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Study on Available Condition of Static Circuit Parameters Applied to Predict the Transmission Characteristics of Step Microstrip Line

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Abstract—The static circuit parameters extracted from the field results of non-uniform microstrip line provides an efficient way to predict dynamic effect of non-uniform structure. The predictable frequency range of the static circuit parameters on prediction of the transmission characteristics of step microstrip line is researched in this paper. The circuit parameters are extracted from the full wave results of step line, respectively, at three frequencies (9 GHz, 15 GHz and 20 GHz). On one hand, the time domain transmission characteristics of step line can be solved from the equivalent circuit constructed by these extracted circuit parameters. On the other hand, the frequency domain S-parameter can be derived by the static distributed characteristic impedance. By comparing these time and frequency domain results obtained from the static circuit parameters with those obtained directly from the full wave method, the available condition of the static circuit parameters of the step microstrip line can be analyzed. This comparison shows that the static circuit parameters can be used in frequency bands from DC up to 20 GHz. To verify the accuracy of the static parameters used to predict the transmission characteristics of step line, the measured S_{11} is also given for comparison.

1. INTRODUCTION

The microstrip line has attracted considerable attention in recent years attributing to its compact size, broad bandwidth, and easy series and shunt connection [1]. As an impedance transformation structure, step microstrip line is widely used in the junctions of microwave integrated circuits (MIC). In a MIC system, the reflection, radiation and coupling caused by the step structure increases proportionally to the operational frequency. When the step line operates at the microwave and millimeter wave bands, conventional circuit theory is not suitable for precise analysis of transmission characteristics of step line [2, 3]. In order to solve this problem, some common full wave methods, such as Finite Difference Time Domain (FDTD), Method of Moment (MOM) and Finite Element Method (FEM) can be utilized [4–9]. Circuit designers, however, may not be familiar with these field methods and are better at constructing equivalent circuits with circuit parameters and working out transmission characteristics from the circuit, although some work has begun to focus on the extraction of distributed circuit parameters from the field results. These work includes studying the generalized transmission line equations (GTLE) method playing bridge role in connecting circuit parameters with field results [10, 11], determining the capacitance and conductance matrices of a lossy shielded coupled microstrip line [12], and the approximate formula derivation for distributed circuit parameters of a bent line by using full wave results and optimization methods [13]. In our previous work, the feasibility of the static circuit parameters (extracted at a certain frequency) applied to predict the transmission characteristics of a non-uniform microstrip line is verified [14–16]. However, what is the frequency range in which we can use

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the static parameters to accurately predict the dynamic transmission characteristics of microstrip line; and what about the available condition of the extracted circuit parameters when extracting frequency changes? Little work on these problems can be found. The motivation of this paper is to do some work on these aspects. In this paper, the field results of step microstrip line are obtained by CST at three frequencies (9 GHz, 15 GHz and 20 GHz), respectively. Three groups of circuit parameters corresponding to three frequencies are extracted from the field results by solving GTLE. The equivalent circuits at three frequencies are then constructed with the extracted circuit parameters [17–20]. The voltage and current response at any point of the step line are solved from the constructed circuits. By comparing the solved voltage waveforms with those obtained directly from CST, the availability of the static circuit parameters used to predict the time domain waveforms on step line can be studied. Moreover, based on the extracted circuit parameters, the reflection coefficient S_{11} can be acquired by means of the transmission line method. By comparing the results with those obtained directly from CST, the available condition of the extracted static circuit parameters can be found. Finally, in order to verify the accuracy of the static parameters on prediction of the transmission characteristics of step line, the measured S_{11} is also given for comparing with S_{11} obtained from the static circuit parameters.

2. THEORIES AND METHODS

2.1. Extraction of the Static Circuit Parameters

CST 3D electromagnetic simulation software is used in this paper to simulate the step microstrip line shown in Fig. 1. The field results around the step line operating at a certain frequency can be calculated. The voltage distribution $u(l)$ and current distribution $i(l)$ along the step line are then obtained by integrating the field results, where l stands for the distance measured from the excitation point. In order to calculate the distributed circuit parameters from the GTLE as expressed by (1), two groups of $u(l)$ and $i(l)$ at different load conditions need to be substituted into the Equation (2) derived from the GTLE. Then the distributed inductance $L(l)$, capacitance $C(l)$, series voltage dependence source coefficient $\alpha(l)$, and shunt current dependence source coefficient $\beta(l)$ at a certain frequency can be obtained. It must be noted that all the extracted circuit parameters above are the parameters per unit length.

$$\begin{cases} \dfrac{\partial u(l)}{\partial l} = -j\omega L(l) i(l) + \alpha(l) u(l) \\ \dfrac{\partial i(l)}{\partial l} = -j\omega C(l) u(l) + \beta(l) i(l) \end{cases} \tag{1}$$

$$\begin{cases} L(l) = -\dfrac{u_1(l)\dfrac{\partial u_2(l)}{\partial l} - u_2(l)\dfrac{\partial u_1(l)}{\partial l}}{j\omega\left[u_1(l)i_2(l) - u_2(l)i_1(l)\right]} \\ C(l) = -\dfrac{i_1(l)\dfrac{\partial i_2(l)}{\partial l} - i_2(l)\dfrac{\partial i_1(l)}{\partial l}}{j\omega\left[i_1(l)u_2(l) - i_2(l)u_1(l)\right]} \\ \alpha(l) = \dfrac{i_2(l)\dfrac{\partial u_1(l)}{\partial l} - i_1(l)\dfrac{\partial u_2(l)}{\partial l}}{u_1(l)i_2(l) - u_2(l)i_1(l)} \\ \beta(l) = \dfrac{u_2(l)\dfrac{\partial i_1(l)}{\partial l} - u_1(l)\dfrac{\partial i_2(l)}{\partial l}}{i_1(l)u_2(l) - i_2(l)u_1(l)} \end{cases} \tag{2}$$

2.2. Calculation of the Time Domain Waveform

Based on the circuit parameters extracted at a certain frequency, the equivalent circuit of step line can be constructed as shown in Fig. 2. The equivalent circuit consists of a number of circuit units; and a circuit unit corresponds to a small segment of microstrip line with length of Δl. In Fig. 2, U_s, Z_s,

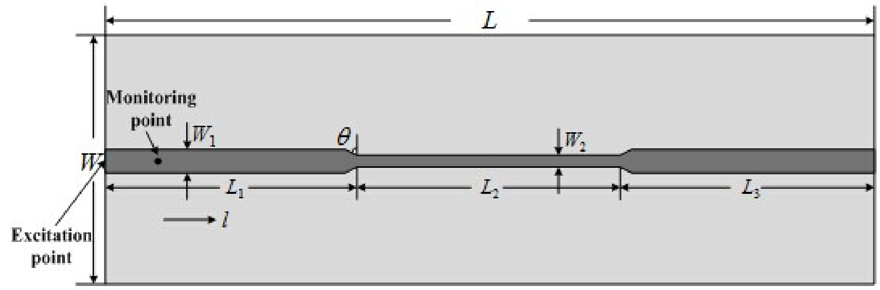

Figure 1. Geometry of the proposed structure.

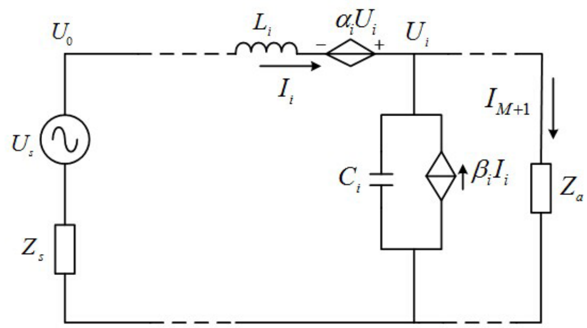

Figure 2. Equivalent circuit of the step microstrip line.

and Z_a are the excitation source, source impedance, and terminal impedance, respectively. C_i, L_i, α_i, and β_i are the distributed circuit parameters of the ith circuit unit; where $C_i = C(l)\Delta l$, $L_i = L(l)\Delta l$, $\alpha_i = \alpha(l)\Delta l$ and $\beta_i = \beta(l)\Delta l$. Through solving the equivalent circuit, the iterative formulas for the voltage and current in time domain can be obtained:

$$I_i^{n+\frac{1}{2}} = \frac{\Delta t}{L_i}\left[U_{i-1}^n - (1+\alpha_i)U_i^n\right] + I_i^{n-\frac{1}{2}}, \quad i = 1, 2, \ldots, M \tag{3}$$

$$U_i^n = \frac{\Delta t}{C_i}\left[(1-\beta_i)I_i^{n-\frac{1}{2}} - I_{i-1}^{n-\frac{1}{2}}\right] + U_i^{n-1}, \quad i = 1, 2, \ldots, M \tag{4}$$

The boundary conditions are:

$$U_0^n = -U_0^{n-1} + U_s^n + U_s^{n-1} - 2Z_s I_1^{n-\frac{1}{2}} \tag{5}$$

$$I_{M+1}^{n+\frac{1}{2}} = -I_{M+1}^{n-\frac{1}{2}} + \frac{2}{Z_a}U_M^n \tag{6}$$

In above equations, Δt is the time interval used to discretize calculation time, m is the cell number, n is the number of time moment, U_0 is the input voltage of circuit, and I_{M+1} is the current of load terminal. In addition, due to the discrete approximation in the iterative formula of the time domain response, the improper value of Z_s and Z_a will result in divergent results. Based on the window effect of time domain signal, we can separate the reflected wave from the whole wave to avoid the effect of Z_s and Z_a if the transmission line is long enough. According to reference [15], Z_s and Z_a are selected as $0\,\Omega$ and $1500\,\Omega$, respectively.

2.3. Calculation of the Reflection Coefficient

Based on the static inductance and capacitance obtained in Section 2.1, the characteristic impedance of each segment along the step line can be calculated by Equation (7).

$$Z_c = \sqrt{\frac{L}{C}} \tag{7}$$

If the input impedance Z_{in} of next segment is regarded as the load Z_l of the current segment, the input impedance of each segment along the step line can be derived by the following formula:

$$Z_{in}^{M-i} = Z_c^{M-i} \frac{Z_{in}^{M+1-i} + jZ_c^{M-i} \tan(\beta\Delta l)}{Z_c^{M-i} + jZ_{in}^{M+1-i} \tan(\beta\Delta l)}, \quad i = 0, 1, 2, \ldots, M \tag{8}$$

For matching load, we can assume that the terminal load Z_l is equal to Z_c of the last segment. The input impedance of the second segment ($i = 1$) can be derived by Equation (8), and the load impedance of the first segment ($i = 0$) is equal to this input impedance. So the reflection coefficient S_{11} (in dB) at the input port can be obtained.

$$|S_{11}| = 20 \lg \left| \frac{Z_{in}^1 - Z_c^0}{Z_{in}^1 + Z_c^0} \right| \tag{9}$$

3. AVAILABILITY OF THE STATIC CIRCUIT PARAMETERS IN TIME DOMAIN

The step microstrip line like that shown in Fig. 1 is analyzed by methods mentioned in Section 2. In the calculation, the substrate is 0.762 mm in thickness, the relative permittivity ε_r is 3.48, and other structure parameters, as shown in Fig. 1, are listed in Table 1. Setting the operating frequency at 9 GHz, 15 GHz and 20 GHz, respectively, three groups of static circuit parameters can be extracted. The equivalent circuits of the step line are then established at the three frequencies. To obtain the time domain response, the equivalent circuit is excited with Gaussian pulse: $U_s(t) = \exp[-g^2(t - t_{\max})^2]$. Here, the pulse peak moment $t_{\max}$ is set to $2.146g^{-1}$.

The voltage pulse at the monitoring point is solved from the equivalent circuit at 9 GHz, and the monitoring point is located at 5 mm from the excitation source. Figs. 3 and 4 show the calculated voltage pulses for cases where Gaussian parameters g are equal to $18 \times 10^9\,\mathrm{s}^{-1}$ and $24 \times 10^9\,\mathrm{s}^{-1}$, respectively. The results obtained directly by CST are also given for comparison. In CST, the length of segment in the direction of l is 1 mm, and the minimum and maximum mesh step are 0.035 mm and 0.220763 mm, respectively. Both of the ports are waveports, and two terminal impedances are perfect match. Furthermore, the microstrip line is perfectly conducting. From the two figures, the following conclusions can be drawn: (1) the extracted static distributed circuit parameters can be used to predict

Table 1. Structure parameters of the step microstrip line (in millimeters).

W	L	W_1	W_2	$L_1(L_2, L_3)$
30	75	1.7	0.8	25

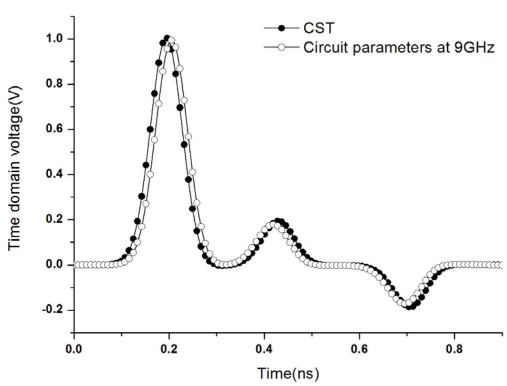

Figure 3. Time domain voltage waveforms at the monitoring point when $g = 18 \times 10^9\,\mathrm{s}^{-1}$.

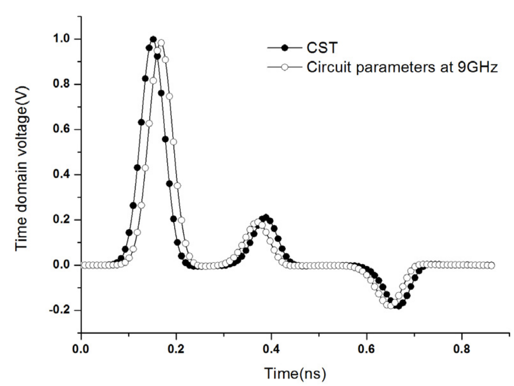

Figure 4. Time domain voltage waveforms at the monitoring point when $g = 24 \times 10^9\,\mathrm{s}^{-1}$.

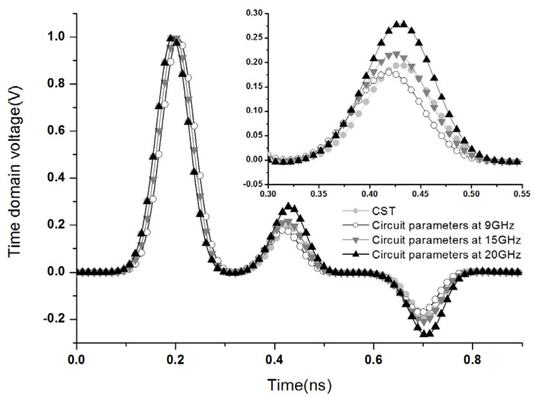

Figure 5. Total voltage waveforms at the monitoring point when $g = 18 \times 10^9\,\mathrm{s}^{-1}$.

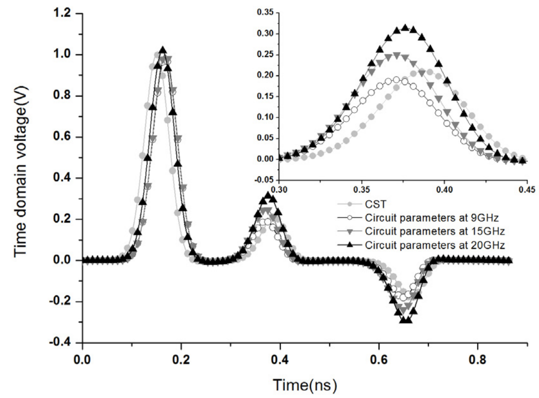

Figure 6. Total voltage waveforms at the monitoring point when $g = 24 \times 10^9\,\mathrm{s}^{-1}$.

Table 2. Relative error of the circuit parameters at three frequencies when $g = 18 \times 10^9\,\mathrm{s}^{-1}$ and $g = 24 \times 10^9\,\mathrm{s}^{-1}$.

Gaussian parameter	$g = 18 \times 10^9\,\mathrm{s}^{-1}$			$g = 24 \times 10^9\,\mathrm{s}^{-1}$		
Extraction frequency	9 GHz	15 GHz	20 GHz	9 GHz	15 GHz	20 GHz
Relative error of the maximum	7.7%	11.8%	36.6%	10.1%	18.6%	46.8%
Relative error of the moment maximum appears	9.6%	4.8%	2.9%	4.1%	3.9%	2.8%
Relative error of the pulse width	8.6%	5.7%	3.8%	8.1%	5.4%	3.4%

the voltage pulses transmitting along the step line; (2) there do exist some differences between the results obtained by means of circuit parameters and by CST both in amplitude and time, and the narrower pulse will cause larger differences. Furthermore, based on the static circuit parameters at 15 GHz and 20 GHz, the voltage pulses are calculated from the equivalent circuit excited by Gaussian pulse with parameter of $g = 18 \times 10^9\,\mathrm{s}^{-1}$ and $g = 24 \times 10^9\,\mathrm{s}^{-1}$. The first reflection peak intercepted from the calculated pulse is given for comparing with that obtained by CST, as shown in Figs. 5 and 6, and the relative error of the maximum, the moment maximum appears, and the pulse width are displayed in Table 2. The figures and table show: (1) because we tend to believe that the voltage pulse from CST is correct, we can regard this result as a reference, and then we can see that the voltage results predicted by circuit parameters at lower frequency is closer to the reference in amplitude, that is, the error is the smallest; (2) on the contrary, the voltage pulse predicted by circuit parameters at higher frequency is closer to the reference in time moment; (3) the higher the extracting frequency of circuit parameters is, the narrower the reflected pulse reconstructed by the circuit parameters is. According to the change of pulse width, there is a certain frequency where circuit parameters are extracted to reconstruct the reflected pulse, and the width of the reflected pulse is the most close to the reference value.

4. PREDICTABLE FREQUENCY SCOPE OF THE STATIC CIRCUIT PARAMETERS

Based on the distributed circuit parameters extracted at 9 GHz, 15 GHz and 20 GHz, the reflection coefficient S_{11} of the step line can be derived by using the transmission line method described in Section 2.3. Fig. 7 shows the $|S_{11}|$ curves from 0 GHz to 30 GHz obtained respectively from the circuit parameters and from CST. In order to compare the results from the above two methods clearly, Fig. 8 displays the difference between the peak of S_{11} curves obtained from the two methods. Figs. 7 and 8 illustrate that: (1) the results of the two methods match well within 0 GHz to 20 GHz, which means that

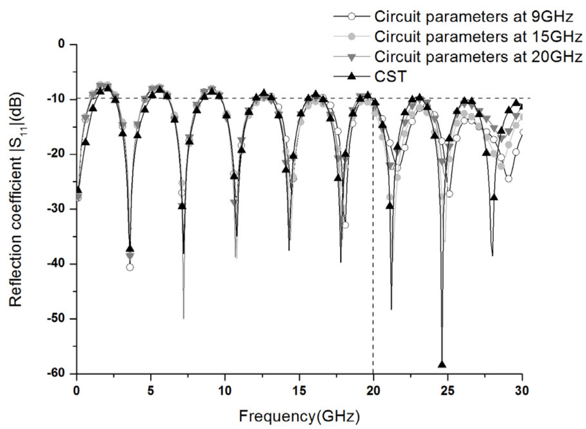

Figure 7. Comparison of the reflection coefficient $|S_{11}|$ obtained from the circuit parameters and CST.

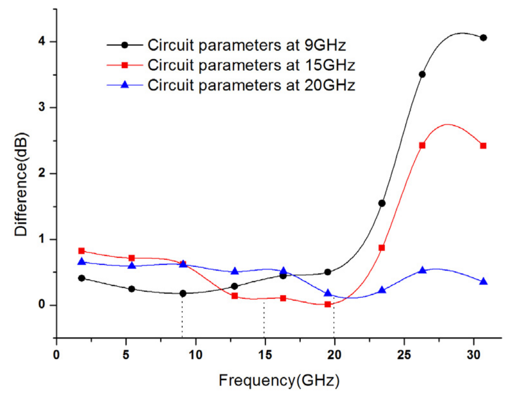

Figure 8. Difference between the peaks of $|S_{11}|$ curve obtained from the circuit parameters and CST.

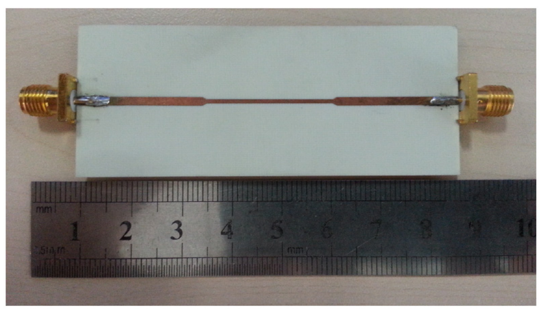

Figure 9. Prototype of the step microstrip line.

the static circuit parameters extracted at a certain frequency among 0 GHz to 20 GHz can effectively predict the reflection characteristics in this frequency range; (2) when the frequency is greater than 20 GHz, the radiation and attenuation of the step line will increase with the increasing of frequency so that the prediction error of the static circuit parameters becomes greater; (3) if f_0 is the frequency where the circuit parameters are extracted, the error of the predicted $|S_{11}|$ curve at f_0 is the smallest, and the prediction error becomes larger as the prediction frequency is far away from f_0. According to Gaussian parameter and the maximum frequency $f_{\max}$ in the predictable frequency range satisfying $g = \sqrt{\pi} f_{\max}$ [21], we can conclude that the static circuit parameters can predict the time domain response on the step line excited by Gaussian signal with parameter of $g \leq 35 \times 10^9\,\mathrm{s}^{-1}$.

In order to verify the availability of the S_{11} prediction by the static circuit parameters, the step microstrip line with parameters listed in Table 1 is fabricated, as shown in Fig. 9. The reflection coefficient S_{11} of the fabricated step line is measured by Agilent E5071C network analyzer. Because of the limitation in the operating frequency of the substrate and SMA connector, the measuring frequency band is set from 100 MHz to 10 GHz. Fig. 10 gives the comparison between the measured and calculated $|S_{11}|$ from the circuit parameters extracted at 9 GHz. From the figure we can see that the two results agree well within 100 MHz to 6 GHz, but when the frequency increases, the difference between the two results becomes greater. The reasons causing this difference are: (1) the result is calculated by means of circuit parameters without considering the effect of the SMA connector and the operating frequency of the connector used in this paper is from 0 GHz to 6 GHz, so the mismatch of the connector will be much more serious when the testing frequency is greater than 6 GHz; (2) the relative permittivity ε_r of the substrate usually exists error, and the loss of the substrate increases as the frequency increases; (3) both the error existing in the fabrication process and the mismatch from the artificial welding point lead to the differences between the measured and calculated results.

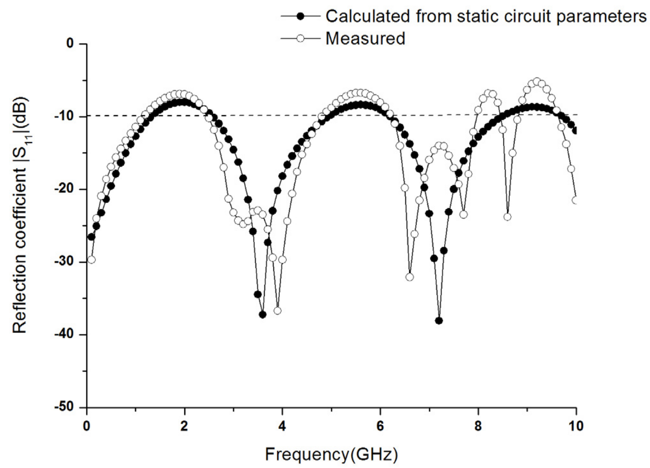

Figure 10. Comparison between the calculated and measured results.

5. CONCLUSIONS

The above analysis and discussion indicate that the circuit parameters extracted from the field results at a certain frequency between 0 GHz and 20 GHz can be used to predict transmission and reflection characteristics of the step microstrip line, both in the time and frequency domains. If the substrate and SMA connector used to fabricate step line can work at a sufficient broad band, the highest frequency of the signal predicted by static circuit parameters can reach 20 GHz. This will make it convenient for circuit designers to use the full wave method results. In the applicable frequency range of the extracted static circuit parameters, they can avoid the complicated EM problems, and directly use these parameters in the performance prediction of the whole integrated circuit.

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