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A Modified Formula for Microwave Measurement of Dielectric Loss Using a Closed Cylindrical Cavity Dielectric Resonator

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Abstract—This paper provides a modified formula for calculating dielectric loss of dielectric resonator of working in $\mathrm{TE}_{01\delta}$ mode in closed cavity. The measurement system is divided into six regions. Based on analyzing the formula of loss tangent published in literatures, a quality factor of a substrate is created, and a modified formula is proposed. Validating the modified formula, with three substrates as supports, the frequencies and unloaded quality factors of dielectric resonators made of two sorts of dielectric materials with permittivity 38 and 75 respectively are measured using a closed cavity method. The measured results are compared with those obtained by other well-known formulas and show a good agreement with the result given by the parallel plate method.

1. INTRODUCTION

Dielectric materials continue to play a very important role in the microwave communication systems. These materials are key in realization of low-loss temperature-stable resonators, filters for satellite and broadcasting equipment, as well as in many other microwave devices [1–3]. The microwave measurements of dielectric properties have recently attracted more and more attention. There are many measurement techniques [4, 5], such as Hakki-Coleman end shorted method [6, 7], $TE_{01\delta}$ mode dielectric resonator method [8, 9], Whispering gallery mode resonators [10–12] and Cavity perturbation method [13]. Hakki-Coleman end shorted method, called parallel plate method, is the most famous resonance technique. However, the method has very strict requirement, for example, measured dielectric sample being standard cylinder with restrict ratio of diameter to height, surface resistance of metal plates to be calibrated before measurement [14, 15]. $\mathrm{TE}_{01\delta}$ mode dielectric resonator method, called closed cavity resonance method, is a simple and novel measurement technique. Putting the dielectric resonator inside a closed metal cavity can completely eliminate the effect of field radiation. Several different theoretical and experimental analyses have been reported in the literatures [16–19]. Accurate prediction of resonant frequency was given in their papers; however the field expressions are usually quite complicated and imperfect. A modified field model has been proposed by Sheen [20, 21], and the dielectric constant was measured with reasonable accuracy; however the effect of a support on loss tangent was neglected. Experiments showed that the support affected the unloaded quality factor [22], but the relationship between the properties of a support and loss tangent of the dielectric resonator has not been studied yet.

This paper adopts the modified field model. A modified formula for calculating the loss tangent of dielectric resonator has been proposed in this paper. To validate the formula, ceramic resonators are measured using the closed cavity method. The results are then compared with those obtained by the open cavity method.

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2. DIELECTRIC LOSS MEASUREMENT

Figure 1 shows the structure model of a closed cylindrical cavity dielectric resonator. The cavity is divided into six regions, where region 1 represents cylindrical dielectric resonator; regions 2 and 4 are substrate; regions 3, 5 and 6 are filled with air.

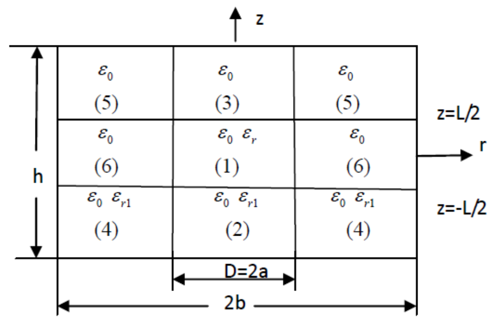

Figure 1. Vertical section view of a closed cavity resonator [20].

When the resonance mode of the dielectric resonator is $TE_{01\delta}$, the magnetic field expressions of the z component in each region, H_{zi} $(i = 1 \sim 6)$, are given by Sheen [20]. Other components of $TE_{01\delta}$ in each region can be derived from literature [23].

There are various formulas for calculating dielectric loss of a resonator, according to the literatures. For example, a few are shown below:

$$\text{Ni [24]:}\ \tan\delta = \frac{1}{Q_d} = \frac{1}{Q_u} - \frac{1}{Q_c} \tag{1}$$

$$\text{Gu [25]:}\ \tan\delta = \frac{1}{Q_d} = \frac{1}{Q_u} - \frac{1}{Q_c} - \frac{1}{Q_r} \tag{2}$$

$$\text{Zhou [23]:}\ \tan\delta = (1+A)\frac{1}{Q_u} - B \tag{3}$$

$$\text{Sheen [21]:}\ \tan\delta = \frac{W_a + W_d}{W_d}\left(\frac{1}{Q_u} - \frac{1}{Q_c}\right) \tag{4}$$

where Q_u is the unloaded quality factor of the whole system, and Q_d, Q_c and Q_r represent the quality factor resulting from the dielectric loss, conducing loss and radiation loss, respectively. W_d and W_a represent the energy stored in the dielectric and surrounding regions. A and B are the ratios related with the stored energy and the dissipated power, respectively [23].

These formulas are valid under certain conditions, which are beyond the scope of this paper.

A modified formula is proposed in this paper, written as:

$$\tan\delta = \frac{W_a + W_d + W_{sub}}{W_d} \cdot \frac{1}{Q_u} - \frac{P_{wall}}{\omega_0 W_d} - \frac{\tan\delta_{sub} \cdot W_{sub}}{W_d} \tag{5}$$

where W_{sub} and W_a represent the energy stored in the substrate used as a support and the air, respectively, and P_{wall} denotes the dissipated power in the conducting wall and $\tan\delta_{sub}$ the dielectric loss of the substrate. Equation (5) can be equivalently written as (6) by mathematical manipulations:

$$\tan\delta = A\left(\frac{1}{Q_u} - \frac{1}{Q_c} - \frac{1}{Q_{sub}}\right) \tag{6}$$

where $A = \frac{W_a+W_d+W_{sub}}{W_d}$, $Q_c = \omega_0\frac{W_a+W_d+W_{sub}}{P_{wall}}$, $Q_{sub} = \frac{W_a+W_d+W_{sub}}{\tan\delta_{sub}\cdot W_{sub}}$.

The values of W_{sub}, W_a, and W_d are calculated by,

$$W_d = \frac{1}{2}\varepsilon_r\varepsilon_0 \iiint |E_{\varphi 1}|^2\, dV \tag{7a}$$

$$W_a = \frac{1}{2}\varepsilon_0 \sum_{i=3,5,6} \iiint |E_{\varphi i}|^2 \, dV \tag{7b}$$

$$W_{sub} = \frac{1}{2}\varepsilon_{r1}\varepsilon_0 \sum_{i=2,4} \iiint |E_{\varphi i}|^2 \, dV \tag{7c}$$

The value of P_{wall} is calculated by [9],

$$P_{wall} = \frac{1}{2}R_s \sum_{i=2}^{5} \iint |H_{ri}|^2 \, dS + \frac{1}{2}R_s \sum_{i=4}^{6} \iint |H_{zi}|^2 \, dS \tag{8}$$

where ε_r and ε_{r1} are relative dielectric constants of resonator and the substrate, respectively; $E_{\phi i}$ and H_{ri} denote circumferential electric field components and radial magnetic field components in each region; R_s is the surface resistance of the metal wall.

Derivation of (5) is described in the Appendix.

3. EXPERIMENTS AND DISCUSS

Two resonators are made of ceramics, whose composites are $(Zr_{0.8}Sn_{0.2})TiO_4$ and $BaO\text{-}TiO_2\text{-}Sm_2O_3$, and whose dielectric constants are about 38 and 75, respectively. The cavity is made from aluminium which has the conductivity of 3.54×10^7 S/m and an inside diameter/height ratio = 35 mm/17.5 mm. There are three substrates made of plastic, alumina ceramics and Teflon glass textile board (TFGT), respectively. Their dielectric constants are 3.68, 6.75 and 2.65; their dielectric losses are 0.043, 0.0018 and 0.012; their heights are 5.44 mm, 4.32 mm and 1.26 mm; respectively. The resonant frequency and the unload quality factor of resonators are measured using Vector Networks Analyzer (Model: Agilent E5071B). Figure 2 illustrates the experimental setup used for the measurement.

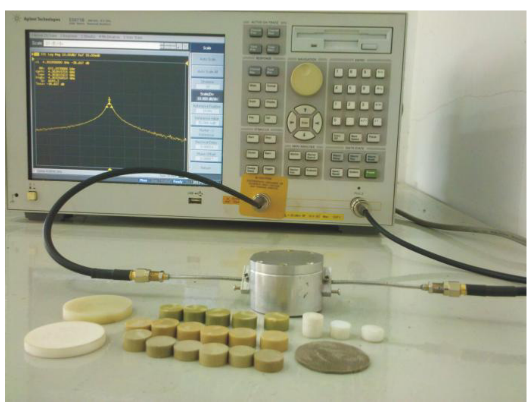

Figure 2. Measurement setup of the closed cylindrical cavity dielectric resonator.

Table 1 shows the measurement results in comparison with the results by the formulas in the literatures. The loss tangent measurements of the same sample from (5) show very close results among the substrates made of plastic, ceramics and TFGT. The results are very close to those measured from the parallel plate method, compared to the results by other formulas. This confirms the accuracy of (5) proposed in this paper.

Loss tangents are same for (3) and (4), because (3) can be changed into (4) by substituting A and B with their expressions and simplifying it. Equations (3) or (4) does not suit for calculating dielectric loss with high dielectric loss substrate as support, because the quality factor of the substrate is much less than that of the cavity. For example, Q_{sub}/Q_c of Sample#1 using plastic as support is 6.51×10^{-3}, shown in Table 2. The higher is the loss tangent of the substrate, the larger is the relative error of loss tangent calculated using (3). As for TFGT, the relative errors are more than 50%, even 90% for Sample#1.

Table 1. Dielectric properties as measured using various formulas.

	Substrate	Freq. (GHz)	Q_u	$\tan\delta$ from (5)	$\tan\delta$ from (3)	$\tan\delta$ from (4)	$\tan\delta$ from (1)	$\tan\delta$ from Parallel plate*
Sample#1	Plastic	4.063	1120	8.33×10^{-5}	9.84×10^{-4}	9.84×10^{-4}	8.84×10^{-4}	
$D = 12.84$ mm	Ceramics	4.023	6800	8.72×10^{-5}	1.54×10^{-4}	1.54×10^{-4}	1.37×10^{-4}	7.856×10^{-5}
$L = 5.50$ mm	TFGT	4.308	6080	8.70×10^{-5}	1.74×10^{-4}	1.74×10^{-4}	1.59×10^{-4}	
Sample#2	Plastic	2.868	1340	3.08×10^{-4}	7.74×10^{-4}	7.74×10^{-4}	7.40×10^{-4}	
$D = 13.02$ mm	ceramics	2.862	2860	3.26×10^{-4}	3.62×10^{-4}	3.62×10^{-4}	3.43×10^{-4}	3.5×10^{-4}
$L = 5.50$ mm	TFGT	3.043	2400	3.29×10^{-4}	3.43×10^{-4}	3.43×10^{-4}	3.31×10^{-4}	

*Loss tangent at certain frequency is derived from the product of frequency and the reciprocal of loss tangent.

Table 2. Calculated parameters.

	Substrate	Q_c	Q_{sub}	Q_{sub}/Q_c	$(W_a+W_{sub})/W_t^{**}$	W_d/W_t	A
Sample#1	Plastic	1.90×10^5	1237	6.51×10^{-3}	0.1019	0.8981	1.1088
$D = 12.84$ mm	Ceramics	1.00×10^5	1.69×10^4	0.169	0.1078	0.8922	1.1210
$L = 5.50$ mm	TFGT	1.60×10^4	4.40×10^4	2.75	0.0847	0.9153	1.0920
Sample#2	Plastic	1.67×10^5	2326	1.39×10^{-2}	0.0441	0.9559	1.0460
$D = 13.02$ mm	ceramics	1.59×10^5	2.92×10^4	0.184	0.0518	0.9482	1.0546
$L = 5.50$ mm	TFGT	1.17×10^5	7.70×10^4	0.658	0.0344	0.9656	1.0356

$^{**}W_t = W_a + W_{sub} + W_d$

It is strange that Q_{sub}/Q_c of Sample#1 using TFGT as support is 2.75, and Q_c value is 1.60×10^4, listed in Table 2, much smaller than others. The reason is that the thickness of TFGT is 1.26 mm, and dielectric constant of Sample#1 is lower, about 38, which make Q_c value smaller. It is inferred that the supports with high dielectric loss have much more effect on the measured Q value of dielectric resonators with low dielectric constant, and the effect of support cannot be ignored and should be taken into account.

4. CONCLUSIONS

The unloaded quality factor of a closed cavity resonator is dependent on the substrate used as a support. The larger is the loss tangent of the substrate, the smaller is the unloaded quality factor of the system. The modified formula proposed in this paper shows that the dielectric loss of a resonator can be calculated by subtracting the conducting loss ($1/Q_c$) and the substrate dissipated loss ($1/Q_{sub}$) from the reciprocal of the unloaded quality factor ($1/Q_u$). The accuracy of the proposed formula is confirmed by the results from the experiments. Compared to other formulas in the literature, the results by the proposed formula show a much closer match with those by the parallel plate method, and the results are robust even with different materials for the substrate.

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APPENDIX A.

Unloaded quality factor (Q_u) of dielectric resonator located in a closed cavity is defined as [2]

$$Q_u = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{Average energy dissipated per cycle}} = \frac{\omega_0 W_t}{p_t} \tag{A1}$$

where ω_0 is angle frequency, W_t the total energy stored in all media, and P_t the total power dissipated by al1 substances.

$$W_t = W_a + W_d + W_{sub} \tag{A2}$$

$$p_t = p_a + p_d + p_{sub} + p_{wall} \tag{A3}$$

where P_d, P_{sub}, P_{wall} and P_a represent the power dissipated in the dielectric, substrate, conductor and the air respectively, and W_a, W_d and W_{sub} are mentioned in Section 2.

Loss tangents of dielectric resonator and substrate are written as

$$\tan\delta_{sub} = \frac{p_{sub}}{\omega_0 W_{sub}} \tag{A4}$$

$$\tan\delta_d = \frac{p_d}{\omega_0 W_d} \tag{A5}$$

By adding (A1)–(A4) into (A5), (A5) is rewritten as

$$\tan\delta_d = \frac{W_t}{W_d Q_u} - \frac{p_a}{\omega_0 W_d} - \frac{\tan\delta_{sub} W_{sub}}{W_d} - \frac{p_{wall}}{\omega_0 W_d} \tag{A6}$$

When we ignore the power dissipated in the air, i.e., $p_a = 0$, (A7) can be obtained.

$$\tan\delta_d = \frac{W_t}{W_d Q_u} - \frac{\tan\delta_{sub} W_{sub}}{W_d} - \frac{p_{wall}}{\omega_0 W_d} \tag{A7}$$

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