Progress In Electromagnetics Research M, Vol. 39, 131–139, 2014

Torque Calculation in Interior Permanent Magnet Synchronous Machine Using Improved Lumped Parameter Models

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Abstract—In this paper, we present improved Lumped-Parameter Models for simulation of a Interior Permanent Magnet Synchronous (IPMS) machine to calculate PM flux linkage, and Q and D-axis inductances which can be used for torque calculation. These improved models include all details of flux barriers and air bridges of rotor and also the effect of saturation in central posts and stator core. To validate the accuracy of these models, results are compared with the Finite Element Method results for a presented IPMS machine.

1. INTRODUCTION

Due to their high efficiency, power density, power factor and torque density, Interior Permanent Magnet Synchronous (IPMS) machines are increasingly being used in various applications, such as variable speed drives, electrical vehicles, and other industrial drives [1, 2]. Compared with Surface Permanent Magnet (SPM) machines, IPMS machines have robust rotor construction, high reluctance torque, and high demagnetization withstand. Also they are suitable for electric vehicles application which requires a wide constant power operating speed range [1, 3].

Since Finite Element Analysis (FEA) is highly time consuming for machine design and optimization process [3], researchers have always looked for analytical methods that can be used for the purpose of machine optimization. The existing methods such as Laplacian or quasi-Poissonian methods solve the field equations for surface permanent magnet machine [4] or inset permanent magnet machine directly and use the conformal mapping for taking the effects of slots into consideration [4, 5]. Anyhow, because of the leakage flux, saturation in different parts and the complicated structure of IPMS machine, it is not possible to use these analytical methods for the modeling and optimization of IPMS machine [6, 7]. The Magnetic Equivalent Circuits (MEC) method used for calculation of no-load and full-load field in inductions [8], switched reluctance [9], salient-pole synchronous [10], SPM [11] and IPMS machines [7] is not an appropriate approach for optimization purposes due to its complexity and modeling efforts. The saturating Lumped Parameter Model (LPM) is one of the most efficient methods for optimization of IPMS machines especially in high-load conditions [12, 13], since it can include machine complex geometry and saturation in stator and rotor cores [14]. Due to its fast and accurate results in calculating machine parameters, LPM can be used in optimization process that needs thousands of iterations for finding an optimal solution [15, 16].

In this paper, three different models of lumped parameter are used for calculation of average torque [6, 12]. First, a linear LPM is used for calculation of PM flux linkage [14, 17]. For simplicity, the reluctances of the rotor yoke and stator yoke can be ignored compared to the reluctance of the air gap. The level of saturation in iron bridges and central posts are assumed constant [18, 19]. Two other LPMs can be used for calculation D and Q-axes inductances. Since the effective air gap in D-axis is large [20], the reluctance of rotor yokes and stator yokes can be ignored for calculation of D-axis inductance, and also the variation under different loading conditions has no effect on the saturation level in iron bridges;

Received 30 September 2014, Accepted 27 October 2014, Scheduled 30 October 2014
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therefore, iron bridge length is added to flux barrier length, but central posts are modeled with variable reluctance. In these models Carter coefficient can be used for taking slot effect into consideration [2, 17]. Since candidate IPMS machine has three layers and has no skew in rotor, cross coupling is neglected from the D-axis magnets and stator excitation. The mechanism of cross coupling depends on particular geometry such as number of layers and rotor skew [21, 22].

In this paper, we have succeeded in accurately calculating PM flux linkage, inductances of Q- and D-axis and the average torque of a three-layer IPMS machine using specially designed LPM method consisting of the effects of saturation of D-axis and stator core with detailed model of flux barriers and central posts. The results of our method are in excellent agreement with those obtained by Finite Element Method (FEM).

Table 1. IPMS machine parameters.

Name	symbol	Quantity
Number of poles	N_p	12
Number of layers	k	3
Winding factor	K_{a1}	0.933
Stator and rotor core material	-	M-19
Core length	L	60 [mm]
Stator inner radius	R_{si}	109 [mm]
Rotor outer radius	R_{ro}	108.4 [mm]
Number of series turns per phase	N_a	24
Number of slots	Q_s	72
Remnant flux density	B_r	0.46 [T]
Magnet span layers	$\alpha_1, \alpha_2, \alpha_3$	8, 17.5, 26.2 [mech.degrees]
Central posts	w_c	1 [mm]
Magnet width	$h_{m1},\ h_{m2},\ h_{m3}$	2, 3, 4 [mm]
Flux barrier width	$d_1,\ d_2,\ d_3$	1, 2, 3 [mm]
Width of air bridges	$w_{b1},\ w_{b2},\ w_{b3}$	0.75, 1.5, 1 [mm]
Length of air gap	g	0.6 [mm]
Magnet length	$w_{m1},\ w_{m2},\ w_{m3}$	10.5, 22, 27 [mm]

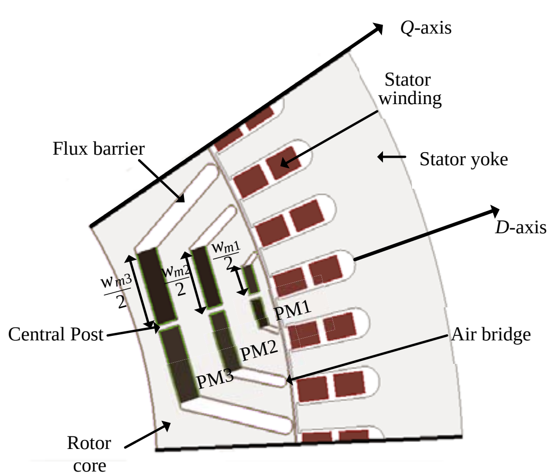

Figure 1. One pole of a 12-pole IPMS machine.

2. ELECTROMAGNETIC TORQUE

Figure 1 shows one pole of a 12-pole IPMS machine. In this configuration, each rotor pole contains three buried magnets. Machine basic parameters are given in Table 1.

The main aim of this paper is to obtain electromagnetic torque analytically. The following equation can be used to calculate the electromagnetic torque [1].

$$T_e = 3\left(\frac{N_p}{2}\right)\left(\lambda_{PM} I_q - \left(L_q - L_d\right) I_q I_d\right) \tag{1}$$

where N_p and λ_{PM} are the number of poles and PMs flux linkage, respectively. L_q, L_d, I_q and I_d are Q and D-axis inductances and current components, respectively. Although Equation (1) is typically used in linear LPM, it can be use for saturated LPM if the inductances are implicit functions of excitation [6]. Therefore, to obtain T_e, one needs to calculate PM flux linkage and Q and D-axis inductances by using the saturated LPM.

3. FLUX LINKAGE CALCULATION

The PM flux linkage, λ_{PM}, can be calculated using linear LPM. In this calculation, assumptions of constant magnetic vector potential in the stator and rotor cores, fixed magnet remanence, and saturated constant flux density iron bridges are made [3]. The PM flux linkage, λ_{PM}, is calculated using Equation (2) and linear magnetic circuit model [1, 3].

$$\lambda_{PM} = \frac{4\sqrt{2} R_{ro} L N_a K_{a1} B_1}{N_p} \tag{2}$$

In this equation, B_1 and K_{a1} are fundamental air gap flux density and fundamental winding coefficient, respectively. Figure 2 shows three-layer IPMS machine designated as PM1, PM2 and PM3 with flux line. The calculated flux lines using FEA clearly shows the locations of reluctances and flux sources. According to the paths of flux line obtained from FEA, LPM can be extracted as shown in Figure 3. In Figure 3, φ_{gk} for ($k = 1$ to 3) are air gap flux densities while the corresponding reluctances are $R_{gk} = \frac{g}{\mu_0 A_{gk}}$, where:

$$A_{gk} = \left(\alpha_{p(k)} - \alpha_{p(k-1)}\right) \frac{2\pi(R_{si} - \frac{g}{2})}{N_p} L, \quad \text{for} \quad (k = 2, 3) \tag{3}$$

$$A_{gk} = \alpha_{p(k)} \frac{2\pi(R_{si} - \frac{g}{2})}{N_p} L, \quad \text{for} \quad (k = 1) \tag{4}$$

where μ_0 is the permeability of air, and ($\alpha_{pk} = \frac{\alpha_k N_p}{2\pi}$) is the pole-arc to pole pitch ratio. $\varphi_{rk} = B_r w_{mk} L$, $\varphi_{mbk} = B_{satbk} L w_{bk}$, and $\varphi_{mbck} = B_{satck} L w_c$ are the flux sources, leakage fluxes of PMs through the

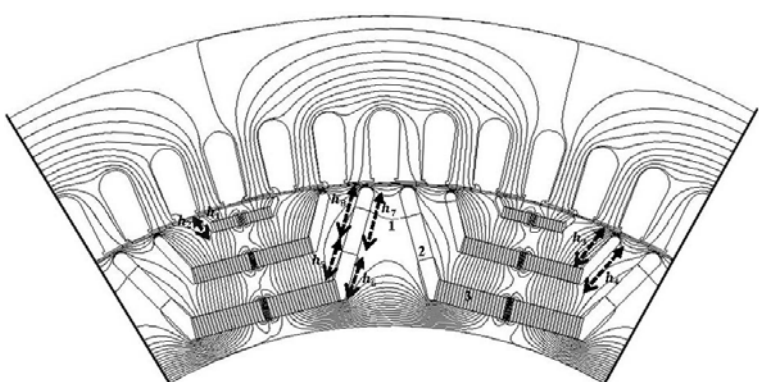

Figure 2. Flux lines for IPMS machine from FEM.

bridges, and leakage fluxes of PMs through the central posts, respectively. B_{satbk} and B_{satck} are the level of saturation in bridges and central posts, respectively. Since a part of flux lines of the third magnet flows through its own flux barrier and a part of it flows commonly through its own flux barrier and the flux barrier of the adjacent magnet, for LPMs, each flux barrier is divided into two reluctances. Hence, in Figure 2 the flux lines 1, 2, and 3 are corresponding to reluctances R_{kml1}, R_{1ml2} and R_{mok}, respectively.

$$R_{mok} = \frac{h_{mk}}{\mu_0 \mu_r w_{mk} L} \tag{5}$$

$$R_{kml1} = \frac{2h_{mk}}{\mu_0 L(h_n + h_{n+1})} \quad (k = 1,\ n = 1), \quad (k = 2,\ n = 3), \quad (k = 3,\ n = 5) \tag{6}$$

$$R_{3ml2} = \frac{4d_3}{\mu_0 L(h_7 + h_8)} \tag{7}$$

where h_n is the length of flux barrier. Since there is no saturation in stator and rotor yokes, R_{s1},

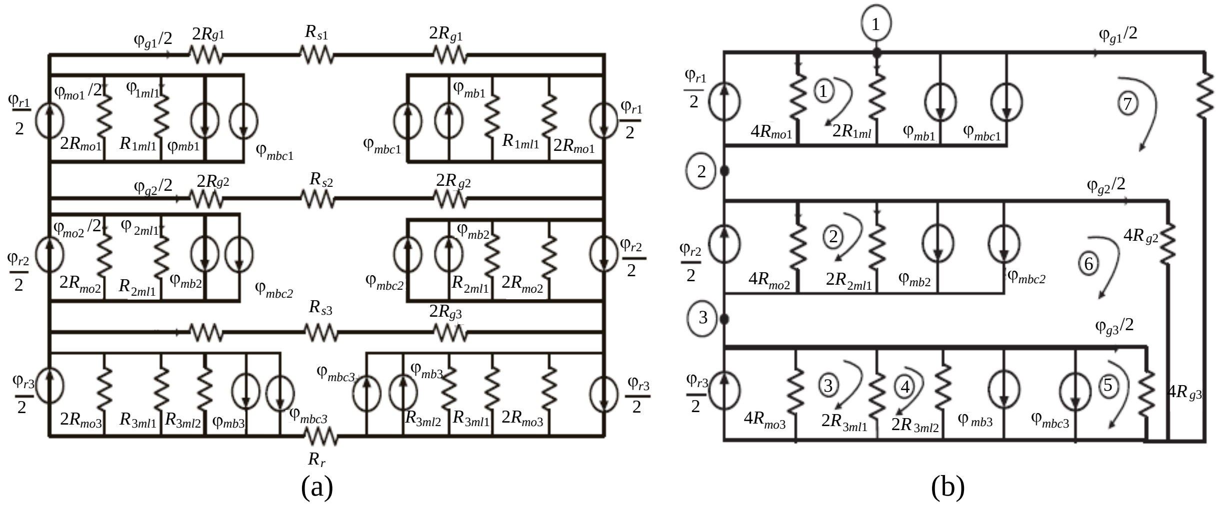

Figure 3. Lumped parameter model. (a) Lumped circuit model of IPMS. (b) Simplified lumped circuit model.

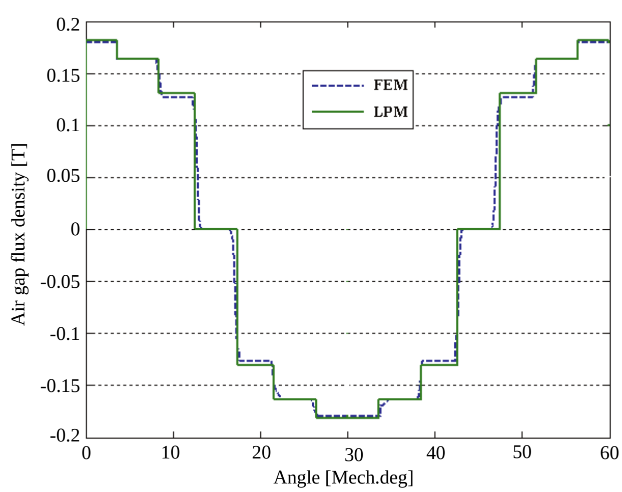

Figure 4. Air gap flux density without slot effect.

R_{s2}, R_{s3}, and R_r can be neglected in Figure 3(a) compared with R_{gk}. Therefore, Figure 3(a) can be simplified to Figure 3(b) due to symmetry. In this figure, the Kirchhoff's law is applied to loops 1–7 and nodes 1–3. So φ_{gk} for $k = 1$ to 3 can be obtained, and the average air-gap flux densities are $B_{g1} = \frac{\varphi_{g1}}{A_{g1}}$, $B_{g2} = \frac{\varphi_{g2}}{A_{g2}}$, $B_{g3} = \frac{\varphi_{g3}}{A_{g3}}$, respectively.

Figure 4 shows the resultant air gap flux without stator slot effects. For calculating λ_{PM}, air gap g is substituted with ($g_e = K_c g$) in all the equations above, where K_c is the Carter coefficient. Flux linkage calculation using LPM yields 9.2 mwb while flux calculation using FEM yields 9.3 mwb.

4. *D*-AXIS INDUCTANCE

The D-axis inductance is the sum of magnetization inductance and leakage inductance [6].

$$L_d = L_{dm} + L_l \tag{8}$$

L_l can be calculated as described in [1]. An analytical relationship between the unsaturated Q-axis inductance and D-axis inductance was developed by Vagati et al. in [23]. Therefore, in this paper the d-axis inductance will be given without detailed derivation of the equations. The D-axis magnetizing inductance is composed of 'through' and 'circulating' components. As described in [6] L_{dt} and L_{dc} are estimated using magnetic circuit analysis based on formulas given in [23].

$$L_{dm} = L_{dt} + L_{dc} \tag{9}$$

Figure 5(a) shows the per-unit magnetic circuit for IPMS machine with three layers, which is solved to determine the inductance components. Several geometric quantities in Figure 5(a) need further definitions. The angle, $\Delta\alpha_k$, is defined as the angular distance at the rotor surface between adjacent magnet flux paths such that $\Delta\alpha_k = \alpha_k - \alpha_{(k-1)}$. The cross-sectional areas, for the total stator air gap surface A_r, stator tooth pitch A_s, each central post A_{ck}, and each magnet A_{mk}, are defined as $A_r = 2\pi R_{si} L$, $A_s = \frac{A_r}{Q_s}$, $A_{ck} = w_c L$, and $A_{mk} = w_{mk} L$.

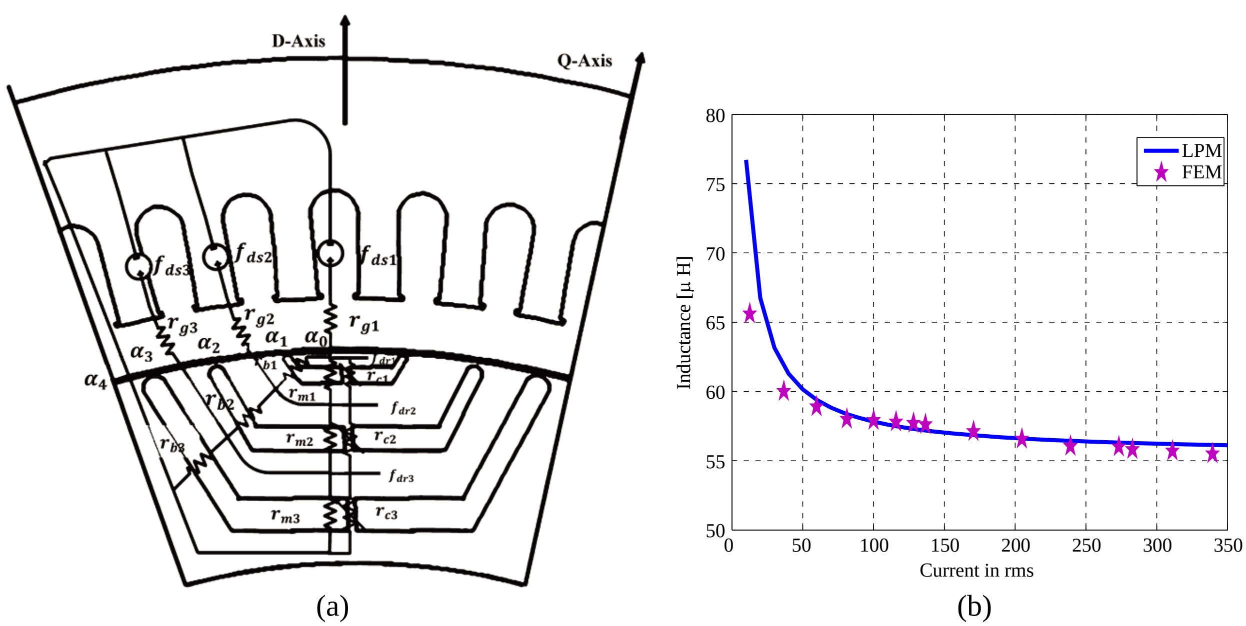

Figure 5. Equivalent circuit and inductance of d-axis. (a) Equivalent d-axis inductance LPM circuit. (b) Comparison of D-axis inductance using FEM and LPM.

Then per-unit circuit reluctance for magnet cavity and air gap segment can be defined as:

$$r_{mk} = \frac{2h_{mk}A_s}{g_e A_{mk}} \tag{10}$$

$$r_{gk} = \frac{\Delta\alpha_s}{\Delta\alpha_k} \tag{11}$$

$$r_{ck} = \frac{2h_{mk}A_s}{g_e A_{ck}}\left(\frac{I_{ds}}{I_b}\right) \tag{12}$$

r_{ck} is per-unit circuit reluctance for central post. Air bridge flux density variation with regard to changes in D-axis current is negligible. However, central post flux varies with the changes in d-axis current. Therefore, $\frac{I_{ds}}{I_b}$ coefficient is considered to compensate for these variations where I_{ds} and I_b are D-axis current and base current, respectively. The stator mmf per unit source for the kth peripheral segment is expressed as [12]:

$$f_{dsk} = \frac{\cos(\alpha_{k-1}) - \cos(\alpha_k)}{\Delta\alpha_k} \tag{13}$$

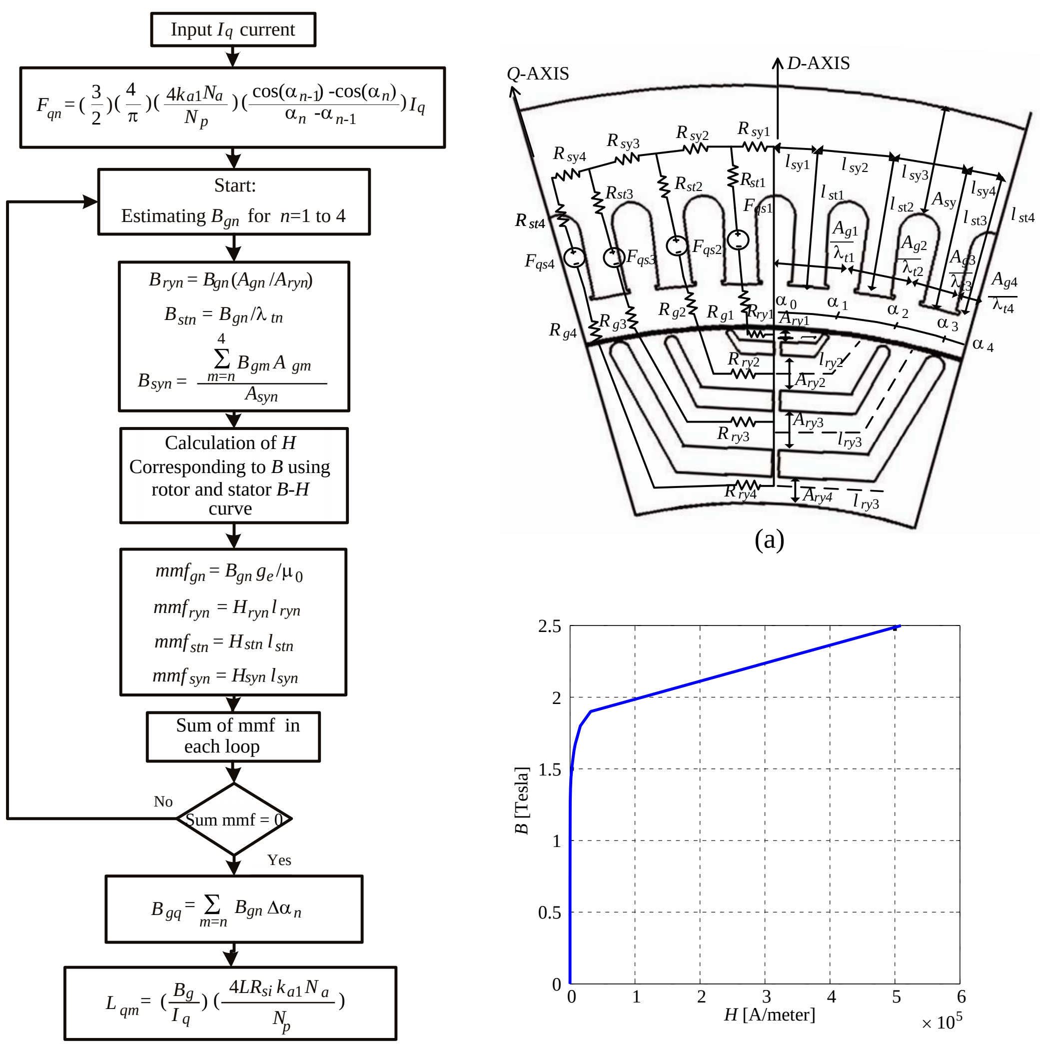

Figure 6. Calculation of Q-axis reluctance. (a) LPM for Q-axis inductances. (b) Flowchart for calculating Q-axis inductance. (c) B-H curve of M-19 steel.

For calculating f_{drk} we can use the Kirchhoff's law [3]. The D-axis inductance is calculated using the following equations [12, 23]:

$$\frac{L_{dc}}{L_{qm}} = 1 - \frac{4}{\pi}\sum_k \Delta\alpha_k f_{dsk}^2 \tag{14}$$

$$\frac{L_{dt}}{L_{qm}} = \frac{4}{\pi}\sum_k f_{dsk}(f_{dsk} - f_{drk})\Delta\alpha_k \tag{15}$$

where L_{qm} is Q-axis magnetization inductance and can be calculated from round rotor air gap inductance [1]:

$$L_{ag} = L_{qm} = \frac{3}{2}\left(\frac{4}{\pi}\right)\frac{\mu_0 N_a^2 K_{a1}^2 L R_{si}}{\left(\frac{N_p}{2}\right)^2 g_e} \tag{16}$$

The D-axis total magnetizing inductance is given by:

$$L_{dm} = \left(\frac{L_{dc}}{L_{qm}} + \frac{L_{dt}}{L_{qm}}\right) L_{qm} \tag{17}$$

Figure 5(b) shows the variation of D-axis inductance ($L_d = L_{dm} + L_l$) versus the current for the LPM and FEM methods, which are in great agreement.

5. Q-AXIS INDUCTANCE

For high-performance applications, the exciting MMF is likely to drive Q-axis path into the saturation region of the core material. To accurately predict L_q under rated load condition, a saturated model of the LPM is used here [6]. Figure 6(a) shows Q-axis reluctance model of the stator and rotor. In this model, the stator is partitioned in a manner similar to rotor. It is assumed that the magnetic flux is carried in a single flux tube from each rotor segment through the stator back iron. The element lengths are defined as the average length through each segment and are shown in Figure 6(a). In this figure, λ_{tn} is the effective teeth area calculated using the air gap area scaled by the slot pitch fraction. The MMF drop across each reluctance is calculated instead of calculating the reluctances. For calculating L_q using this model, the LPM is solved iteratively along each branch using the flowchart shown in Figure 6(b). Figure 6(c) shows B-H curve of M-19 steel for stator and rotor core.

Figure 7(a) shows the Q-axis inductances ($L_q = L_{qm} + L_l$) obtained from FEM and LPM which again shows great agreement.

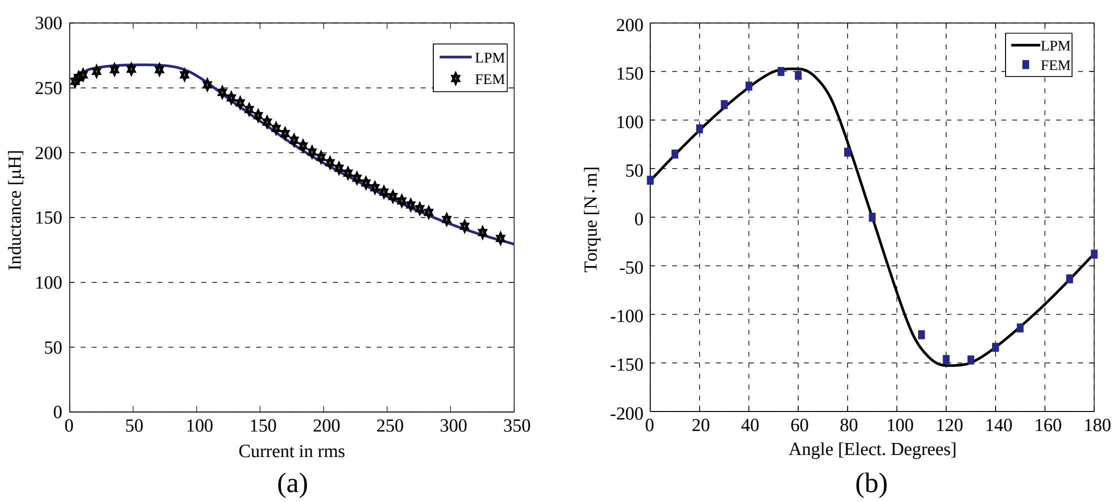

Figure 7. Comparison of Q-axis inductance and torque calculated. (a) Comparison of Q-axis inductance calculated using FEM and LPM. (b) Comparison of electromagnetic torque calculated using FEM and LPM.

6. TORQUE CALCULATION

We are interested in calculating the electromagnetic torque produced by the IPMS machine using LPM analysis. To validate the model, the results are compared with FEM ones for a candidate IPMS machine. Figure 7(b) shows electromagnetic torque calculated using FEM and LPM versus torque angle with constant rms phase current, which shows good accuracy of LPM method.

7. CONCLUSION

Comparison of the results obtained by the two methods (LPM and FEA) for inductances of D and Q-axes and electromagnetic torque shows the accuracy of the LPM method for three-layer IPMS machine having flux barrier and central posts. The improvement employed in the calculation of the D-axis inductance has resulted in more accuracy of D-axis inductance in lower currents. Because of the small number of elements used for calculation of inductances in the proposed LPM method, it can be used as one of the most efficient methods in the design optimization process. The model is modified to include flux barriers, central posts, and saturation at heavy loading conditions.

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