\left(\tilde{\tilde{k}}_1 L\right)},$$

$$C_y = \frac{\tilde{\tilde{k}}_1 \tilde{R}\sin\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] + \sin\left(\tilde{\tilde{k}}_1 L\right)\left[2\sin\left(\tilde{\tilde{k}}_1\tilde{R}\right) + \tilde{\tilde{k}}_1\tilde{R}\cos\left(\tilde{\tilde{k}}_1\tilde{R}\right)\right]}{2\sin\left(\tilde{\tilde{k}}_1 L\right) - \tilde{\tilde{k}}_1\tilde{R}\cos\left(\tilde{\tilde{k}}_1 L\right)}.$$

Following from (17), the solution for the current in the impedance monopole is valid for both resonant ($|\tilde{\tilde{k}}_1 L| = n\frac{\pi}{2}$, $n = 1, 2, \ldots$) and nonresonant ($|\tilde{\tilde{k}}_1 L| \neq n\frac{\pi}{2}$) vibrators, i.e., for radiators of arbitrary electrical length.

With regard to (13), expression (17) can be represented as

$$J_0(s) = \frac{\alpha i\omega\varepsilon_1 V_0}{k_1}\left\{\frac{\sin\left(\tilde{\tilde{k}}_1 s\right) - C_j\sin\left[\tilde{\tilde{k}}_1\left(\tilde{R}+s\right)\right] + C_y\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+s\right)\right]}{\tilde{\tilde{k}}_1\left(\tilde{R}+s\right)}\right\}, \tag{18}$$

where, for ease of comparison, transition to local coordinate $s \in [0; L]$ was made. The structure of (18) coincides with the trinomial formula of King and Wu defining the current on the thin impedance vibrator in the free space [14] (with identical expressions for $\tilde{\tilde{k}}$)

$$J(s) = -\alpha_K V_0 \frac{i\omega}{2\tilde{\tilde{k}}} \frac{\sin\tilde{\tilde{k}}\left(L-|s|\right) + F_{K1}\left(\cos\tilde{\tilde{k}}s - \cos\tilde{\tilde{k}}L\right) + F_{K2}\left(\cos\frac{ks}{2} - \cos\frac{kL}{2}\right)}{\cos\tilde{\tilde{k}}L},$$

where α_K is a small parameter, and the coefficients F_{K1} and F_{K2} are found approximately by converting Pocklington integral equation for the Hallen linearized equation [12] and using the properties of its kernel. As can be seen, the formula King-Wu requires an alternative current presentation for resonant and nonresonant vibrators.

4. RADIATION FIELDS OF THE RADIAL IMPEDANCE VIBRATOR ON THE PERFECTLY CONDUCTING SPHERE

The actual current distribution (17) allows calculation of all electrodynamic characteristics of the impedance vibrator on the sphere. In accordance with our model of a spherical surface antenna (Fig. 1) consisting of the impedance vibrator and a metal spherical scatterer, its total radiation field is determined by the radial component of the electric Hertz pseudovector $\tilde{\Pi}_\rho(\vec{r}) = k_1\rho\Pi_\rho(\vec{r})$. To find the fields, the current distribution $J(\rho) = J_0(\rho)$ along the impedance monopole in the form (17) should be substituted into the formula (1). Then, the expressions for the total field components based on the use of rotary formulas will be:

$$\begin{gathered}
E_\rho(\vec{r}) = \frac{k_1\partial^2\left[\rho\Pi_\rho(\vec{r})\right]}{\partial\rho^2} + k_1^3\left[\rho\Pi_\rho(\vec{r})\right], \quad E_\theta(\vec{r}) = \frac{k_1}{\rho}\frac{\partial^2\left[\rho\Pi_\rho(\vec{r})\right]}{\partial\rho\partial\theta}, \quad E_\varphi(\vec{r}) = \frac{k_1}{\rho\sin\theta}\frac{\partial^2\left[\rho\Pi_\rho(\vec{r})\right]}{\partial\rho\partial\varphi}, \\
H_\rho(\vec{r}) = 0, \quad H_\theta(\vec{r}) = \frac{ikk_1\varepsilon_1}{\sin\theta}\frac{\partial\Pi_\rho(\vec{r})}{\partial\varphi}, \quad H_\varphi(\vec{r}) = -ikk_1\varepsilon_1\frac{\partial\Pi_\rho(\vec{r})}{\partial\theta}.
\end{gathered} \tag{19}$$

Formulas (19) allow us to find the radiation electromagnetic field at any distance from the antenna, i.e., at arbitrary $\rho \geq \tilde{R}$. For homogeneous lossless media ε_1 has the real value, and the formulas (19) for the antenna far field ($\rho \gg \lambda$) can be simplified, since the terms proportional to coefficient $1/\rho^2$ may be omitted.

As an example, we derive the explicit expression for the magnetic field components of spherical antenna radiation by substituting the expressions (1) and (17) into formula (19)

$$\begin{aligned} H_\theta(\vec{r}) &= \frac{kk_1}{\omega \sin\theta} \int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho') \frac{\partial}{\partial \varphi} G^e_{\rho\rho'}(\rho, \theta, \varphi;\ \rho', \theta_0, \varphi_0) d\rho', \\ H_\varphi(\vec{r}) &= -\frac{kk_1}{\omega} \int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho') \frac{\partial}{\partial \theta} G^e_{\rho\rho'}(\rho, \theta, \varphi;\ \rho', \theta_0, \varphi_0) d\rho'. \end{aligned} \tag{20}$$

Since in this case $\rho > \rho'$, we obtain

$$\begin{aligned} H_\theta(\vec{r}) &= -\frac{kk_1}{\omega \sin\theta} \sum_{n=0}^{\infty} \frac{n+1/2}{2\pi} k_1 h_n^{(2)}(k\rho) \frac{\partial P_n(u)}{\partial \varphi} \int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho') \left[h_n^{(2)}(k_1\rho') Q_n \left[y_n\left(k_1 \tilde{R}\right) \right] - y_n\left(k_1\rho'\right) \right] d\rho', \\ H_\varphi(\vec{r}) &= \frac{kk_1}{\omega} \sum_{n=0}^{\infty} \frac{n+1/2}{2\pi} k_1 h_n^{(2)}(k_1\rho) \frac{\partial P_n(u)}{\partial \theta} \int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho') \left[h_n^{(2)}(k_1\rho')\, Q_n \left[y_n\left(k_1 \tilde{R}\right) \right] - y_n(k_1\rho') \right] d\rho', \end{aligned} \tag{21}$$

where $\frac{\partial P_n(u)}{\partial \varphi} = \frac{n+1}{u^2-1}[P_{n+1}(u) - uP_n(u)]\frac{du}{d\varphi}$, $\frac{\partial P_n(u)}{\partial \theta} = \frac{n+1}{u^2-1}[P_{n+1}(u) - uP_n(u)]\frac{du}{d\theta}$.

In the far-field region expression (21) can be easily converted, since for $k_1\rho \to \infty$ and $|k_1\rho| \gg n$ the spherical Hankel functions of the second kind have the known asymptotic $h_n^{(2)}(k_1\rho) \approx (i)^{n+1} \frac{e^{-ik_1\rho}}{k_1\rho}$.

If the vibrator radiator is oriented along $\{0x\}$ axis, i.e., if $\varphi' = 0$ and $\theta' = \pi/2$, we obtain $u = \sin\theta \cos\varphi$. Than the field in the equatorial plane ($\theta = \pi/2$) can be written as:

$$\begin{aligned} H_\theta(\vec{r}) = &- \frac{kk_1}{\omega \sin\varphi} \frac{e^{-ik\rho}}{\rho} \sum_{n=1}^{\infty} \frac{n+1/2}{2\pi} (i)^{n+1} (n+1) \left[P_{n+1}(\cos\varphi) - P_n(\cos\varphi)\cos\varphi \right] \\ &\times \int\limits_{\tilde{R}}^{\tilde{R}+L} J(\rho') \left[h_n^{(2)}\left(k_1\rho'\right) Q_n[y_n(k_1\tilde{R})] - y_n\left(k_1\rho'\right) \right] d\rho', \\ H_\varphi(\vec{r}) = &\ 0. \end{aligned} \tag{22}$$

Summation in formula (22) begins with $n = 1$, since for $n = 0$ the polynomial difference in brackets is zero.

5. INPUT IMPEDANCE OF THE RADIAL VIBRATOR ON THE SPHERE

To find the input impedance of the monopole $Z_{in} = R_{in} + iX_{in}$ or the input admittance $Y_{in} = 1/Z_{in} = G_{in} + iB_{in}$ at the point of voltage supply, the well-known relation can be used

$$Z_{in}[O_M] = V_{act}\left(\tilde{R}\right) / J_{act}\left(\tilde{R}\right). \tag{23}$$

Of course, operating voltage in (23) is defined by the formula $V_{act}(\tilde{R}) = V_0$. However, it is known from the literature [1, 12] that the use of the zero approximation for the current does not always provide the required accuracy of input impedance calculation for the dipole radiators as opposed to their integral characteristics. On the other hand, obtaining analytical formulas for the subsequent approximations by the method of successive iterations may be very cumbersome. Therefore, it is appropriate to define the input impedance of the vibrator by the method of induced EMF with basis functions defined as zero-order approximation for the current. Now let us solve the integral Equation (4) by this method.

On the basis of the expressions (15), using formula (13), after identity transformations, we obtain

$$J_0(\rho) = C_1 \frac{\sin\left[\tilde{\tilde{k}}_1\left(\rho - \left(\tilde{R}+L\right)\right)\right]}{\tilde{\tilde{k}}_1\rho\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]} - \frac{\alpha i\omega\varepsilon_1 V_0\left[\cos\left(\tilde{\tilde{k}}_1\rho\right)\sin\left(\tilde{\tilde{k}}_1 L\right)+\sin\left[\tilde{\tilde{k}}_1\left(\tilde{R}-\rho\right)\right]\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]\right]}{k_1\tilde{\tilde{k}}_1\rho\cos\left[\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]}. \tag{24}$$

Let us call expression (24) by improved zero approximation, and leave only the first term in (24), as the basis function. Thus, we exclude terms proportional to the small parameter α, until constant C_1 will be determined. Then we combine the multiplier $\cos[\tilde{\tilde{k}}_1(\tilde{R}+L)]$ in the denominator and the unknown quantity J_0 and get

$$J(\rho) = J_0 f(\rho) = J_0 \frac{\sin\left[\tilde{\tilde{k}}_1\left(\rho - \left(\tilde{R}+L\right)\right)\right]}{\tilde{\tilde{k}}_1\rho}. \tag{25}$$

Then the current amplitude, according to the method of induced EMF, is $J_0 = F/Z_\Sigma$, where $F = -\frac{i\omega\varepsilon_1 V_0}{2k_1^2}\int_{\tilde{R}}^{\tilde{R}+L} f(\rho)\delta(\rho-\tilde{R})\,d\rho|_{E_{0\rho}(\rho)=V_0\delta(\rho-\tilde{R})} = -\frac{i\omega\varepsilon_1 V_0\sin(\tilde{\tilde{k}}_1 L)}{2k_1^2(\tilde{\tilde{k}}_1\tilde{R})}$, and Z_Σ is dimensionless coefficient defined by the expression

$$\begin{aligned}
Z_\Sigma =& \frac{i\varepsilon_1\bar{Z}_S}{\sqrt{\varepsilon_1\mu_1}\left(\tilde{\tilde{k}}_1 r\right)}\left\{\begin{array}{l} -\frac{\sin^2\left(\tilde{\tilde{k}}_1 L\right)}{\left(\tilde{\tilde{k}}_1\tilde{R}\right)} - \cos\left[2\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]\left[Si\left[2\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] - Si\left(2\tilde{\tilde{k}}_1\tilde{R}\right)\right] \\ +\sin\left[2\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right]\left[Ci\left[2\tilde{\tilde{k}}_1\left(\tilde{R}+L\right)\right] - Ci\left(2\tilde{\tilde{k}}_1\tilde{R}\right)\right]\end{array}\right\} \\
&+\left(k_1^2-\tilde{\tilde{k}}_1^2\right)\int_{\tilde{R}}^{\tilde{R}+L}\rho f(\rho)\left(\int_{\tilde{R}}^{\rho} f(\rho')G_2(\rho,\rho')d\rho' + \int_{\rho}^{\tilde{R}+L} f(\rho')G_1(\rho,\rho')d\rho'\right)d\rho \\
&+\frac{2\sin\left(\tilde{\tilde{k}}_1 L\right)}{\left(\tilde{\tilde{k}}_1\tilde{R}\right)}\int_{\tilde{R}}^{\tilde{R}+L} f(\rho')G_1(\tilde{R},\rho')d\rho' - \frac{2}{\tilde{\tilde{k}}_1}\int_{\tilde{R}}^{\tilde{R}+L}\frac{\left(\tilde{\tilde{k}}_1\rho\right)\cos\left[\tilde{\tilde{k}}_1\left(\rho-(\tilde{R}+L)\right)\right]-\sin\left[\tilde{\tilde{k}}_1\left(\rho-(\tilde{R}+L)\right)\right]}{\rho^2} \\
&\times\left(\int_{\tilde{R}}^{\rho} f(\rho')G_2(\rho,\rho')d\rho' + \int_{\rho}^{\tilde{R}+L} f(\rho')G_1(\rho,\rho')d\rho'\right)d\rho - \int_{\tilde{R}}^{\tilde{R}+L} f(\rho')G_2(\tilde{R}+L,\rho')d\rho' \\
&+\frac{\sin\left(\tilde{\tilde{k}}_1 L\right)}{\tilde{\tilde{k}}_1}\int_{\tilde{R}}^{\tilde{R}+L} f(\rho')\left.\frac{dG_1(\rho,\rho')}{d\rho}\right|_{\rho=\tilde{R}}d\rho' + \cos\left(\tilde{\tilde{k}}_1 L\right)\int_{\tilde{R}}^{\tilde{R}+L} f(\rho')G_1(\tilde{R},\rho')d\rho'.
\end{aligned} \tag{26}$$

Here $Si(z) = \int_0^z \frac{\sin z}{z}dz$ and $Ci(z) = -\int_{-\infty}^{z}\frac{\cos z}{z}dz$ are integral sine and cosine functions of complex argument;

$$G_1(\rho,\rho') = -k_1\sum_{n=0}^{\infty}(2n+1)h_n^{(2)}(k_1\rho')\left\{h_n^{(2)}(k_1\rho)Q_n\left[y_n(k_1\tilde{R})\right] - y_n(k_1\rho)\right\}P_n\left(\cos\frac{r}{\tilde{R}+L/2}\right);$$

$$G_2(\rho,\rho') = -k_1\sum_{n=0}^{\infty}(2n+1)h_n^{(2)}(k_1\rho)\left\{h_n^{(2)}(k_1\rho')Q_n\left[y_n(k_1\tilde{R})\right] - y_n(k_1\rho')\right\}P_n\left(\cos\frac{r}{\tilde{R}+L/2}\right);$$

$$\left.\frac{dG_1(\rho,\rho')}{d\rho}\right|_{\rho=\tilde{R}} = -k_1^2\sum_{n=0}^{\infty}(2n+1)h_n^{(2)}(k_1\rho')P_n\left(\cos\frac{r}{\tilde{R}+L/2}\right)$$
$$\times\left\{\left(\frac{n}{k_1\tilde{R}}h_n^{(2)}(k_1\tilde{R})-h_{n+1}^{(2)}(k_1\tilde{R})\right)Q_n\left[y_n(k_1\tilde{R})\right]-\left(\frac{n}{k_1\tilde{R}}y_n^{(2)}(k_1\tilde{R})-y_{n+1}^{(2)}(k_1\tilde{R})\right)\right\}.$$

Thus, using the formula (23), we obtain the expression for the input impedance of the radial impedance monopole on the sphere

$$Z_{in} = \frac{30i\tilde{\tilde{k}}_1\left(\tilde{\tilde{k}}_1\tilde{R}\right)Z_\Sigma}{\varepsilon_1 k_1\sin^2\left(\tilde{\tilde{k}}_1 L\right)}\,[\text{Ohm}]. \tag{27}$$

The voltage standing-wave ratio in the antenna feeder line is $\text{VSWR} = \frac{1+|S_{11}|}{1-|S_{11}|}$, where $S_{11} = \frac{Z_{in}-W}{Z_{in}+W}$ is reflection coefficient in the feeder, and W is wave resistance of the feeder line. Due to imperfections of monopole excitation model, under the condition $(1/k_1\tilde{R}) \ll 1$ the calculation results of the input impedance may be improved by using relations obtained in [7]. Then the expression (25) takes the form

$$J(\rho) = J_0 f(\rho) = J_0\left\{\frac{\sin\left[\tilde{\tilde{k}}_1(\rho-(\tilde{R}+L))\right]}{\tilde{\tilde{k}}_1\rho}+\frac{\cos\left[\tilde{\tilde{k}}_1(\rho-(\tilde{R}+L))\right]}{\tilde{\tilde{k}}_1\rho}\left[\frac{1}{\left[\tilde{\tilde{k}}_1(\tilde{R}+L)\right]}-\frac{1}{\tilde{\tilde{k}}_1\rho}\right]\right\}. \tag{28}$$

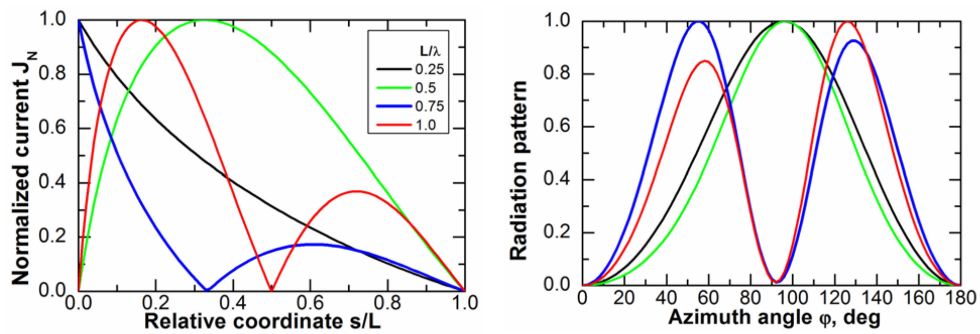
To exclude the singularity defined by $\sin(\tilde{\tilde{k}}_1 L) = 0$ in the denominator of (27), numerical calculation should be performed adding a small losses ($\bar{R}_S \approx 0.0001$) into $\tilde{\tilde{k}}_1$.

6. NUMERICAL RESULTS

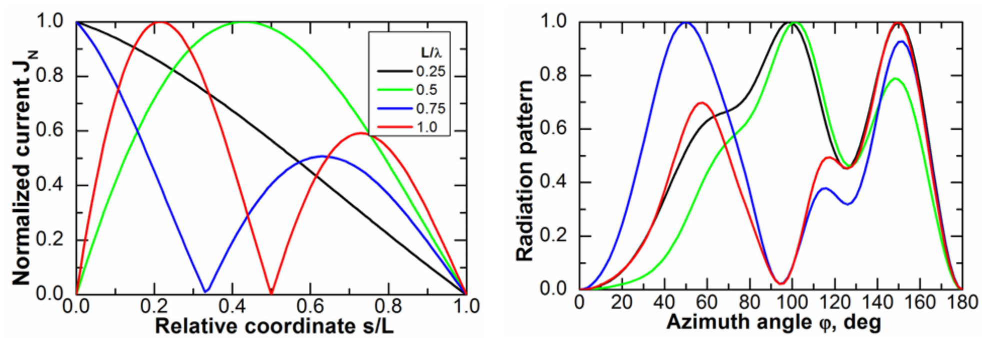
Various vibrator characteristics, such as radiation fields in any observation zone, can be determined using function $\sin[k_1(L-|s|)]$ approximating current along the cylindrical vibrator [1, 12, 13]. If required quantities are integral characteristics of the current distribution function, small approximation errors do not give significant contributions to the result. However, one must keep in mind that the vector potential for the spherical antenna at a given point on the monopole is determined not only by the local value of the current at this point, but by cumulative effects of the currents induced on other parts of the monopole and spherical scatterer. This fact allows us to consider a system consisting of a monopole and perfectly conducting sphere as a spherical antenna. Thus, it can be expected that the resulting current distribution (17) can differ appreciably from the sinusoidal distribution. This difference is the greater, the smaller diffraction radius of the sphere $k\tilde{R}$, i.e., the greater is the difference in interaction between the monopole and scatterer as compared with that between the monopole and infinite screen. This fact is true for both perfectly conducting and impedance monopoles. Formulas for calculating the vibrator surface impedances for various geometric and electrical parameters are given in [1].

Figure 2 shows the calculation results for the normalized current distribution on the radial perfectly conducting monopole located on the conducting sphere and the radiation pattern (RP) by power in the equatorial plane $\theta = \pi/2$ for this antenna. The calculations were made using the formulas (17) and (22) for the local longitudinal coordinate $s = \rho - \tilde{R}$ and parameters $\bar{R}_S = 0.0001$, $r/\lambda = 0.0033$ and $\lambda = 10\,\text{cm}$. Analysis of the plots shown in Fig. 2, confirms the assumption that the spherical scatterer of the small and resonant diffraction radii has a greater influence upon the current distribution on the monopole than that of the larger electrical size. For small spheres, such influence leads to shortening of the monopole electrical length. When the monopole length is varied, the RP of the spherical antenna changes in a greater extent if spheres have small diffraction radii. For example, for the sphere radius $\tilde{R} = 0.1\lambda$ both maximum of the single-lobed RP for the quarter-wave monopole and minimum for two-lobed RP for monopoles $0.75 \le L \le \lambda$ can be observed in the direction $\varphi = \pi/2$. In the latter case, the form of RP is defined by anti-phase areas in the current distribution.

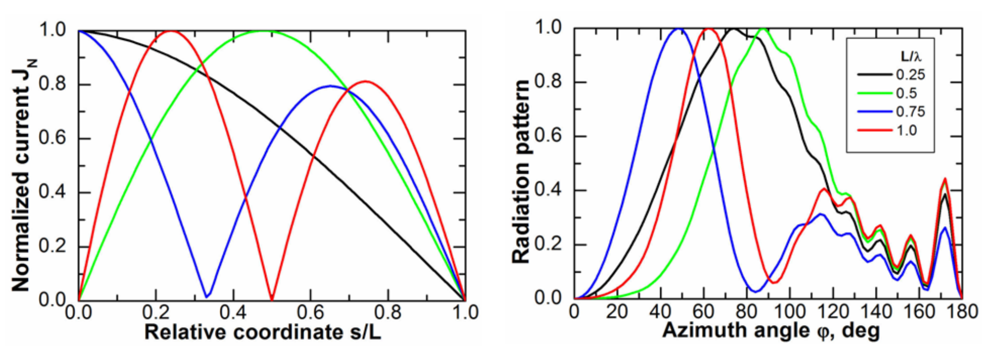
Figure 3 shows the normalized current distributions and the RP for the quarter-wave monopole, $L = \lambda/4$, characterized by surface impedance ($\bar{Z}_S = i\bar{X}_S$) of inductive ($\bar{X}_S > 0$) or capacitive ($\bar{X}_S < 0$)



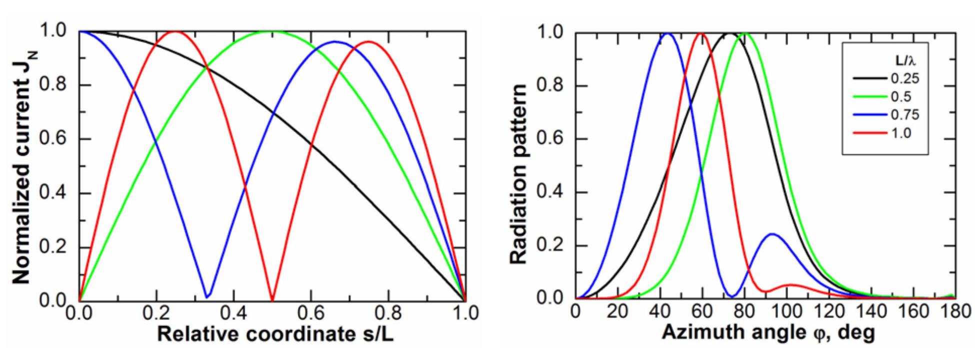
(a)



(b)



(c)



(d)

Figure 2. The current distributions and RP for the spherical antennas: (a) $\tilde{R} = 0.1\lambda$ ($k\tilde{R} = 0.2\pi$); (b) $\tilde{R} = 0.5\lambda$ ($k\tilde{R} = \pi$); (c) $\tilde{R} = 2\lambda$ ($k\tilde{R} = 4\pi$), (d) $\tilde{R} = 12\lambda$ ($k\tilde{R} = 24\pi$).

type. The diffraction radius of the spherical antenna is $k\tilde{R} = \pi$. As can be seen from the plots, there exist fundamental differences between the resonant characteristics for perfectly conducting and impedance monopoles. If the distributed surface impedance is used, the resonant length of impedance

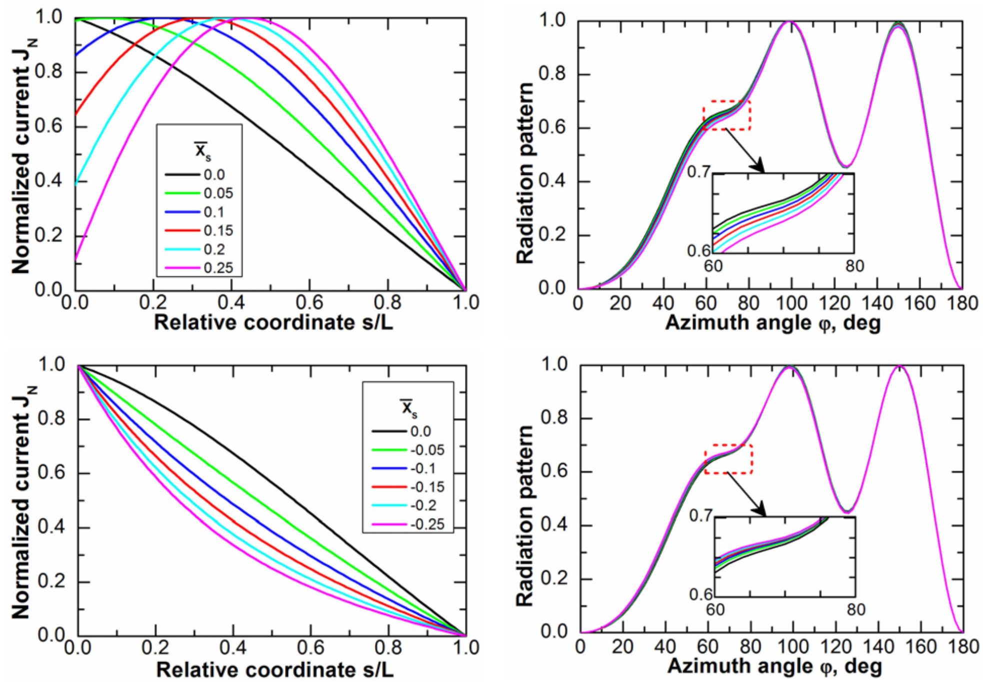


Figure 3. The current distributions and RP for impedance monopoles: $L = \lambda/4$, $k\tilde{R} = \pi$.

monopoles can be lesser or greater $\lambda/4$, i.e., the monopole shortening for $\bar{X}_S < 0$ or lengthening for ($\bar{X}_S > 0$) is observed.

Note that the monopole lengthening defined by the inductive impedance can be compensated by influence of the spherical scatterer. Variation in current distribution caused by the impedance monopole upon the antenna pattern is very small, since the physical length of the monopole remains constant when its electrical length varies. The calculation results show that these trends remain similar for spherical scatterers of any dimension.

Now let us pay attention to contribution of the spherical scatterer upon the antenna RP. To do this, let us analyze the basic principles of RP formation for the spherical antenna in free space. If $k\tilde{R}$, the diffraction radius of the sphere is increased, the RP for the magnetic field component $H_\theta(\vec{r})$ in the equatorial plane according to expression (22) becomes irregular and multi-lobed. As can be seen from Fig. 4, the amplitude oscillations occurs mainly in the geometrical shadow region, $\varphi > \pi/2$ if $\varphi_0 = 0$.

The observed oscillations may be explained by the fact that the waves propagating along the spherical scatterer surface arrive into shadow zone along the meridians in the forward and backward directions. The oscillations of the field amplitude are the result of interference of these waves. The larger is $k\tilde{R}$, the greater is the number of standing waves on the sphere surface, and the greater is the number of side lobes in the RP. The deepest oscillations are observed near the "dark pole", $\theta = \pi/2$, $\varphi = \varphi_0 + \pi$, where the amplitude of the interfering waves are almost the same. As the distance from the "dark pole" grows larger, the propagation path of the forward wave decreases, and that of the backward wave increases. Therefore, the difference in the degree of damping of these waves increases, and the oscillations amplitude decreases. As expected, screening by the sphere increases if $k\tilde{R}$ grows larger. Thus, if the sphere is large, RP of spherical antenna in the front half-space approaches that of a vertical monopole over an infinite perfectly conducting plane. In the rear half-space the amplitude of radiation field is significantly reduced. The directivity of the spherical antenna has been studied previously in [4, 5] by the eigen-wave method. As seen from Fig. 4, the calculated data agree well with the data presented in these articles. The small discrepancies between the data can be explained by a different number of summation terms in expressions similar to (21) and (22).

The effectiveness of the spherical antenna is defined not only by the RP, but also by its input impedance. Therefore, additional investigation of the antenna input impedance is required. First, the

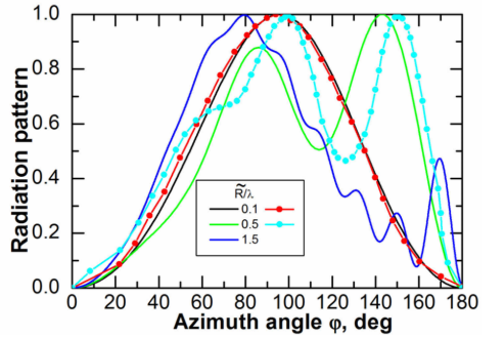


Figure 4. The RP of the spherical antenna for $L = \lambda/4$; the calculation results from [5] are labeled by circles.

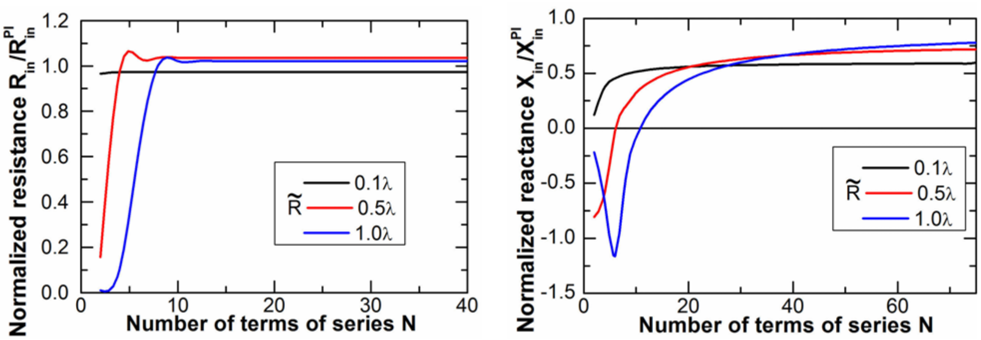


Figure 5. The series convergence in (27) for the monopole input impedance, normalized to the impedance of the monopole over infinite plane $Z_{in} = R_{in}^{PI} + iX_{in}^{PI}$, for $L = \lambda/4$.

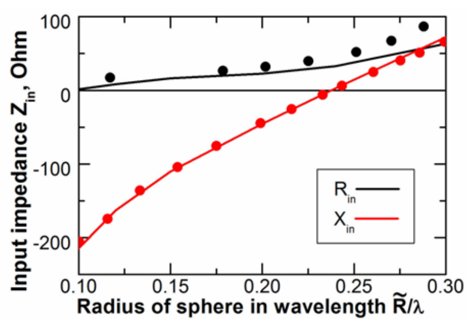


Figure 6. The input impedance of the monopole on a perfectly conducting sphere versus wavelength: $\Omega = 2\ln(L/r) = 8.2$, $L = \tilde{R}$; the results obtained by the method of moments [5] are marked by circles.

convergence of complex impedance $Z_{in} = R_{in}+iX_{in}$ calculation depending on the number N of spherical harmonics in the formula (27) was analyzed. Data for a perfectly conducting quarter-wave monopole are presented in Fig. 5. Quite good convergence achieved in this case depends upon the fact that in (26) the inversion of differential operator was realized, allowing integrability for the singular kernel of the integral Equation (4). Calculating the input impedance $Z_{in} = R_{in} + iX_{in}$ one must perform the summation in Equation (26), and substitute the results into (27). As can be seen from Fig. 5, the stabilization R_{in} and X_{in} are achieved for $N = 40$ and $N = 75$, respectively.

Such a choice of summation terms allows the numerical integration in (26) with required precision not only for the quarter-wave monopoles. This conclusion is confirmed by Fig. 6, where the input impedance of the spherical antenna as a function of wavelength is compared with data obtained by the method of moments [5]. Satisfactory agreement between the results shown in Fig. 6 also confirms the validity of function (25) usage and, in general, the implementation of more effective method of the input impedance calculation as compared with that proposed in [5].

The calculations were carried out so that the equality $L = \tilde{R}$ holds. The wavelength dependence thus obtained allows us to treat the antenna design as a hybrid heterobruchial vibrator which arms are the straight wire segment and the conductive sphere. As seen from Fig. 6, such hybrid vibrator can be resonantly tuned if $L = \tilde{R} \approx 0.235\lambda$. It is self-evident that the spherical antenna with other relationship between L and $\tilde{R}$ may be resonant at other wavelengths. Therefore, the study of wavelength dependence of Z_{in} for the spherical antenna consisting of a fixed length monopole and sphere of variable radius is of practical interest. As seen from Fig. 7, the calculated values of real and imaginary parts of the input impedance for perfectly conducting monopoles have an oscillating character. The amplitude oscillations are decreasing when the sphere radius is increased and the input

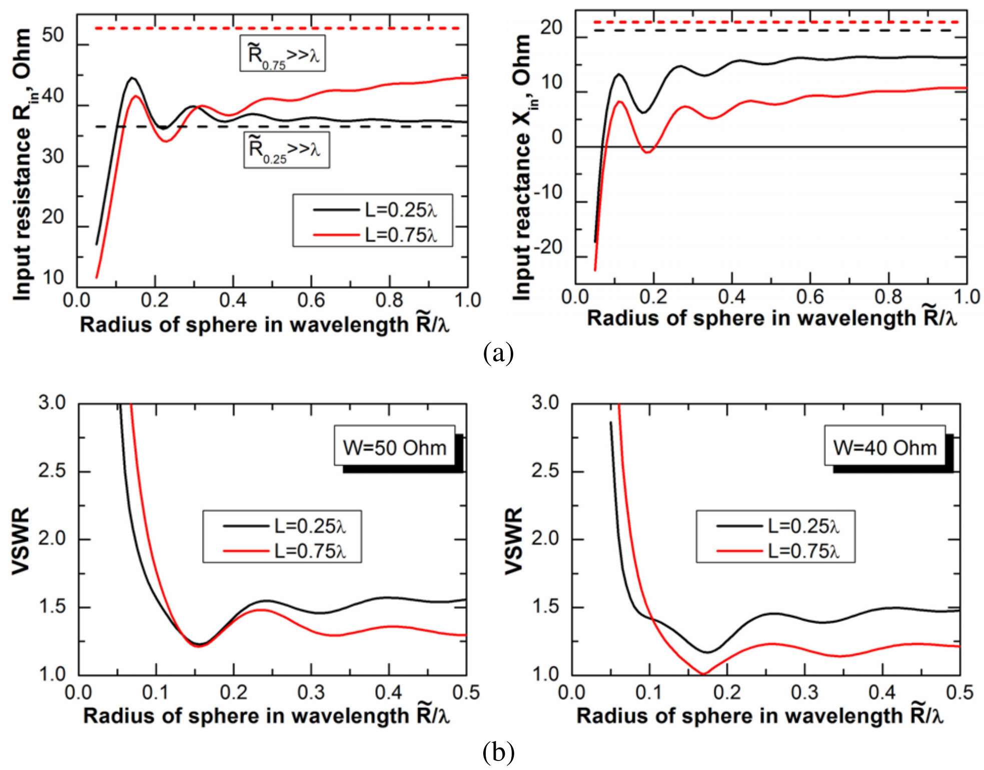


Figure 7. The input impedance and voltage standing-wave ratio (VSWR) for the spherical antenna versus the sphere radius; (a) the dashed lines correspond to the input impedance $Z_{in} = R_{in}^{PI} + iX_{in}^{PI}$ of the monopole over perfect conducting plane.

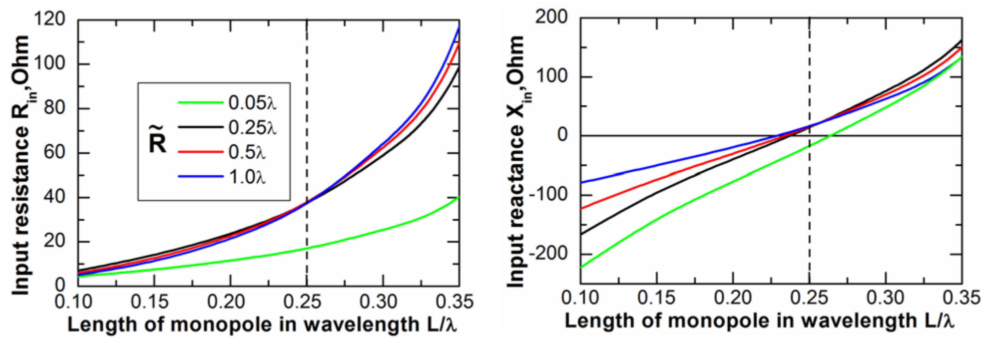


Figure 8. The antenna input impedance versus the monopole electrical length.

impedance asymptotically approaches its limiting value. For example, the input impedance approaches to $Z_{in} = R_{in}^{PI} + iX_{in}^{PI} = 36.5 + i21.25$ [Ohm] and $Z_{in} = 52.75 + i22.75$ [Ohm] corresponding to the input impedances of the quarter-wave and the three quarter-wave monopoles over a perfectly conducting plane, respectively. The plots $Z_{in} = f(\tilde{R}/\lambda)$ for the quarter-wave and three quarter-wave monopoles are significantly different. The three quarter-wave monopole can be resonantly tuned for two different sphere radii, since there exist two points where the imaginary part of the input impedance X_{in} is equal to zero.

Resonant tuning is important for matching the antenna and feeder line. The closer are the values of the wave impedance W of feeder line and the real part of the antenna input impedance R_{in} (when $X_{in} \approx 0$), the better is matching. This assertion is clearly illustrated in Fig. 7(b), where the VSWR in the antenna feeder line is shown for two values of the wave impedance, $W = 50$ Ohm and $W = 40$ Ohm.

The electrical length of the monopole significantly affects the input impedance of the spherical antenna. This is confirmed by direct calculations shown in Fig. 8. The computation has also shown

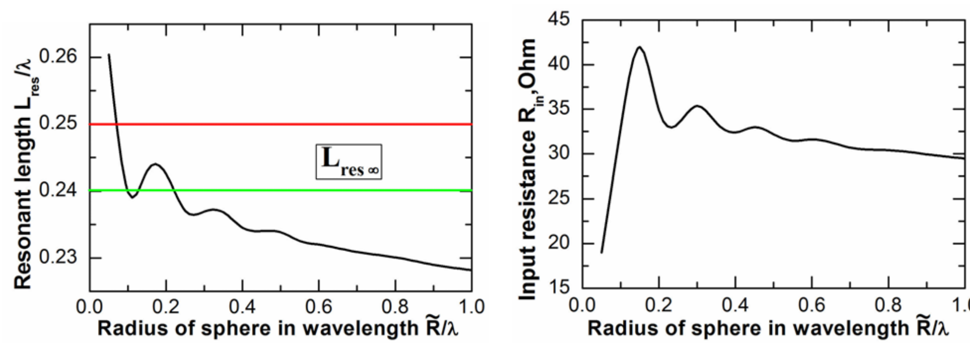


Figure 9. The monopole resonant length and input resistance in resonance of the spherical antenna versus the sphere radius; $L_{res\,\infty}$ corresponds to the monopole over an infinite plane.

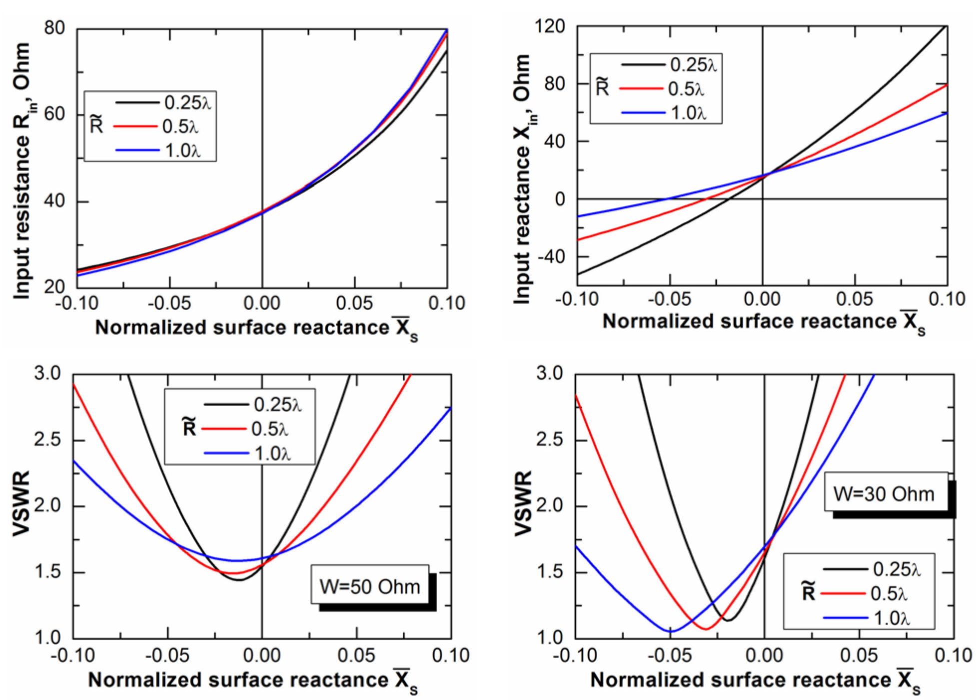


Figure 10. Dependences Z_{in} and VSWR upon the imaginary part of normalized surface impedance for various $\tilde{R}$ and $L = 0.25\lambda$.

that spherical antennas of small diffraction radii $\tilde{R} < 0.1\lambda$ are characterized by lengthening of their resonant length. If the sphere radius is increased up to the limiting case $\tilde{R} \to \infty$, the shortening of the resonant length is observed. The values of the lengthening and shortening span relative to $L = 0.25\lambda$ are comparable (Fig. 9).

The electrical length of the monopole can be varied in limited extent by superimposing the reactive impedance at its surface. Therefore, it is important to estimate the influence of the surface impedance upon Z_{in}. The computation has shown that the value of X_{in} for the spherical antenna can be varied in sufficiently wide limits by varying the imaginary part of the surface impedance $\bar{X}_S$ (Fig. 10). The monopole impedance effectively influences the input impedance X_{in} only for spherical antennas with small diffraction radii. The values of $\bar{X}_S$ necessary for antenna resonant tuning essentially depends upon the sphere radius. For spheres with small diffraction radius, matching between the antenna and feeding line achieved by varying the wave impedance W leads to a narrowing of the VSWR range.

7. CONCLUSION

The physically correct analytical solution of the integral equation for the current in the thin radially oriented impedance monopole located on the sphere was built by the successive iteration method for arbitrary spherical antenna dimensions. The antenna input impedance was determined by the method of induced EMF using zero-order approximation of the solution. The current distribution in the vibrator, radiation fields, and input impedance of the spherical antenna were investigated by numerical simulation. The comparison with the results obtained earlier for the spherical antenna with perfectly conducting vibrator confirmed the accuracy of the calculated data. It was shown that the impedance monopoles on spherical scatterers, may have different resonant wavelengths depending upon the geometric parameters and the value of the monopole surface impedance. On the one hand, controlling the resonant length of the monopole within wide limits is made possible, and on the other hand, allows resonant tuning of the constant length monopole on the sphere of given radius.

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