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A New Hybrid MoM-GEC Asymptotic Method for Electromagnetic Scattering Computation in Waveguides

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Abstract—This paper presents a new hybridization between MoM-GEC and some asymptotic methods. In fact, a new hybrid current function based on Physical Optic (PO) and a modal method is developed. The approach consists in approximating the total current on an invariant metallic pattern on two parts; the inside of metal is governed by PO method; however, the edges are modeled by infinite cylinders and described by Hankel functions (modal method). The considered single test function is required then by MoM method to replace a lot of sinusoidal or triangular test functions, in order to get a rapid convergence and less computational time. For validation purposes, the new developed hybrid approach is applied to compute scattering in different structures. The obtained input impedances, currents and fields distributions are in agreement with those obtained by MoM method. Considerable gain in computational time and memory resources is achieved.

1. INTRODUCTION

Microwave devices applications cover a wide band of the electromagnetic domain and have numerous functions and dimensions according to their frequency range of use. To model such devices and their interactions, there is a set of methods that solves a number of problems, in some scope. Maxwell's equations cannot be analytically solved in most cases. Hence, many modeling methods and various techniques have been developed to solve electromagnetic problems in time or frequency domain. They can be classified as different categories according to their formulation and their analysis domain. Some are called full wave methods such as MoM, FEM [1, 2], FDTD [3–5], TLM [6, 7]. They provide an approximate solution by numerically solving Maxwell's equations, in differential or integral form. Others are called asymptotic methods. They are based on fields such as GO, GTD [8–10], UTD [11, 12] or on currents such as PO [13, 14], PTD [15, 16]. They provide an approximate solution of Maxwell's equations, which tends to the exact solution at very high frequency. These last methods are used to model electrically large objects (objects of very large dimensions compared to the wavelength) without containing fine details. Other so-called hybrid methods are based on one or more coupling between two families of the previously mentioned methods. This allows extending the range of analysis and field of applications of numerical methods.

Full wave methods require a complete spatial meshing of the computational space (surface or volume), joined by a temporal meshing in the case of temporal methods. Then, the analysis space of modeled device will be greatly limited. Full wave methods are also used to model electrically small objects whose dimensions do not exceed a few tens of wavelength. Consequently, we can understand the difficulty in modeling large areas of analysis. Among full wave methods, integral ones are the most suitable to achieve an electromagnetic study of planar microwave structures. Indeed, integral methods can reduce the size of the problem since they write the initial boundary conditions in the form of an integral equation defined on the obstacle's surface. However, when the structure's complexity increases,

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the resolution becomes complicated. The equivalent circuits have been introduced in the development of integral methods formulation with equivalent circuit problems (V, I) instead of field problems (E, H). This constitutes the concept of MoM-GEC that consists of MoM method combined to the Generalized Equivalent Circuit (GEC). Its key idea is the transposition of field problems in GEC which are simpler to treat [17–19, 21, 24–26].

A major feature of this method is reducing the spatial degree size of problems. The study of a volume structure is solved by a surface approach which makes this method particularly suitable for finite areas in infinite medium. Determining the electromagnetic field radiated by an object in a certain volume of calculation is reduced to determine the current on its surface, and this problem requires a surface meshing and not a volume one, so that we can find the electromagnetic field radiated by the structure in the entire space [20]. MoM-GEC is a general method that transforms a functional (differential or integral equation) into a linear equations system that can be solved by matrix techniques [27]. The unknown function f is expressed as a linear combination of known test functions f_n, weighted by unknown coefficients α_n

$$f = \sum \alpha_n f_n \tag{1}$$

The procedure of the MoM-GEC method is based on minimizing the residual error on basis and test functions in order to get the convergence of the solution. Indeed, a judicious choice of test functions can quickly provide the solution while a wrong choice can greatly complicate the problem or not solve it. Generally, sinusoidal, triangular or polynomial test functions are often used to accurately find out the solution. In these cases, it is necessary to use a roughly high number of test functions. As the number of test functions is high, we need a very high number of basis functions to get convergence [21, 25]. This leads to manipulating matrices with great sizes. Consequently, the needed memory resources and computational time to solve such problems will be considerably increased. In this context, numerous researches have been interested in accelerating the resolution of these kinds of problems and alleviating the electromagnetic study of these structures. They are based generally on different methods such as the multi-scale approaches [28–32] and the conjugate gradient [24]. Some others are based on the hybridization between MoM and asymptotic methods, especially PO method (MoM-PO), to achieve various electromagnetic studies such as the study of radiation problems of antenna mounted on a large platform [33] and the analysis of slot arrays enclosed in an electrically large Radome [34]. The MoM-PO has been also used to solve large electromagnetic scattering problems utilizing Characteristic Basic Functions (CBFs), special functions, to reduce the number of unknowns and the size of matrices [35].

The work presented in this paper is directly related to reducing the number of required test functions by using physical approximations based on an hybridization between MoM-GEC method and some asymptotic methods. Thus, we try to develop a single test function to solve a scattering problem. The proposed approach is based on approximating the current test function to obey physical laws satisfying boundary conditions inside metal and on its edges. In fact, we propose to compose the test function on two parts. The first one is the current on metal edges, modeled by infinite cylinders that provide a diffracted field approximated by Hankel functions [36–38]. The second is the current inside metal based on asymptotic approximation by the Physical Optic (PO) method. The developed asymptotic test function will be required in MoM-GEC to replace a lot of triangular or sinusoidal ones. This original idea constitutes an hybridization of MoM-GEC method, PO method and a modal method that approximates the diffracted field on metal edges by Hankel functions.

The paper is presented as follows. In Section 2, we present the problem formulation and the principle of the proposed hybridization. In fact, the concept of MoM-GEC is presented and the new approximated test function developed. In Section 3, numerical results, compared with MoM method, are given and discussed. Finally, benefits guaranteed by the new hybrid approach, to reduce the computational time and the numbers of test and basis functions needed for obtaining the convergence are appreciated.

2. PROBLEM FORMULATION

2.1. Structures of Study

In the beginning, we present the structures on which we apply the new hybrid approach. They are depicted in Figure 1. The first one consists of one microstrip in a cross section of a rectangular

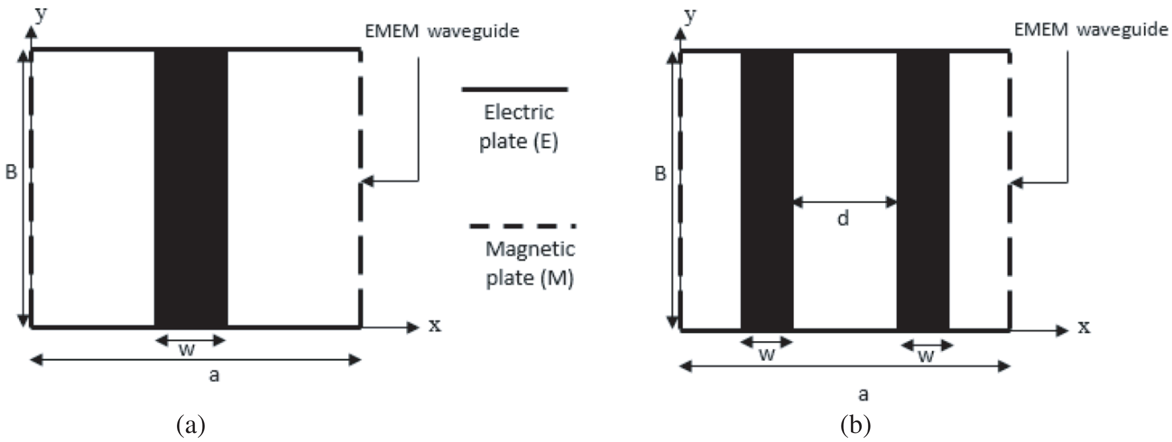

Figure 1. Studied structures (a) Structure 1: one micro-strip in a rectangular electric magnetic electric magnetic (EMEM) waveguide. (b) Structure 2: two micro-strips in a rectangular (EMEM) waveguide. $a = 20\,\text{mm}$, $B = 9\,\text{mm}$, $w = 1\,\text{mm}$ and $d = 5\,\text{mm}$.

waveguide. The second one consists of two microstrips in a rectangular waveguide. The considered waveguides are infinite, lossless and symmetric through the discontinuity planes. They are associated to EMEM boundaries: Two perfect magnetic plates (M) on the lateral sides and two perfect electric ones (E) on the bottom and the top. The dimensions of the structures are: $a = 20\,\text{mm}$, $B = 9\,\text{mm}$, $w = 1\,\text{mm}$ and $d = 5\,\text{mm}$.

Each EMEM waveguide is associated to a modal basis f_m [22–24]. The studied structures are invariant in y direction, and the waveguides excitations are assured by their fundamental TEM modes. There is no y dependency, so only TEM and Transverse Electric (TE) modes exist. Consequently, the considered modal basis is as the following:

$$\begin{cases} f_0 = \dfrac{1}{\sqrt{a}} \\ f_m = \sqrt{\dfrac{2}{a}} \cos\left(\dfrac{m\pi x}{a}\right) \end{cases} \tag{2}$$

2.2. Principle of MoM-GEC Approach Application

2.2.1. Basic Concept of MoM-GEC

Integral methods are the most appropriate for electromagnetic study of planar microwave structures. Since they write initial boundary conditions in form of integral equations defined on the obstacle surface, these methods permit the reduction of the problem's dimension. But, the resolution will become more complicated as the structure's complexity increases. In this context, the equivalent circuits are introduced for the development of integral method formulation based on the transposition of field problems in generalized equivalent circuit that are simpler to treat.

In fact, for alleviating the resolution of Maxwell's equations, the method of Generalized Equivalent Circuit (MGEC) was proposed [17–19, 21, 24–26] in order to represent integral equations by equivalent circuits that express the unknown electromagnetic boundary conditions. The equivalent circuit presents a true electric image of the studied structures for describing the discontinuity and its environment. In the discontinuity plane, the electromagnetic state is described by generalized test functions that are modeled by virtual sources not storing energy. The discontinuity environment is expressed by an impedance operator or admittance operator that represents boundary conditions on each side of discontinuity surface. However, the wave exciting the discontinuity surface is represented by a real field source or a real current source because it delivers energy.

Generally, the electromagnetic modeling with GEC extends the Kirchhoff's laws used in (V, I) concept to the Maxwell's formalism (E, H). In order to apply Kirchhoff's laws accurately, we should substitute the magnetic field by the current density J defined as $\vec{J} = \vec{H} \wedge \vec{n}$ where $\vec{n}$ is the normal vector to the discontinuity surface. It is noted that these generalized equivalent circuits are associated to perfect interfaces, which are characterized by the fact that electric field and current density are defined on complementary domains.

The modeling of the considered structures 1 and 2 is assured by the generalized equivalent circuit illustrated in Figure 2. Let's $(f_m)_{m\in(1,2,\ldots,M)}$ be the modal basis corresponding to this waveguide described in (2).

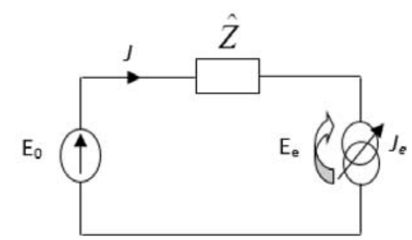

Figure 2. Generalized equivalent circuit for the proposed structures.

The impedance operator $\hat{Z}$ is defined in function of higher order modes as $\hat{Z} = \sum_m |F_m > z_m < F_m|$. Je is the virtual source defined on the metallic domain of the discontinuity surface, and it is the problem's unknown expressed as a series of known functions g_p weighted by unknown coefficients $(x_p)_{p\in(1,2,\ldots,Ne)}$.

$$J_e = \sum x_p g_p \tag{3}$$

Then, the application of the generalized Ohm and Kirchhoff laws to the GEC in Figure 2 leads to obtaining the equation system (4):

$$\begin{cases} J = -J_e \\ E_e = E_0 + \hat{Z} J_e \end{cases} \tag{4}$$

The current J is expressed in modal basis functions $(f_m)_{m\in(1,2,\ldots,M)}$ weighted by unknown coefficients $(I_m)_{m\in(1,2,\ldots,M)}$.

$$J = \sum I_m f_m \tag{5}$$

Therefore, the application of the Galerkin's method and Kirchhoff's theorem leads to obtaining the simplified matrix representation as follows:

$$\begin{pmatrix} I \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -A^T \\ A & B \end{pmatrix} \begin{pmatrix} V_0 \\ X \end{pmatrix} \tag{6}$$

where: $A(p, 1) = \langle f_0, g_p \rangle$ and $B(p, q) = \sum_m \langle g_p, f_m \rangle z_m \langle f_m, g_q \rangle$. z_m and ω correspond respectively to the mode impedance (TE mode) and the wave pulsation.

Thus, from Equation (6), we obtain the equations system:

$$\begin{cases} I = -A^T X \\ 0 = AV_0 + BX \end{cases} \tag{7}$$

The resolution of the equations system (7) leads to calculating the structure's input impedance as:

$$Z_{in} = \frac{1}{A^T B^{-1} A} \tag{8}$$

It also permits to determine weighting coefficients (x_p), current distribution J and diffracted field E_e in the discontinuity surface.

2.2.2. Virtual Sources

Let's consider D a discontinuity plane formed by metallic and dielectric patterns ($D = D_M + D_D$). Based on current and field properties on D, D_M is the metallic sub-domain on which the field is null, and its dual current is not null. However, D_D is the dielectric domain on which the current is null, and its dual field is not null. The concept of virtual source is to assemble all field and current representations in an only one which will be valid in all points of the domain D. In this problem, J_e is a current virtual source defined on metallic domain of structures discontinuities. It is the problem's unknown expressed as a series of known test functions weighted by unknown coefficients as mentioned in Equation (1). According to the MoM-GEC method, the choice of test functions is of great interest to obtain the convergence of the solution, whereas an inadequate test function choice can complicate the problem or not solve it. Generally, a roughly high number of sinusoidal and triangular test functions is used in MoM-GEC method to get the solution's convergence. In this work, we propose a new test function which is only used to replace a lot of sinusoidal or triangular test functions, in order to attain the same results in less time. The proposed test function is based on physically approximating the current on metal domain in two parts. The first part consists of current in the inside of metal. It is governed by PO method. And the second one consists of current on the metal edges. It is modeled by infinite cylinders that provide a diffracted field described by Hankel functions [36–38]. This approach, based on asymptotic methods (PO and modal method) and MoM method, constitutes a challenge for hybridization of methods to optimize the convergence and the computational time in electromagnetic problems.

2.3. Approximation of the Current Test Function

2.3.1. Approximation of the Current inside Microstrips: Physical Optic Current (J_{PO})

Physical approaches are based on surface current created in structure's surface illuminated by an electromagnetic wave. The PO method consists in approximating the electric current density $\vec{J}_{PO}$ on the surface of an object induced by an incident magnetic field $\vec{H}_{in}$ [13, 14]. The PO method relates approximatively the surface's current of a conductor to the incident magnetic field as:

$$\vec{J}_{PO} = 2\vec{H}_{in} \wedge \vec{z} \tag{9}$$

In our problem, $\vec{H}_{in}$ is the incident magnetic field on the waveguides, and $\vec{z}$ corresponds to the unit vector normal to the discontinuity that coincides with the propagation direction of the waveguides. $\vec{H}_{in}$ is related to the incident electric field as follows:

$$\vec{rot}\vec{E} = -j\omega\mu_0\vec{H}_{in} \tag{10}$$

Consequently,

$$\vec{J}_{PO} = \frac{2\beta}{\omega\mu_0}\sqrt{\frac{1}{a}}\vec{y} \tag{11}$$

where β is the waveguide constant of propagation, ω the wave pulsation, and μ_0 the air permeability.

2.3.2. Approximation of the Current on Microstrip Edges: Modal Current (J_M)

To determine the current $\vec{J}_M$ created by microstrip's edges, we propose to study the diffraction of a plane wave by the edge. It is modeled by an infinite cylindric metallic plate in y direction. Let's consider an incident field normal to the considered cylinders as shown in Figure 3. It is polarized in y direction. The induced current is in the same direction as follows:

$$\vec{A} = A_y(\rho, \varphi, y)\vec{y} \tag{12}$$

This approach approximates that the excited modes by the cylinders are expressed as follows:

$$\vec{A} = (C_1 H_m^1(\beta\rho) + D_1 H_m^2(\beta\rho))(C_2\cos(m\phi) + D_2\sin(m\phi))(A_z\exp(-j\beta z) + B_z\exp(j\beta z))\vec{y} \tag{13}$$

where H_m^1 and H_m^2 are respectively the Hankel functions of first and second kind of order m, and C_1 and D_1 are unknown scattering coefficients.

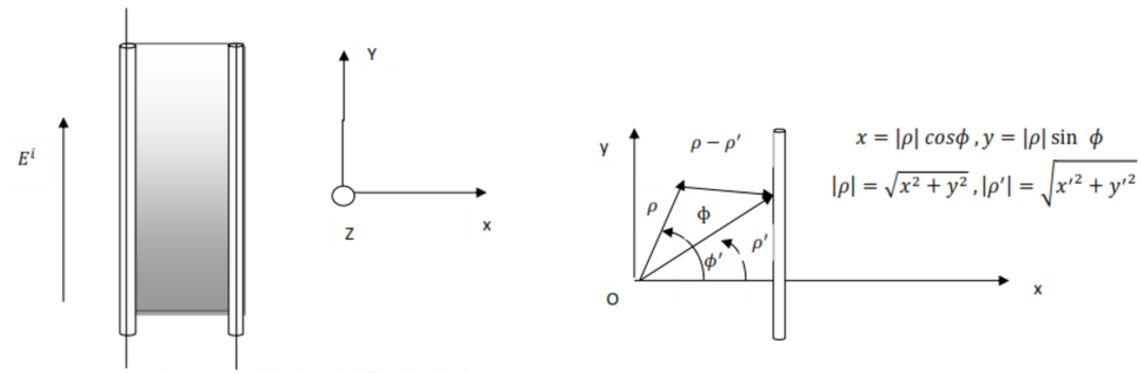

Figure 3. Cylinders modeling metallic edges and coordinate of an observation point. ($R_0 \ll \lambda$).

In our problem, radius of each cylinder modeling an edge is neglected compared to the wavelength ($R_0 \ll \lambda$). This implies that the diffraction phenomenon is governed only by the Hankel function of the second kind of order zero [36–38].

Due to the structure's invariance in y direction, $\beta_y = 0$ thus $\beta = \beta_z$.

Consequently:

$$\vec{A} = A_0 H_0^2(\beta\rho)\vec{y} \tag{14}$$

As shown in Figure 4, $R = |\vec{\rho} - \vec{\rho'}| = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')}$.

Where $\vec{\rho} = \vec{OP}$ and $\vec{\rho'} = \vec{OM}$ relate the origin O respectively to the observation point P and the source point M.

For the first cylinder, we have $\vec{\rho} = \vec{x_1}$, $\vec{\rho'} = \vec{x'}$ and $R = |\vec{\rho} - \vec{\rho'}| = \sqrt{x_1^2 + x'^2 - 2x_1 x'}$.

The cylinder is assumed as infinite and the observation point P assumed on the cylinder surface while the source point M is assumed on its axis. Taking into consideration the boundary condition by setting $x' = x_1 + R_0$.

$$E^i + E^d = 0 \tag{15}$$

$$E^i + (-j\omega A_0^1 H_0^2(\beta(|x_1 - x'|))) = 0 \tag{16}$$

$$E^i = j\omega A_0^1 H_0^2(\beta(|x_1 - x'|)) \tag{17}$$

where $\vec{E^i}$ is the waveguide's TEM mode ($\vec{E^i} = E^i \vec{y} = \sqrt{\frac{1}{a}}\vec{y}$).

Thus, the constants A_0^1 and A_0^2 are expressed by:

$$A_0^1 = A_0^2 = \frac{-\sqrt{\frac{1}{a}}}{j\omega H_0^2(\beta R_0)} \tag{18}$$

Then, the diffracted electric field is expressed as:

$$\begin{cases} \vec{E}_1^d = \dfrac{-\sqrt{\frac{1}{a}}}{H_0^2(\beta R_0)} H_0^2(\beta(|x - x'|))\vec{y}; x' = x_1 + R_0 \\ \vec{E}_2^d = \dfrac{-\sqrt{\frac{1}{a}}}{H_0^2(\beta R_0)} H_0^2(\beta(|x - x''|))\vec{y}; x'' = x_2 + R_0 \end{cases} \tag{19}$$

Consequently, the magnetic field can be deduced as:

$$\begin{cases} \vec{H}_1^d = \dfrac{\beta}{\mu_0} \dfrac{-\sqrt{\frac{1}{a}}}{j\omega H_0^2(\beta R_0)} H_1^2(\beta(|x - x'|))\vec{x}; x' = x_1 + R_0 \\ \vec{H}_2^d = \dfrac{\beta}{\mu_0} \dfrac{-\sqrt{\frac{1}{a}}}{j\omega H_0^2(\beta R_0)} H_1^2(\beta(|x - x''|))\vec{x}; x'' = x_2 + R_0 \end{cases} \tag{20}$$

The modal current is finally concluded by the following expression:

$$\vec{J}_M = -(A_0^1 \frac{\beta}{\mu_0} H_1^2(\beta|x-x'|) + A_0^2 \frac{\beta}{\mu_0} H_1^2(\beta|x-x''|))\vec{y} \tag{21}$$

Finally, the current $\vec{J}_e$ is composed of two parts $\vec{J}_{PO}$ and $\vec{J}_M$ ($\vec{J}_e = \vec{J}_{PO} + \vec{J}_M$), and it will be used as a single test function in MoM-GEC method.

$$\vec{J}_e = -\left(\left(A_0^1 \frac{\beta}{\mu_0} H_1^2(\beta|x-x'|) + A_0^2 \frac{\beta}{\mu_0} H_1^2(\beta|x-x''|) + \frac{2\beta}{\omega\mu_0}\sqrt{\frac{1}{a}}\right)\right)\vec{y} \tag{22}$$

To avoid the singularity effect of the electric field caused by the nature of Hankel functions, the observation point is assumed very close to the edge (cylinder axis) but remains spaced from it about R_0. Hence, this constitutes the key idea of using the cylinder radius lower than λ to model the edges ($R_0 \ll \lambda$). This permits to approximate the integrals by a series as shown in the following.

According to the Galerkin's method, to compute a structure's input impedance and distributions of field and current, a number of projections between test and basis functions $\langle g_p, f_m\rangle$ must be achieved as in the matrix B (Equation (6)). However, considering our approach, the single test function is divided into two parts ($J_e = J_{PO} + J_M$). Hence, its projection on the waveguide's basis functions leads to obtaining the two integrals in the following:

$$\langle J_e, f_m\rangle = \langle J_M, f_m\rangle + \langle J_{PO}, f_m\rangle = \int_{x_1}^{x_2} f_m J_M dx + \int_{x_1}^{x_2} f_m J_{PO} dx \tag{23}$$

The projection of J_{PO} on sinusoidal basis functions provides a simple integral that can be analytically calculated. However, the edges of metal are governed by modal current J_M described by Hankel functions. Their projection on sinusoidal waveguide's modes leads to obtain an integral that can not be analytically solved. Consequently, this integral calculation is solved numerically using the rectangle method (midpoint method) as expressed in the follows:

$$\int_a^b f(x)dx = \frac{b-a}{N}\sum_{i=1}^{N} f(x_i) \tag{24}$$

In this problem $f(x)$ is given by:

$$f(x) = \sqrt{\frac{2}{a}} cos(\frac{m\pi x}{w})(-(A_0^1 \frac{\beta}{\mu_0} H_1^2(\beta|x-x'|) + A_0^2 \frac{\beta}{\mu_0} H_1^2(\beta|x-x''|))) \tag{25}$$

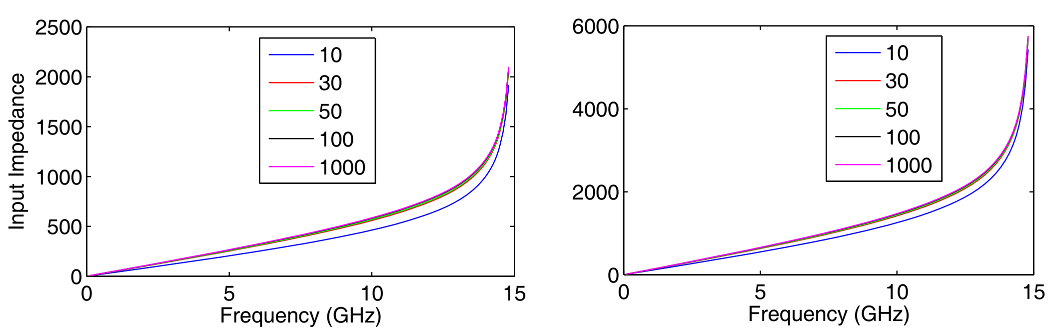

Figure 4. Study of input impedances convergence. Input impedances versus frequency for various used modal basis functions number. (a) Structure 1 (only one microstrip). (b) Structure 2 (two microstrips).

3. NUMERICAL RESULTS AND DISCUSSIONS

3.1. Convergence Study

In this problem, there is only one test function, so the convergence study is done as a function of number of used modal basis functions. Figure 4 shows the input impedance (Z_{in}) of the two considered structures against number of modes of waveguides in the frequency range [0,1–15 GHz]. It is obvious that for all structures, the Z_{in}'s convergence is obtained for 100 modes of waveguides. We note that the convergence is rapidly attained compared to cases using sinusoidal or triangular test functions.

This conforms with the law ($\frac{P}{N} < \frac{Metal\ Surface}{Total\ Surface}$) [21, 25] in a domain of discontinuity, where P is test functions number and N is basis functions number.

3.2. Comparison between the Hybrid Approach and the MoM Method

In order to evaluate our approach, we compare, for both structures 1 and 2, input impedances, currents, and electric fields distributions to these obtained by MoM method using sinusoidal test functions. As a start, the input impedances of the two structures are computed for the frequency range [0.1–15 GHz] and drawn in Figure 5.

For both structures, the obtained results by the two methods are in agreement for all frequencies. We note that the numbers of sinusoidal test and basis functions used for MoM method are respectively 30 and 2000 for both the structures, whereas our approach consists in using only one hybrid test function with 100 basis functions.

The convergence is quick since the number of test and basis functions are immensely reduced.

The current distributions for the two studied structures are drawn in Figure 6 and compared to those obtained with MoM method.

A good agreement between the two methods is assured. It is also obvious that the current on edges, approximated by Hankel functions, well satisfies boundary conditions: it is maximum on the microstrips edges. This proves the development of the proposed test function.

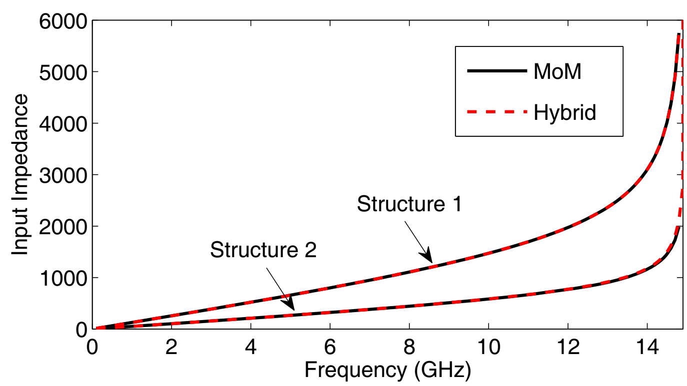

Figure 5. Input impedances of both structures (1) and (2) against frequency, obtained by hybrid approach and compared to MoM method.

Similarly, Figure 7 shows the electric fields distributions in the two structures, 1 and 2. Like input impedances and currents, there is a perfect accordance between the two methods with verification of boundary conditions. In addition, the number of basis functions is reduced too, and a good accuracy of the new hybrid approach is observed.

Let's remind that this hybrid method which this work is based on, requires less numerical complexity and needs less storage than the MoM, which makes it more convenient. In fact, using a single test function, the size of manipulated matrices becomes 1×1 instead of 30×30 in case of sinusoidal test functions. Hence, there is no need to inverse nor to stock matrices. Then, the convergence is quickly reached.

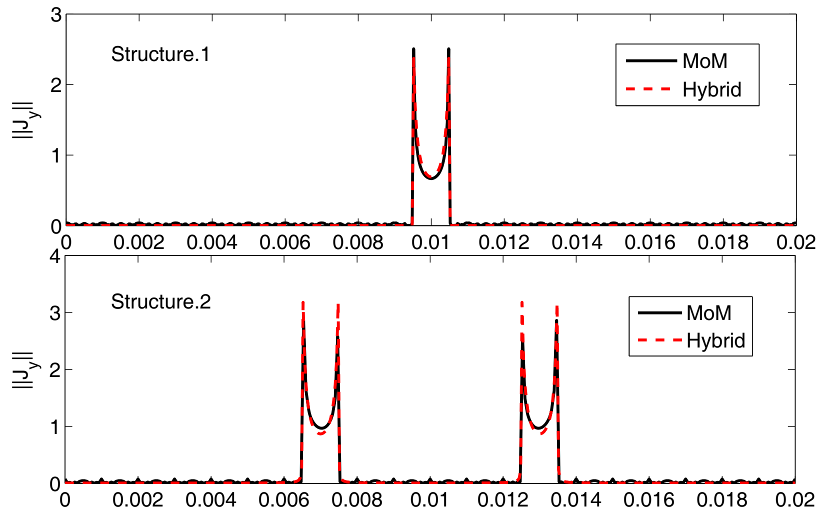

Figure 6. Current distributions of both structures (1) and (2) obtained by Hybrid Approach and compared to MoM method.

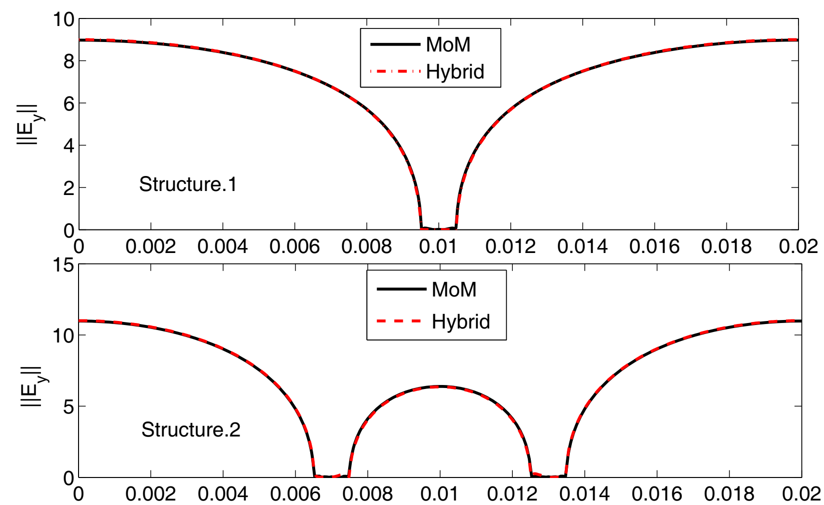

Figure 7. Field distributions of both structures (1) and (2) obtained by hybrid approach and compared to MoM method.

3.3. Validity of the New Approach As a Function of Cylinder Radius R_0

The study of the MoM-GEC-Asymptotic method as a function of the cylinders radius R_0 seems very interesting to show the different domains of validity of the considered hybrid test function. Figure 8 shows the error η ($\eta = 100 \times \frac{||Zin_{MoM}| - |Zin_{Hybrid}||}{|Zin_{MoM}|}$) of the input impedance of the structure 1 given by the hybrid approach compared to that obtained by the MoM method in a frequency $f = 5\,\text{GHz}$ versus $\frac{R_0}{\lambda}$.

It is noted that the obtained curve is divided into three parts. There is only one domain when the considered approach is able to be applied. In this domain, when $0.5 * 10^{-3}\lambda < R_0 < 2.5 * 10^{-3}$, the relative error $\eta < 1\%$, whereas in both of the other domains, when R_0 is close to 0 and $R_0 > 2.5 * 10^{-3}\lambda$, the hybrid approach cannot be applied since it gives considerable error.

In order to confirm these results, the model's validity is also studied in terms of field and current

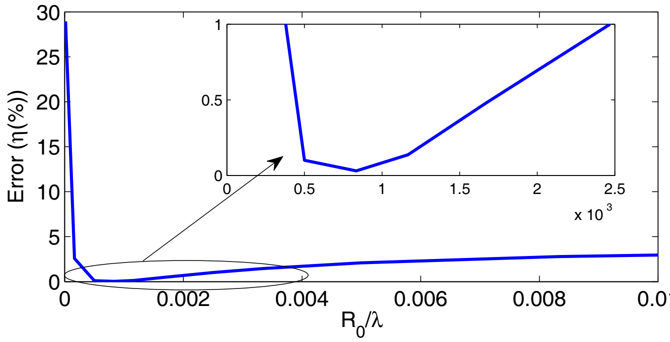

Figure 8. Variation of the relative error between the new hybrid approach and the MoM method with the ratio $\frac{R_0}{\lambda}$. Domain of validity of the proposed model against R_0.

distributions. Figure 9 shows the current and field distributions in various domains of $\frac{R_0}{\lambda}$.

The obtained results conform with those obtained for the input impedance in a function of $\frac{R_0}{\lambda}$. The field and current distributions behave similarly to the input impedance in the three different domains of $\frac{R_0}{\lambda}$.

In fact, when R_0 is very close to 0 or $R_0 > 2.5 * 10^{-3}\lambda$, the field and current distributions obtained by the new hybrid approach are different from those obtained by MoM method. Thus, the new hybrid test function is not adequate in these domains.

Elsewhere, when $0.5 * 10^{-3}\lambda < R_0 < 2.5 * 10^{-3}$, the obtained results show accuracy with MoM method. Only in this domain, we can apply the proposed model.

Consequently, the application of this model is limited by the ratio $\frac{R_0}{\lambda}$. This result satisfies the proposed approach because we considered $\frac{R_0}{\lambda} \ll 0$. However, we should avoid the values of cylinders radius very close to 0.

4. GAIN IN CONVERGENCE AND COMPUTATIONAL TIME ACHIEVED BY THE NEW APPROACH

Finally, to show the new approach's advantages, we present the gain in convergence and computation time compared to MoM method based on sinusoidal test functions. Because it is a single test function, the calculation is much less expensive, and the convergence becomes faster than the MOM method. Consequently, the size of manipulated matrices is enormously reduced. This leads to reducing the numerical complexity and memory resources required by the considered problems.

The mentioned advantages of our approach are proved by Tables 1 and 2. Table 1 represents the number of test and modal basis functions needed to obtain convergence of the input impedances in the range of [0.1–15 GHz] (drawn in Figure 6). The required time is given by the same table. Then, a comparison with MoM method is achieved. In fact, to calculate the considered input impedances, using the new approach, structures 1 and 2 need respectively 17.784 s and 120.078 s instead of 215.32 s and 1241.765 s when using MoM method with sinusoidal test functions. In addition, using the hybrid approach for the two considered structures, only 100 waveguide's modes are required to obtain the convergence. However, the MoM method needs 30 test functions and 3000 waveguides modes for both structures to obtain the convergence.

Similarly, Table 2 gives the computational time and the number of test and basis functions needed to get the convergence for both of the structures when computing the currents and the electric fields on discontinuities. Then, a comparison between the new approach and the MoM method is achieved.

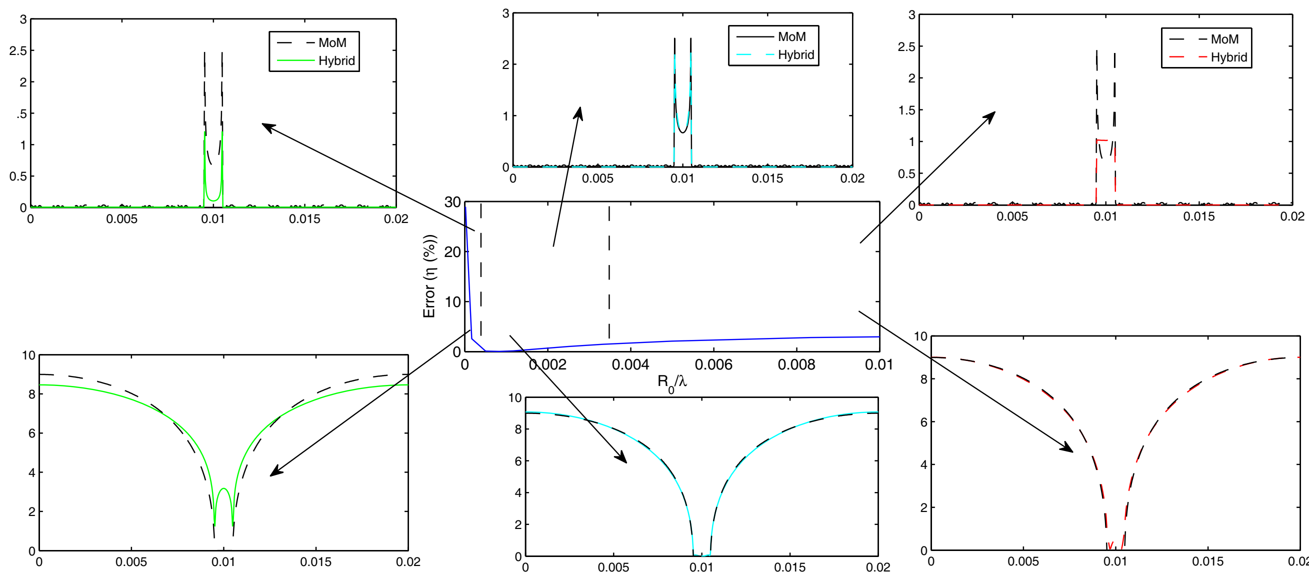

Figure 9. Variation of the relative error, field and current distributions with the ratio $\frac{R_0}{\lambda}$. Domain of validity of the proposed model against R_0.

Table 1. Computational time consumed by the new Hybrid Approach to compute the input impedances in the range [0.1–15 GHz] compared to each consumed by the MoM method. (P: Number of test functions and N: Number of basis functions).

Method	Time consumed (Str. 1)	Time consumed (Str. 2)	Convergence (Str. 1)	Convergence (Str. 2)
MoM	215.32	1241.765	$P = 30$ $N = 3000$	$P = 30$ $N = 3000$
Hybrid Approach	17.784	120.087	$P = 1$ $N = 100$	$P = 1$ $N = 100$

Table 2. Computational time consumed by the new hybrid approach to compute field and current distributions compared to each consumed by the MoM method. (P: Number of test functions and N: Number of basis functions).

Method	Time consumed (Str. 1)	Time consumed (Str. 2)	Convergence (Str. 1)	Convergence (Str. 2)
MoM	24.38	132.693	$P = 70$ $N = 10000$	$P = 70$ $N = 10000$
Hybrid Approach	3.538	25.33	$P = 1$ $N = 100$	$P = 1$ $N = 100$

Consequently, the proposed hybrid approach provides a considerable reduction and enhancement of the processing time. Besides, the number of modes to obtain convergence is greatly reduced. These two advantages are guaranteed by the use of a single test function. Indeed, when the number of test functions decreases, sizes of manipulated matrices decrease too. Thus, their filling and inversion require less time. Therefore, the computational time will be reduced exponentially.

5. CONCLUSION

In this work, we propose an approach to approximate a single test function, which alone is able to describe discontinuity. Under this approach, the current on a metallic strip is separated on two parts. The first runs on the edges given by Hankel functions, and the second is the current on the inside of metal approximated by the PO method. Then, for validation purposes, the proposed test function is subsequently used to solve a problem of diffraction using the method of moments combined with generalized equivalent circuits.

Obtained results, compared to MoM with sinusoidal test functions, prove very satisfactory. The input impedance of the studied structures, currents and fields found by the two methods are in agreement over a wide frequency range.

The new approach is of great interest and has several advantages:

- ⊡ A rapid convergence, since it requires only one test function and a limited number of basis functions compared to MoM when using sinusoidal or triangular test functions.
- ⊡ A very substantial gain in computational time indicated by Tables 1 and 2.
- ⊡ A significant reduction in required memory resources since the manipulated matrices are greatly reduced in size.

A precision in computation of scattering compared to MoM:

- Accuracy in the calculation of the input impedance over a wide frequency band.
- Accuracy in the calculation of the current distribution and verification of boundary conditions.
- Accuracy in the calculation of the electric field presented on both of the proposed structures.

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