The Extended Gauge Transformations

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Abstract—In this work, new "extended gauge transformations" involving current and fields are presented. The transformation of Maxwell's equations under these gauges leads to a massive boson field (photon) that is equivalent to Proca field. The charge conservation equation and Proca equations are invariant under the new extended gauge transformations. Maxwell's equations formulated with Lorenz gauge condition violated give rise to massive vector boson. The inclusion of London supercurrent in Maxwell's equations yields a massive scalar boson satisfying Klein-Gordon equation. It is found that in superconductivity Lorenz gauge condition is violated, and consequently massive spin-0 bosons are created. However, the charge conservation is restored when the total current and charge densities are considered.

1. INTRODUCTION

We have shown recently [1] that a nonzero electric conductivity (σ) of vacuum leads to a nonzero mass for the photon. The possibility of a nonzero photon mass has been considered by several authors [2, 3], and the relation between this mass, m, the medium conductivity and the astrophysical implications were studied and outlined by Kar et al. [3]. The mass is found to be associated with an existence of a vacuum wave, which arises from a condensate state (scalar particle) satisfying Klein-Gordon equation. It is found that a generalized current that involves a vector potential can induce a mass term in Maxwell's equations. The additional term in the generalized current is related to Cooper pairs supercurrent [4]. The longitudinal wave will acquire a mass if the Lorenz gauge is not satisfied. Bass and Schrodinger pointed that a longitudinal wave will be generated whenever the vector potential is parallel to the electric field [5]. It thus pertinent to say that the gauge transformation is equivalent to introducing a vector potential parallel to the electric field. This additional term modifies the ordinary Maxwell's equations. The resulting equations are dubbed Maxwell-Proca equation [6]. While Maxwell's equations are invariant under gauge transformations, Proca equations are not. It is argued that the vector and scalar potentials in which the electric and magnetic fields are expressed are more genuine than the electric and magnetic fields, and not mere mathematical constructs as are generally presumed. This feature is already manifested in the Aharonov and Bohm experiment [7]. The gauge transformations of the generalized current lead to the appearance of a mass term in Maxwell's equations. We call these gauge transformations the "extended gauge transformations." The charge conservation equation is shown to be invariant under these transformations. It is widely thought that the inclusion of mass term in Maxwell's equations breaks the gauge invariance; however, the gauge symmetry is restored under the extended gauge transformations. This is a very interesting merit of the new gauge transformations. Thus, Maxwell's equation should be modified to incorporate interactions experienced by electromagnetic field inside a medium.

In field theoretic models a formal paradigm to generate a mass term for interacting particles is the Higgs mechanism [8]. The Higgs mechanism can be considered as the superconductivity in the vacuum.

It occurs when all of space is filled with a sea of charged particles, i.e., when these charged particles acquire a nonzero vacuum expectation value. The interactions with the quantum fluid comprised of these particles filling the space would prevent certain forces from propagating over long distances.

In a superconductor, however, electric charges move with no dissipation, and this allows for permanent surface currents, not only surface charges. When magnetic fields are introduced at the boundary of a superconductor, they produce surface currents which exactly neutralize them. The Meissner effect is due to currents in a thin surface layer whose thickness is the London penetration depth (λ_L) which can be calculated from a simple model. This simple model is introduced by Landau and Ginzburg, where they treated superconductivity as a charged Bose-Einstein condensate [9]. In an actual superconductor, the charged particles are electrons. So in order to have superconductivity, the electrons need to somehow bind into Cooper pairs. The charge of the condensate q is therefore twice the electron charge e. The pairing in a normal superconductor is due to lattice vibrations, i.e., the pairs are very loosely bound. Such a formulation has been performed by Bardeen, Cooper and Schrieffer [4].

The possibility and the effects of a non-zero photon rest mass is incorporated into the Proca equations [6]. An upper limit for the photon rest mass is $\sim 10^{-50}$ kg [10, 11]. If massive electromagnetic field possesses a Black-Body radiation spectra since the time of Big Bang to the present, then the photon mass can be estimated to be $\sim 10^{-68}$ kg. This is fifteen orders of magnitude smaller than the upper limit obtained by experimental techniques employed by Goldhaber and Nieto [12]. One of the consequences of a hypothesis of a massive photon is that the speed of the photon would depend on its frequency, as it is apparent from the theory of relativity [13].

This paper is organized as follows: we introduce the extended gauge transformations in Section 2. The London supercurrent ansatz is related to current-density transformations. Charge conservation is preserved under these transformations. It seems that in an elaborated electrodynamics theory Lorenz gauge condition is tantamount to charge conservation. If anyone is violated, a massive scalar particle is librated. The photon acquires a mass not in vacuum, but in a medium, e.g., a superconductor. In Section 3 we show that the application of the charge — current densities transformations in Maxwell's equations yield Maxwell-Proca equations. In Section 4 we show that van Vlaenderen equations can be obtained from Maxwell equations by applying the extended gauge transformations [14]. These formulations are found to lead to Klein-Gordon equation of spin-0 bosons. In Section 5 we show that the London's current can be associated with a superfield (and supercharge density) that follows Klein-Gordon equation. The scalar and vector potentials in Maxwell theory are shown to be of physical significance in Proca formulation. We have shown that in superconductivity the Lorenz gauge condition is violated, and consequently scalar (Higgs) particles with zero spin are librated as a result of converting electromagnetic energy to these massive bosons. The paper ends with some concluding remarks.

2. THE EXTENDED GAUGE TRANSFORMATIONS

We extend here ordinary gauge transformations to include charge and current densities. Further, we use the London's supercurrent to introduce charge-current densities transformations. We show that the charge conservation is preserved under these transformations. Maxwell's equations are [15]

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \tag{1}$$

and

$$\vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}. \tag{2}$$

Maxwell equations are invariant under the ordinary gauge transformations,

$$\vec{A}' = \vec{A} + \vec{\nabla}\Lambda, \quad \varphi' = \varphi - \frac{\partial \Lambda}{\partial t}. \tag{3}$$

We have shown earlier that the generalized continuity equation is invariant the transformations [16, 17]

$$\vec{J}' = \vec{J} + \vec{\nabla}\Lambda', \quad \rho' = \rho - \frac{1}{c^2}\frac{\partial \Lambda'}{\partial t}, \tag{4}$$

where Λ' is a scalar function satisfying the wave equation.

London proposed that electrons that took part in the formation of Cooper pairs have a ground state with zero total momentum. In this case, the supercurrent is given by [4]

$$\vec{J}_s = -\frac{n_s e^2}{m} \vec{A}, \tag{5}$$

where n_s is the number density of the Cooper pairs. The above relation suggests that the total conduction current in Maxwell's equation should be corrected by this additional term. Since this current contains the vector potential, $\vec{A}$, it should transform under gauge transformation in a similar manner to $\vec{A}$. Note that Maxwell's equations are invariant under the ordinary gauge transformation, Equation (3).

Equation (5) urges us to consider the charge-current densities transformations,

$$\vec{J} \to \vec{J} - \alpha \vec{A}, \quad \rho \to \rho - \frac{\alpha}{c^2} \varphi, \tag{6}$$

and

$$\vec{J}' = \vec{J} + \alpha \vec{\nabla}\Lambda, \quad \rho' = \rho - \frac{\alpha}{c^2} \frac{\partial \Lambda}{\partial t}, \tag{6a}$$

where $\alpha\Lambda = \Lambda'$. We call Equations (3) and (6a) the extended gauge transformations. It is apparent that the transformation in Equation (6) mixes the polar and axial current. We encounter this situation in the electroweak theory, where neither the polar current nor the axial current is conserved separately, but the difference is a conserved current. Equations (5) and (6) suggest that one may choose, $\alpha = \frac{n_s e^2}{m}$. The extra current and charge densities in Equation (6) may be attributed to the influence of the electromagnetic fields in the vacuum. Such fields may destabilize (polarize) the vacuum creating avalanche of particles producing the above charge and current densities. It is interesting to see that if we apply Equation (6) in Maxwell's equation we will get Proca equation for massive vector field (see Section 3).

The charge and current densities in Equation (6) satisfy the continuity equation

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0, \tag{7}$$

provided that the Lorenz gauge

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0, \tag{8}$$

is satisfied. Moreover, the continuity equation (charge conservation) in Equation (7) is invariant under the combined gauge transformations in Equation (6a) provided that Λ satisfies the wave equation

$$\frac{1}{c^2} \frac{\partial^2 \Lambda}{\partial t^2} - \nabla^2 \Lambda = 0. \tag{9}$$

Hence, Λ is a scalar function satisfying the wave equation with zero mass. It seems that in a complete electrodynamics theory, the charge conservation is tantamount to Lorenz gauge condition.

3. THE MASSIVE FIELD MAXWELL EQUATIONS

We show here that the application of the charge-current densities in Maxwell's equations yields Maxwell-Proca equations. Let us now apply the current-density transformation in Equation (6) in Maxwell equations, i.e., Equations (1) and (2), to get

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \alpha\mu_0\varphi, \tag{10}$$

and

$$\vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \alpha\mu_0 \vec{A}, \tag{11}$$

where $\alpha\mu_0 = \mu^2$. Maxwell-Proca equations are known not to be gauge invariant. However, it is interesting to see that the Proca equations, Equations (10) and (11), are invariant under the extended gauge transformations, Equations (3) and (4).

Now Proca equations, Equations (10) and (11), are invariant under the extended gauge transformations in Equation (6a), together with Equation (3). The generalized continuity equations are expressed as [17],

$$c^2\vec{\nabla}\rho + \frac{\partial\vec{J}}{\partial t} = 0, \quad \vec{\nabla}\times\vec{J} = 0, \quad \vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0. \tag{a}$$

The continuity equations are also invariant under the extended gauge transformations provided that

$$\frac{1}{c^2}\frac{\partial^2\Lambda}{\partial t^2} - \nabla^2\Lambda = 0. \tag{b}$$

This is indeed the case for Λ as in the original gauge transformations.

In terms of the electromagnetic tensor,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{12}$$

Equations (10) and (11) can be derived from the Lagrangian

$$\mathcal{L} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} - J_\mu A^\mu + \frac{1}{2}\alpha A_\mu A^\mu. \tag{13}$$

The extended transformations, Equations (3) and (6a), can be expressed in a covariant form as

$$J^{\mu\prime} = J^\mu - \alpha\,\partial^\mu\Lambda, \quad A^{\mu\prime} = A^\mu - \partial^\mu\Lambda. \tag{6b}$$

Here, $\alpha\mu_0 = \frac{n_s e^2 \mu_0}{m} = \frac{1}{\lambda_L^2}$, where λ_L is London penetration depth of the superconductor. The latter relation would imply that the photon mass is deeply connected with λ_L via the relation, $m_\gamma = \frac{\hbar}{\lambda_L c} \propto n_s^{1/2}$. This is a quite interesting result which suggests that the phenomenon of superconductivity is intimately related to massive vector boson (photon). Thus, the denser the Cooper pairs, the heavier the photon. It is argued by some authors that the photon mass resulted from the longitudinal part of the electromagnetic field, while the mass resulting from the transverse component is zero. Unlike massless photons, massive photons do interact with each other. A new estimate for photon mass can be obtained by assuming that the Wein's displacement law ($\lambda_{\max}T = 2.9 \times 10^{-3}\,\mathrm{mK}$) still holds at cosmic level (with universe radius of $10^{26}\,\mathrm{m}$). In this case the photon temperature today will be $T \sim 10^{-29}\,\mathrm{K}$ in comparison with a massive photon that had a Planck mass at Planck's time. Its mass at cosmic level will be $\sim 10^{-68}\,\mathrm{kg}$. This limit is fifteen orders of magnitude less compared to some recent limits. Consequently, two interacting spin-1 massive photons can give rise to effective spin-0 and spin-2 bosons. These result in creating scalar, and tensor particles like gravitons. Scalar particles obey Klein-Gordon equation, and spin-2 particles obey Einstein general relativity equations.

4. MAXWELL — KLEIN-GORDON WAVE EQUATIONS

We show here that van Vlaenderen equations can be obtained from Maxwell's equations by applying the extended gauge transformations [14]. However, if the Lorenz gauge condition is not satisfied, we obtain the Klein-Gordon equation. Let us now apply the transformations, Equation (6a), in Maxwell equations, Equations (1) and (2), to obtain

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} - \alpha\mu_0\frac{\partial\Lambda}{\partial t}, \tag{14}$$

and

$$\vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} + \alpha\,\mu_0\vec{\nabla}\Lambda, \tag{15}$$

These equations are recently obtained by extending Maxwell's equations to include scalar wave [18]. If we compare these equations with Equations (10) and (11) we deduce that

$$\frac{\partial \Lambda}{\partial t} = \varphi, \quad \vec{\nabla}\Lambda = -\vec{A}, \tag{16}$$

Equation (16) satisfies Lorenz gauge condition provided Λ satisfies the wave equation (with zero mass), Equation (9). However, if the Lorenz gauge condition is not satisfied and in particular if

$$\frac{1}{c^2}\frac{\partial \varphi}{\partial t} + \vec{\nabla}\cdot\vec{A} = -\alpha\mu_0\Lambda, \tag{17}$$

then Equation (16) yields

$$\frac{1}{c^2}\frac{\partial^2 \Lambda}{\partial t^2} - \nabla^2\Lambda + \mu^2\Lambda = 0, \tag{18}$$

where $\mu^2 = \alpha\mu_0$. Taking the divergence of the second equation in Equation (15), using Equations (16) and (17), yield

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = -\alpha^2\mu_0\Lambda. \tag{c}$$

Subtracting Equation (c) from Equation (17) yields

$$\frac{\partial \rho_T}{\partial t} + \vec{\nabla}\cdot\vec{J}_T = 0, \tag{d}$$

where $\vec{J}_T = \vec{J} - \alpha\vec{A}$ and $\rho_T = \rho - \frac{\alpha}{c^2}\varphi$. Hence, the total charge is conserved. This result is consistent with Equation (6). Equation (18) is the Klein-Gordon equation of a massive spin-0 particle (scalar field, Λ). Therefore, whenever Lorenz gauge condition is broken, a massive scalar field is generated. Thus, the massless field (Λ) associated with satisfaction of Lorenz gauge condition becomes massive when this condition is broken. The scalar mass is $m_\Lambda = \hbar\mu/c$. It is worth to mention that Proca equations satisfy Klein-Gordon equation for a massive vector field. Equation (16) together with Equation (3) implies that $\vec{A}' = \varphi' = \vec{E}' = \vec{B}' = 0$. It seems from Equation (16) that the electromagnetic potentials (energy) representing the spin-1 bosons are transferred to the massive spin-0 scalar Λ (by giving it energy and momentum). In field theory, whenever a local symmetry is broken a massive field is generated. This known as Higgs mechanism. We argue that such a scalar wave can result from the vacuum charges movements. And since the vacuum is neutral, only condensate states of fermions with zero spin can be produced. Such states can then be propagated as spin-0 scalar with a mass that appears to be associated with the photon. We therefore emphasize that the Lorenz gauge is broken in superconductivity. These condensate states may mimic the Cooper pairs in superconductors. In this case the mass m_Λ will be twice the mass of the electron, i.e., $m_\Lambda = 2m_e$ so that $n_s = 7.58 \times 10^{38}\,\text{m}^{-3}$. The length scale associated with α is then $\lambda_L = 0.0019\,\text{Å}$. Interestingly, this number density coincides with that of white dwarfs where electrons form degenerate electron gas. This allows us to assume that the white dwarfs are in state of Cooper pairs condensation. The scalar wave emanating from condensation of Cooper pairs permeates the whole space even inside the material object. This is nothing but the *ether* that had been hypothesized to fill the whole space! Thus, the ether is back again.

In a medium characterizes by its polarization $\vec{P}$ and magnetization $\vec{M}$ Maxwell's equations read

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla}\cdot\vec{E} = \frac{1}{\varepsilon_0}\left(\rho - \vec{\nabla}\cdot\vec{P}\right), \tag{19}$$

and

$$\vec{\nabla}\cdot\vec{B} = 0, \quad \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \frac{\partial \vec{P}}{\partial t} + \vec{\nabla}\times\vec{M}\right) + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}, \tag{20}$$

where $\frac{\partial \vec{P}}{\partial t}$ and $\vec{\nabla}\times\vec{M}$ represent the polarization and magnetization current densities, respectively. We can make an analogy between Equations (14) & (15) and Equations (19) & (20). It implies that

$$\frac{\alpha}{c^2}\frac{\partial \Lambda}{\partial t} = \vec{\nabla}\cdot\vec{P}, \tag{21}$$

and

$$\alpha \vec{\nabla} \Lambda = \frac{\partial \vec{P}}{\partial t} + \vec{\nabla} \times \vec{M}. \tag{22}$$

Now differentiate Equation (21) partially with respect to time and subtract it from the divergence of Equation (22) to obtain

$$\frac{1}{c^2} \frac{\partial^2 \Lambda}{\partial t^2} - \nabla^2 \Lambda = 0. \tag{23}$$

Let us take the gradient of Equation (21) and subtract it from the partial derivative with respect to time of Equation (22) to get

$$\frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} - \nabla^2 \vec{P} = \vec{\nabla} \times \left(\vec{\nabla} \times \vec{P} - \frac{1}{c^2} \frac{\partial \vec{M}}{\partial t} \right). \tag{24}$$

This equation shows clearly that the propagation of an electromagnetic wave in a material medium induces a polarization wave that reflects the response of the medium to this perturbation. Moreover, the polarization and magnetization are coupled to each other.

Now consider a medium for which

$$\vec{\nabla} \times \vec{P} = \frac{1}{c^2} \frac{\partial \vec{M}}{\partial t}, \tag{25}$$

leads to

$$\frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} - \nabla^2 \vec{P} = 0, \quad \frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} - \nabla^2 \vec{M} = 0, \tag{26}$$

where $\vec{\nabla} \cdot \vec{M} = 0$. Equation (25) can be compared with Equation (2) (in free space) if we make the analogy $\vec{P} \rightarrow \vec{B}$ and $\vec{M} \rightarrow \vec{E}$ and that $\vec{E} = c\vec{B}$ so that $\vec{M} = c\vec{P}$, too. Hence, a time variation of magnetization induces polarization. Equation (26) states that for such a medium both $\vec{P}$ and $\vec{M}$ induce effects that travel with speed of light. We therefore, see that the application of charge-current density transformation in Maxwell's equation is like introducing a medium with polarization and magnetization in Maxwell's equations.

5. KLEIN-GORDON EQUATION AND SUPERCONDUCTING FIELD

Let us take the divergence of Equation (5) and use Equation (8) to obtain

$$\vec{\nabla} \cdot \vec{J}_s + \frac{\partial}{\partial t} \left(-\frac{n_s e^2}{mc^2} \varphi_s \right) = 0. \tag{27}$$

If we compare this with Equation (7), we find out that

$$\rho_s = -\frac{n_s e^2}{mc^2} \varphi_s. \tag{28}$$

Now if we define the superelectric field as

$$\vec{E}_s = -\frac{\partial \vec{A}_s}{\partial t} - \vec{\nabla} \varphi_s, \tag{29}$$

then Gauss's law, Equation (1), reads

$$\frac{1}{c^2} \frac{\partial^2 \varphi_s}{\partial t^2} - \nabla^2 \varphi_s + \frac{n_s e^2}{mc^2 \varepsilon_0} \varphi_s = 0, \tag{30}$$

using Equations (7) and (8). This implies the ρ_s or φ_s that is associated with the supervector potential satisfies the Klein-Gordon equation of a field with mass (m_s)

$$m_s = \frac{\hbar e}{c} \sqrt{\frac{n_s \mu_0}{m}}. \tag{31}$$

This is equal to m_Λ obtained above for spin-0 particles. This can be related to London penetration depth, λ_L, by

$$\lambda_L = \frac{\hbar}{m_s c}, \tag{32}$$

which is a Compton-like relation. For ordinary conductors, $n \sim 10^{28}\,\mathrm{m}^{-3}$, one has, $m_s \sim 10^{-35}\,\mathrm{kg}$. Notice that m_s is higher for higher number density. Hence, superconductivity is a manifestation of a system of spin-0 scalar particles. Moreover, the inclusion of London supercurrent is equivalent to introducing a massive scalar field satisfying Klein-Gordon equation.

6. ENERGY CONSERVATION

The electromagnetic fields satisfy the energy conservation equation

$$\vec{\nabla} \cdot \vec{S} + \frac{\partial u}{\partial t} = -\vec{J} \cdot \vec{E}, \tag{33}$$

where

$$\vec{S} = \frac{1}{\mu_0} \left(\vec{E} \times \vec{B} \right), \quad u = \frac{1}{2} \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right). \tag{34}$$

Applying the density-current transformations, Equation (6), in Equation (33) yields the same equation as Equation (33), but with

$$\vec{S} = \frac{1}{\mu_0} \left(\vec{E} \times \vec{B} + \mu^2 \varphi \vec{A} \right), \quad u = \frac{1}{2} \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 + \varepsilon_0 \mu^2 \varphi^2 + \frac{\mu^2}{\mu_0} A^2 \right), \tag{35}$$

where we have employed the definition $\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}$, in the right-hand side in Equation (33). Equation (35) implies that the fields, φ and $\vec{A}$ are dynamical having both energy and flux density. Therefore, these fields are not mere mathematical constructs, but real and physical entities. Equation (35) is the energy conservation equation obtained for Proca equations [10]. Equations (16) and (18) which state that the electromagnetic energy carried by the field are converted into massive scalar particles (Λ). Hence, Equation (35) for $\vec{E} = \vec{B} = 0$ implies that the created scalar particles have energy and flux densities, $viz.$, $\vec{S} = -\mu^2 \varphi (\vec{\nabla}\Lambda)$, and $u = \frac{1}{2}(\varepsilon_0 \mu^2 (\frac{\partial \Lambda}{\partial t})^2 + \frac{\mu^2}{\mu_0}(\vec{\nabla}\Lambda)^2)$. The dissipation term ($\vec{J} \cdot \vec{E} = 0$) vanishes so that the scalar particles travel with high penetrating energy.

7. CONCLUSIONS

We have studied in this paper the implications of introducing the extended gauge transformations. The non invariance of Lorenz gauge condition in Maxwell's theory is shown to lead to Maxwell-Proca equations. Maxwell-Proca equations can also be derived from Maxwell equations by applying Equations (3) and (6a), or by applying the charge-current densities transformations inspired by London ansatz. Proca equations are known not to be invariant under the ordinary gauge transformation. However, Proca equations are now invariant under the extended gauge transformations we presented here. Abandoning the Lorenz gauge condition in Maxwell's equations also leads to an emergence a longitudinal (electroscalar) wave having a nonzero mass. The ordinary gauge transformations are equivalent to decomposing the vector potential into parallel and transverse components. Moreover, the spatial and temporal variations of the scalar wavefunction, Λ, are the parallel to $(\vec{A})$ and proportional to (φ), respectively. The introduction of London supercurrent in Maxwell equations is responsible for the existence of a massive scalar field satisfying Klein-Gordon equation.

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