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An Efficient Mode Reduction Technique for Modeling of Waveguide Gratings

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Abstract—In this paper, an efficient mode reduction technique for eigenmode expansion method is developed to analyze 2–D waveguide grating structures which are a special class of piecewise uniform waveguides. To take advantage of the periodicity property of the structure, the eigenmode expansion method (EEM) is used with the scattering matrix method and a recursive-doubling procedure. In this situation, our proposed mode reduction technique achieves a significant speedup for gratings with large number of periods. Comprehensive numerical examples on the waveguide gratings are studied to validate the efficiency of our proposed mode reduction technique.

1. INTRODUCTION

Waveguide grating structures that are piecewise uniform and periodic in the longitudinal direction are widely used in many modern optical devices. Numerical simulations of the wavefields are essential in the design and analysis of these structures. Existing methods for analyzing such structures with multiple longitudinal discontinuities include various eigenmode expansion methods (EEM) [1–7], the bidirectional beam propagation method (BiBPM) [8–12], the finite difference time domain (FDTD) method [13] and the quation integral method [14, 15]. Efficient numerical techniques are in great need for modeling the wavefields of the considered structures.

In various EEM, the wave field in each longitudinally uniform segment is expanded in terms of the eigenmodes of the transverse operator. For an open waveguide, the transverse direction has to be truncated into finite interval by adopting absorbing boundary conditions, such as the perfectly matched layers (PMLs) [16, 17]. After that, the eigenmodes can be calculated semi-analytically by solving a nonlinear equation, if the refractive index profile in each segment is piecewise constant in the transverse direction. On invoking the PMLs, the propagation constants of eigenmodes may be complex. Hence, it is not a simple task to compute all solutions from this nonlinear equation in the complex domain. In this situation, eigenmodes can be efficiently calculated by numerical methods such as the finite element method [1], the finite difference method [4], and Fourier series [6, 7]. If transverse direction is discretized with N grid points, eigenmodes of each segment can be computed by $O(N^3)$ operations. Since the waveguide grating is periodic, the eigenvalue problem only needs to be solved in two segments. Meanwhile, EEM can be implemented in serval different ways. One of the widely used and robust method is the scattering matrix method [6, 18]. For a 2D waveguide grating with m periods, scattering matrix method needs $O(mN^3)$ operations. By adopting the recursive-doubling procedures, the number of operations can be reduced to $O(N^3\log_2 m)$.

The BiBPMs rely on the rational approximation of a square root operator and its exponential. For deeply etched waveguides with large index-contrast, BiBPMs are very difficult to model the radiation

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and evanescent modes. FDTD [13] is a widely used method in the simulation of optical structures. But it requires a small grid size to resolve large index-contrast and a small time step to ensure numerical stability. Integral equation method [14, 15] offers an efficient way to analyze bandgaps of photonic crystals and reflectance of waveguides. Furthermore, on invoking the integral equation method, only the surface needs to discretize and avoids the discretization of the whole computation domain, and therefore significantly reduce the number of unknowns [14]. Meanwhile, integral equation method also can efficiently simulate very complicate geometries. In [15], an integral equation method was extensively studied, and then adopted to analyze the optical response from PEC waveguide composed of two periodic and one-dimensional rough surfaces.

In this paper, we develop a mode reduction technique for the analysis of 2D waveguide grating with large number of periods. After the process that the scattering matrix of one period is calculated using the full set of eigenmodes, our approach can significantly reduce the number of modes used in the construction of the scattering matrix of the whole structure. By using a recursive doubling procedure, our approach can reduce the total number of operations to $O(N^3) + O(N_*^3 \log_2 m)$, where N_* ($N_* < N$) is the number of retained modes. For structures with large m, a significant speedup could be achieved. The error induced by our mode reduction technique is small even the number of retained eigenmodes N_* is less than one third of N. In the future, it is possible to extend our approach to analyze 3D waveguides and 2D slab photonic structures.

2. PROBLEM FORMULATION

Let $\{x, y, z\}$ be a Cartesian coordinate system, we consider the structure and the electromagnetic field are both independent of z. Then, the z–components of the time harmonic field satisfy the following Helmholtz equation

$$\rho \frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial u}{\partial x}\right) + \rho \frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial u}{\partial y}\right) + k_0^2 \varepsilon \mu\, u = 0, \tag{1}$$

where k_0 is the free space wavenumber, ε the relative permittivity, and μ the relative permeability. $u = E_z$ and $\rho = \mu$ correspond to the transverse electric (TE) polarization. $u = H_z$ and $\rho = \varepsilon$ correspond to the transverse magnetic (TM) polarization.

A piecewise uniform waveguide with the main propagation direction in x is a structure with discontinuity points $x_1 < x_2 < \ldots < x_m$, such that ε and μ depend only on y, i.e., $\varepsilon = \varepsilon^{(l)}(y)$ and $\mu = \mu^{(l)}(y)$, in the lth uniform segment given by $x < x_1$ if $l = 1$, $x_{l-1} < x < x_l$ if $1 < l \leq m$, and $x > x_m$ if $l = m + 1$. We are particularly interested in waveguide gratings for which m is an even integer, the segments with odd l correspond to the original waveguide. The segments with even l are modifications (such as grooves) etched on the original waveguide. The structure is periodic with period $L = x_3 - x_1$ for $x_1 \leq x \leq x_m$. An example with five grooves ($m = 10$) is depicted in Fig. 1. For such a structure, the standard problem is to calculate the transmitted and reflected wave fields for incident waves given in the semi-infinite uniform segments $x < x_1$ and $x > x_m$.

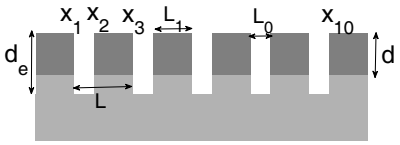

Figure 1. A schematic view of a piecewise uniform waveguide with 5 grooves ($m = 10$). The thickness of waveguide core is d and the deepness of grooves is d_e. The width of the "teeth" and "grooves" are L_1 and L_0, respectively. The period is $L = L_0 + L_1$.

3. EIGENMODE EXPANSION METHOD

The EEM is popular for 2D piecewise uniform waveguides, since it avoids the discretization of x variable (along the waveguide axis) and is relatively efficient. To avoid the continuous spectra of open waveguides

and to model the radiation modes, PMLs [16, 17] can be used to truncate the y variable. Let the y variable be truncated to the interval (y_0, y_*), then, the eigenmodes of the l-th uniform segment satisfy

$$\frac{\rho^{(l)}}{s}\frac{d}{dy}\left[\frac{1}{s\rho^{(l)}}\frac{d\phi^{(l)}}{dy}\right] + k_0^2\varepsilon^{(l)}\mu^{(l)}\phi^{(l)} = [\beta^{(l)}]^2\phi^{(l)}, \quad y_0 < y < y_*, \tag{2}$$

where $\phi^{(l)} = \phi^{(l)}(y)$ is an eigenfunction, $[\beta^{(l)}]^2$ the corresponding eigenvalue, $\beta^{(l)}$ the propagation constant of the eigenmode, and $s = s(y)$ a complex function related to the PMLs ($s \neq 1$ only in the PMLs). Furthermore, the eigenfunctions must satisfy some simple homogeneous boundary conditions at y_0 and y_*, that is,

$$\phi^{(l)}(y_0) = \phi^{(l)}(y_*) = 0. \tag{3}$$

The above eigenvalue problem has an infinite sequence of eigenpairs which are denoted as $\phi_j^{(l)}$ and $\beta_j^{(l)}$ for $j = 1, 2, \ldots$. We assume that the fundamental mode of the l-th segment is $\phi_1^{(l)}$ with propagation constant $\beta_1^{(l)}$.

In each uniform segment, the wave field can be decomposed as the sum of forward and backward components, and they can be expanded in the transverse eigenmodes. In the l-th segment where $1 < l \leq m$, the wave field is given as

$$u(x,y) = \sum_{j=1}^{\infty} \phi_j^{(l)}(y)\left[a_j^{(l)}e^{i\beta_j^{(l)}(x-x_{l-1})} + b_j^{(l)}e^{i\beta_j^{(l)}(x_l-x)}\right], \quad x_{l-1} < x < x_l. \tag{4}$$

For the first and last segments, the expansions have the following different expressions:

$$u(x,y) = \sum_{j=1}^{\infty} \phi_j^{(1)}(y)\left[\tilde{a}_j^{(1)}e^{i\beta_j^{(1)}(x-x_1)} + b_j^{(1)}e^{i\beta_j^{(1)}(x_1-x)}\right], \quad x < x_1, \tag{5}$$

$$u(x,y) = \sum_{j=1}^{\infty} \phi_j^{(m+1)}(y)\left[a_j^{(m+1)}e^{i\beta_j^{(m+1)}(x-x_m)} + \tilde{b}_j^{(m+1)}e^{i\beta_j^{(m+1)}(x_m-x)}\right], \quad x > x_m. \tag{6}$$

In the above, the coefficients $\tilde{a}_j^{(1)}$ and $\tilde{b}_j^{(m+1)}$ are given, and they represent the incident waves in the left and right semi-infinite waveguides, respectively. The unknown coefficients $\{a_j^{(l)} : \ 2 \leq l \leq m+1, \ j = 1, 2, \ldots\}$ and $\{b_j^{(l)} : \ 1 \leq l \leq m, \ j = 1, 2, \ldots\}$ can be solved from the linear system

$$\mathbf{A}\begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{a}_2 \\ \vdots \\ \boldsymbol{b}_m \\ \boldsymbol{a}_{m+1} \end{bmatrix} = \boldsymbol{f}, \tag{7}$$

where $\boldsymbol{a}_l$ and $\boldsymbol{b}_l$ are column vectors for $a_j^{(l)}$ and $b_j^{(l)}$, respectively. $\boldsymbol{f}$ is related to the incident waves, and $\mathbf{A}$ is a matrix with a banded structure. Equation (7) can be established from the continuity conditions of u and $\rho^{-1}\partial_x u$ at $x = x_l$ for $1 \leq l \leq m$.

Many different implementations of EEM exist, and they correspond to different ways for computing the transverse eigenmodes and enforcing the continuity conditions. If $\varepsilon^{(l)}(y)$ and $\mu^{(l)}(y)$ are piecewise constant, then, the eigenvalue problem (2) and (3) can be solved analytically. In this sense, the eigenfunctions are given analytically, while the eigenvalues must be solved numerically as the zeros of a transcendental function. This approach is not so convenient, as it is not easy to systematically find all zeros of a function in the complex plane, and it is also not clear how to choose the first N eigenmodes. Here, N is the truncation order for the expansions. On the other hand, it is very simple to solve the 1D eigenvalue problem by a numerical method. In that case, Eqs. (2) and (3) are approximated by a matrix eigenvalue problem with a $N \times N$ matrix, where N now corresponds to the number of discretization points for y. The matrix eigenvalue problem gives rise to N numerical eigenmodes. If we use all these modes, then $\boldsymbol{a}_l$ and $\boldsymbol{b}_l$ are vectors of length N. Since the coefficient matrix $\mathbf{A}$ has a banded structure, the linear system (7) can be solved in $O(mN^3)$ operations.

4. SCATTERING MATRIX METHOD

The scattering matrix method [6, 18] is often used with EEM to avoid the large linear system (7) and reduce the memory requirement from $O(mN^2)$ to $O(N^2)$. For a general piecewise uniform waveguide, the scattering matrix method cannot reduce the required number of operations. But if the structure is periodic, it can be accelerated by a recursive-doubling procedure. Then, the number of operations can be reduced to $O(N^3 \log_2 m)$. The scattering matrix method calculates the scattering matrix $\mathbf{S}(x_1^-, x_m^+)$ satisfying

$$\mathbf{S}(x_1^-, x_m^+) \begin{bmatrix} \tilde{\boldsymbol{a}}_1 \\ \tilde{\boldsymbol{b}}_{m+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{m+1} \\ \boldsymbol{b}_1 \end{bmatrix}, \tag{8}$$

with $\tilde{\boldsymbol{a}}_1$ and $\tilde{\boldsymbol{b}}_{m+1}$ as column vectors for coefficients $\tilde{a}_j^{(1)}$ and $\tilde{b}_j^{(m+1)}$ of the incident waves. If the structure is periodic, the matrix $\mathbf{S}(x_1^-, x_m^+)$ can be computed from the scattering matrix for a single period. A brief summary of the scattering matrix method and the recursive-doubling procedure in given in Appendix A.

5. MODE REDUCTION PROCEDURE

First, we calculate the scattering matrices of individual segments using all numerical eigenmodes. Next, we reduce the size of these scattering matrices, and use them to find a reduced version of the final scattering matrix $\mathbf{S}(x_1^-, x_m^+)$. For a waveguide grating with periodic feature, we only need to consider the segment given by $x_1 < x < x_2$. The scattering matrix for the considered segment, denoted as $\mathbf{S}(x_1^-, x_2^+)$, satisfies

$$\mathbf{S}(x_1^-, x_2^+) \begin{bmatrix} \tilde{\boldsymbol{a}}_1 \\ \tilde{\boldsymbol{b}}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_3 \\ \boldsymbol{b}_1 \end{bmatrix}, \tag{9}$$

where $\tilde{\boldsymbol{b}}_3$ is a column vector, and coefficients $\tilde{b}_j^{(3)}$ are used in the following expansion

$$u(x,y) = \sum_{j=1}^{\infty} \phi_j^{(3)}(y) \left[a_j^{(3)} e^{i\beta_j^{(3)}(x-x_2)} + \tilde{b}_j^{(3)} e^{i\beta_j^{(3)}(x_2-x)} \right], \quad x > x_2. \tag{10}$$

In the fully discretized version, $\mathbf{S}(x_1^-, x_2^+)$ is a $(2N) \times (2N)$ matrix and can be calculated in $O(N^3)$ operations. If we only keep N_* modes, then $\mathbf{S}(x_1^-, x_2^+)$ is approximated by a $(2N_*) \times (2N_*)$ matrix, and it can be used to find a $(2N_*) \times (2N_*)$ approximation of the final scattering matrix $\mathbf{S}(x_1^-, x_m^+)$ in $O(N_*^3 \log_2 m)$ operations. Therefore, the total number of operations is $O(N^3) + O(N_*^3 \log_2 m)$. In particular, for large m, a significant speedup may be achieved.

To select the modes, we first normalize the eigenmodes such that

$$\int_{y_0}^{y_*} \frac{s(y)}{\rho^{(l)}(y)} \left[\phi_j^{(l)}(y) \right]^2 dy = 1. \tag{11}$$

For an incident fundamental mode $\phi_1^{(1)}(y) e^{i\beta_1^{(1)}(x-x_1)}$ (i.e., $\tilde{\boldsymbol{a}}_1 = [1, 0, \ldots, 0]^T$ and $\tilde{\boldsymbol{b}}_3 = \mathbf{0}$), the coefficients of the transmitted and reflected waves of one period are the first column of matrix $\mathbf{S}(x_1^-, x_2^+)$. Let $\boldsymbol{g}$ be the vector containing the first N elements of the first column of matrix $\mathbf{S}(x_1^-, x_2^+)$. We select the modes based on the magnitudes of the elements of $\boldsymbol{g}$. More precisely, for an integer $N_* < N$, we choose N_* elements with the largest magnitude in $\boldsymbol{g}$, and find the mode indices for all these elements, and finally get the N_* modes that we need to keep. Based on the N_* selected modes, by keeping the corresponding row and column elements in $\mathbf{S}(x_1^-, x_2^+)$, we could find a $(2N_*) \times (2N_*)$ matrix approximating $\mathbf{S}(x_1^-, x_2^+)$.

6. NUMERICAL EXAMPLES

The first example is a modeling exercise of COST 268 [19]. It is about an optical waveguide with a deeply etched short Bragg grating as shown in Fig. 1. The original waveguide is formed by a Si_3N_4 layer of the thickness $d = 0.5\,\mu\text{m}$ deposited onto a SiQ_2 substrate. The refractive indices of waveguide core

and the substrate are frequency dependent and given in [19]. The Bragg grating is composed of 1024 rectangular grooves with a grating period of $L = 0.43\,\mu m$. The widths of the "tooth" and "groove" are chosen to be equal, i.e., $L_0 = L_1 = 0.215\,\mu m$. The groove depth is $d_e = 0.625\,\mu m$. In all examples, we assume that an incident fundamental mode is coming from $x < x_1$. In numerical implementation, the transverse y direction is truncated by PMLs to an interval with length $4.5\,\mu m$ with $2.5\,\mu m$ in the substrate and $1.5\,\mu m$ in the air, and discretized by a fourth-order FD scheme [20] with $N = 180$ grid points. Fig. 2 shows the absolute values of the first 180 elements in the first column of $\mathbf{S}(x_1^-, x_2^+)$ at wavelength $\lambda = 1.55\,\mu m$. We can see that the values decay very fast with the increasing mode index for both TE and TM cases. Under certain error tolerance, we can discard the modes corresponding to elements with small magnitudes in constructing an approximation to scattering matrix $\mathbf{S}(x_1^-, x_m^+)$.

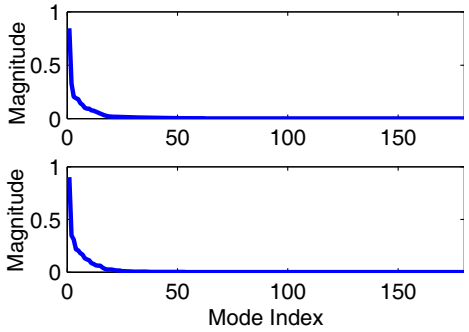

Figure 2. The absolute value of the first 180 elements in the first column of $\mathbf{S}(x_1^-, x_2^+)$ in the first example with wavelength $\lambda = 1.55\,\mu m$. Top and bottom pictures correspond to TE and TM polarizations, respectively.

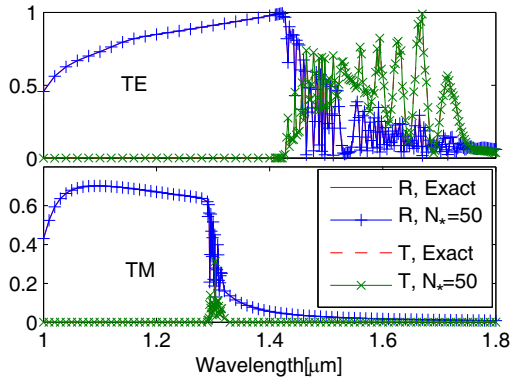

Figure 3. The reflection and transmission spectra calculated by EEM with and without mode reduction technique in the first example. Top figure corresponds to the TE polarization; bottom figure corresponds to the TM polarization. $N_* = 50$ modes are used in the mode reduction technique.

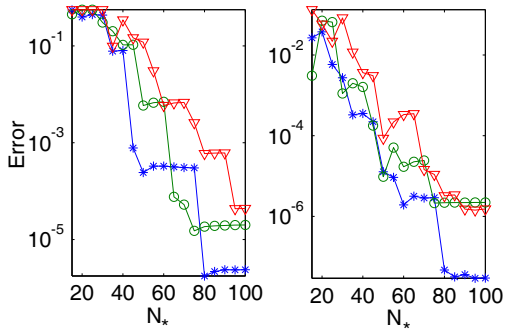

Figure 4. Error of transmission and reflection computed by EEM with mode reduction technique for different N_* and N in the first example. Left figure is for transmission of TE polarization at wavelength $\lambda = 1.48\,\mu m$; right figure is for reflection of TM polarization at wavelength $\lambda = 1.29\,\mu m$. Blue star, green circle and red triangle are for $N = 180$, $N = 360$ and $N = 720$, respectively.

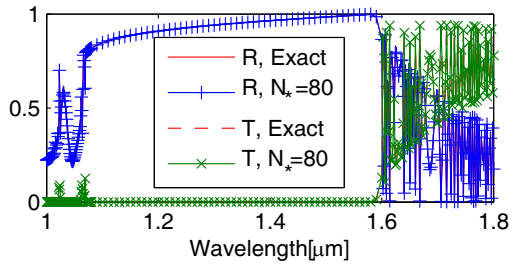

Figure 5. The reflection and transmission spectra for TE polarization calculated by EEM with and without mode reduction technique in the second example. $N_* = 80$ modes are used in the mode reduction technique.

To demonstrate the effectiveness of the simulation of wavefields from the proposed mode reduction technique, Fig. 3 shows the reflection and transmission spectra calculated by EEM with and without mode reduction technique. The number of retained modes in mode reduction technique is $N_* = 50$. In this paper, the "exact" results refer to the results computed by EEM without mode reduction technique. We can see that the mode reduction technique can produce very accurate results with less than 30% of the total modes. The largest maximum absolute error of reflection and transmission for TE polarization is 4.3×10^{-3} and 7.0×10^{-4}, respectively. The largest maximum absolute error for reflection and transmission of TM polarization is 7.0×10^{-4} and 6.0×10^{-4}, respectively. At two fixed wavelengths, error of transmission and reflection computed by EEM with different N_* and N is shown in Fig. 4. Generally, high accuracy can be achieved by using less than 30% of the modes.

The second example is a high index contrast optical waveguide as shown in Fig. 1. The thickness of the waveguide core is $d = 0.22\,\mu$m and the depth of the "grooves" is $d_e = d$. The width of the "teeth" and "grooves" are $L_1 = 0.175\,\mu$m and $L_0 = 0.135\,\mu$m, respectively. There are 1024 periods. The refractive indexes of the waveguide core and substrate are $n_2 = 3.4$ and $n_3 = 1.45$, respectively. The superstrate is air. In numerical calculation, the transverse y direction is truncated by PMLs to an interval of length $3.08\,\mu$m with $1.54\,\mu$m in substrate and $1.32\,\mu$m in superstrate and discretized by $N = 280$ grid points. The reflection and transmission spectra for TE polarization calculated by EEM with and without mode reduction technique are given in Fig. 5. The number of retained modes in mode reduction technique is $N_* = 80$. The largest maximum absolute error for reflection and transmission is 2.5×10^{-4}. Fig. 6 shows the error of reflection computed by EEM with mode reduction technique with different N_* and N at wavelength $1.62\,\mu$m for TE polarization.

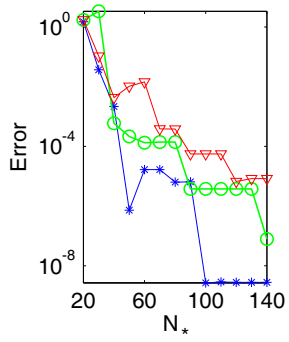

Figure 6. Error of reflection computed by EEM with mode reduction technique with different N_* and N in the second example at wavelength $\lambda = 1.62\,\mu$m for TE polarization. Blue star for $N = 280$, green circle for $N = 560$ and red triangle for $N = 1120$.

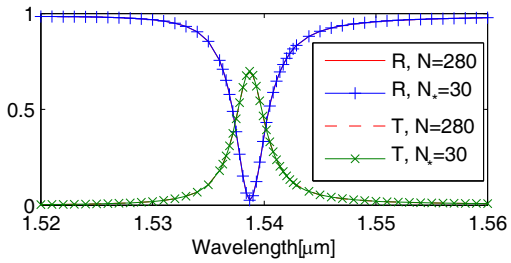

Figure 7. The reflection and transmission spectra of the defect waveguide calculated by EEM with and without mode reduction technique for TE polarization in the third example. $N_* = 30$ modes are used in the mode reduction technique.

In the third example, we consider a defect structure that has been studied in Section 5.3 in [21]. The defect waveguide contains 8 grooves with the central "tooth" be $1.515\,\mu$m width. Other parameters are the same as that in the second example. The reflection and transmission spectra calculated by EEM with and without mode reduction technique for TE polarization are shown in Fig. 7. The transverse direction is discretized by $N = 280$ grid points, and $N_* = 30$ modes are used in the mode reduction technique. The largest maximum absolute error for both reflection and transmission is 1.3×10^{-3} and 3.8×10^{-3}, respectively. For such short structures, mode reduction technique could gain high accuracy with about 10% of the total modes.

7. CONCLUSION

We have developed a mode reduction technique for eigenmode expansion method (EEM) to analyze 2–D waveguide grating structures. Our mode reduction is based on the scattering matrix method and contains three steps. First, eigenmodes of two different segments are calculated by a fourth-order finite

difference method after the transverse direction is truncated by PMLs. This step costs $O(N^3)$ operations with N as the number of discrete grid points in transverse direction. Second, the scattering matrix of a single period is computed using the full set of eigenmodes with $O(N^3)$ operations. Finally, the most important N_* ($N_* < N$) eigenmodes are selected elegantly and used to construct the scattering matrix of the whole structure by a recursive-doubling procedure. This requires $O(N_*^3 \log_2 m)$ operations for a structure with m periods. The total number of operations is $O(N^3)+O(N_*^3 \log_2 m)$. Numerical examples demonstrate that we can obtain a high accuracy with N_* less than one third of N.

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APPENDIX A. SCATTERING MATRIX METHOD WITH RECURSIVE-DOUBLING

In this section, a brief summery of how to use recursive-doubling procedure to calculate the scattering matrix $\mathbf{S}(x_1^-, x_m^+)$ of periodic structures is given in [6, 18]. Since the structure is symmetry, the scattering matrix $\mathbf{S}(x_1^-, x_2^+)$ for segment $x_1 < x < x_2$ can be written in blocks form as

$$\mathbf{S}(x_1^-, x_2^+) = \begin{bmatrix} \boldsymbol{T} & \boldsymbol{R} \\ \boldsymbol{R} & \boldsymbol{T} \end{bmatrix}. \tag{A1}$$

To compute matrices $\boldsymbol{T}$ and $\boldsymbol{R}$, we first compute the scattering matrix of a single interface. With expressions (5) and (4) truncated to N terms and using the continuity conditions of u at interface $x = x_1$, a $(2N) \times (2N)$ scattering matrix $\mathbf{S}(x_1^-, x_1^+)$ for interface $x = x_1$ can be obtained as

$$\mathbf{S}(x_1^-, x_1^+) \begin{bmatrix} \tilde{\boldsymbol{a}}_1 \\ \tilde{\boldsymbol{b}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(1)} & \mathbf{S}_{12}^{(1)} \\ \mathbf{S}_{21}^{(1)} & \mathbf{S}_{22}^{(1)} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{a}}_1 \\ \tilde{\boldsymbol{b}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_2 \\ \boldsymbol{b}_1 \end{bmatrix}, \tag{A2}$$

where $\tilde{\boldsymbol{b}}_2 = \boldsymbol{\Gamma}_2 \boldsymbol{b}_2$, and $\boldsymbol{\Gamma}_2$ is a $N \times N$ diagonal matrix with jth diagonal $e^{iL_0\beta_j^{(2)}}$ for $j = 1, 2, \ldots, N$. Similarly, form Equations (4) and (10) and the continuity conditions of u at $x = x_2$, we have a scattering matrix $\mathbf{S}(x_2^-, x_2^+)$ at interface $x = x_2$ as

$$\mathbf{S}(x_2^-, x_2^+) \begin{bmatrix} \tilde{\boldsymbol{a}}_2 \\ \tilde{\boldsymbol{b}}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(2)} & \mathbf{S}_{12}^{(2)} \\ \mathbf{S}_{21}^{(2)} & \mathbf{S}_{22}^{(2)} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{a}}_2 \\ \tilde{\boldsymbol{b}}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_3 \\ \boldsymbol{b}_2 \end{bmatrix}, \tag{A3}$$

where $\tilde{\boldsymbol{a}}_2 = \boldsymbol{\Gamma}_2 \boldsymbol{a}_2$. Eliminating $\tilde{\boldsymbol{a}}_2$ and $\tilde{\boldsymbol{b}}_2$ form Equations (A2) and (A3), we have

$$\boldsymbol{T} = \mathbf{S}_{22}^{(1)} \boldsymbol{\Gamma}_2 \boldsymbol{P} \mathbf{S}_{22}^{(2)}, \quad \boldsymbol{R} = \mathbf{S}_{21}^{(1)} + \mathbf{S}_{22}^{(1)} \boldsymbol{\Gamma}_2 \boldsymbol{P} \mathbf{S}_{21}^{(2)} \boldsymbol{\Gamma}_2 \mathbf{S}_{11}^{(1)}, \tag{A4}$$

where $\boldsymbol{P} = (I - \mathbf{S}_{21}^{(2)} \boldsymbol{\Gamma}_2 \mathbf{S}_{12}^{(1)} \boldsymbol{\Gamma}_2)^{-1}$ and I is the identity matrix.

To compute scattering matrix $\mathbf{S}(x_1^-, x_m^+)$ of the whole periodic structure, we first rewrite it as

$$\mathbf{S}(x_1^-, x_m^+) = \begin{bmatrix} \boldsymbol{T}_m & \boldsymbol{R}_m \\ \boldsymbol{R}_m & \boldsymbol{T}_m \end{bmatrix}. \tag{A5}$$

If the number of segments m is a power of 2, i.e., $m = 2^K$, let $\boldsymbol{T}_1 = \boldsymbol{T}$ and $\boldsymbol{R}_1 = \boldsymbol{R}$, the recursive-doubling procedure is given as

$$\boldsymbol{T}_{2^{k+1}} = \boldsymbol{T}_{2^k} \boldsymbol{\Gamma}_1 \boldsymbol{P}_{2^k} \boldsymbol{T}_{2^k}, \quad \boldsymbol{R}_{2^{k+1}} = \boldsymbol{R}_{2^k} + \boldsymbol{T}_{2^k} \boldsymbol{\Gamma}_1 \boldsymbol{P}_{2^k} \boldsymbol{R}_{2^k} \boldsymbol{\Gamma}_1 \boldsymbol{T}_{2^k}, \tag{A6}$$

for $k = 0, 2, \ldots, K-1$, where $\boldsymbol{\Gamma}_1$ is a $N \times N$ diagonal matrix with the jth diagonal element $e^{iL_1\beta_j^{(1)}}$ for $j = 1, 2 \ldots, N$ and $\boldsymbol{P}_{2^k} = (I - \boldsymbol{R}_{2^k} \boldsymbol{\Gamma}_1 \mathbf{R}_{2^k} \boldsymbol{\Gamma}_1)^{-1}$. The total number of operations is $O(N^3 K)$, i.e., $O(N^3 \log_2 m)$.

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