Progress In Electromagnetics Research, Vol. 149, 109–118, 2014

Magnetic Field Induced by Wake of Moving Body in Wind Waves

Xiao-Jian Zhu[1] and Ming-Yao Xia[2, *]

(*Invited Paper*)

Abstract—A general procedure to evaluate the electromagnetic fields generated by moving seawater through the geomagnetic field is proposed. It contains two essential steps: modeling of velocity vector of seawater according to its dynamic mechanism, and solution of Maxwell equations under a stratified ocean configuration. Two kinds of motions are considered in this work, wind-driven waves and wakes due to a moving body. The ocean is taken to be infinitely deep at the moment. Both the velocity vector and magnetic field are expressed as superposition of sinusoidal waves. Simulation results show that the magnetic fields produced by moderate wind waves or a typical size body moving at moderate speed are on the order of a few hundred pico-Tesla near the sea level. The spectrum characteristics of the two kind magnetic anomalies are distinct.

1. INTRODUCTION

It is known that moving seawater through the geomagnetic field can generate electromagnetic field as predicted by Faraday in 1832 and verified by Wollaston in 1851 [1]. Many studies have been conducted since, largely concentrated on wind waves or swells [2–6]. Ocean currents, internal oceanic waves and tsunami were also reported producing magnetic fields [7–9]. How to use the observable magnetic fields to investigate oceanic hydrodynamics is an open topic. It would subject to two conditions, one is high sensitivity magnetometer and the other is theoretical model that relates the sensed magnetic fields to the physics quantities under examinations. Nowadays, atomic magnetometer possesses a sensitivity of a few pico-Tesla (pT), and the superconducting quantum interference device (SQUID) has reached a sensitivity of a few femto-Tesla (fT). These sensitivities should be sufficient for many oceanic investigations. However, there lacks adequate study on bridging the characteristics of measured magnetic fields to specific hydrodynamic factors such as wind vector, ship's size and speed.

The principle that moving seawater could generate magnetic field consists in seawater cutting the geomagnetic field, which would produce weak electric currents within the medium. These induced electric currents could generate an observable magnetic field as far as tens of kilometers away from the seacoast [10]. As a result, to evaluate the magnetic field, a two-step procedure should be followed. The first step is to resolve the velocity vector by resorting to fluid mechanics and find the electric currents in the seawater. The second step is to solve the Maxwell equations under an appropriate oceanic geologic configuration.

For wind-driven waves, in order to simplify the resolution of velocity vector, many researchers took the motion of sweater as a monochromatic sinusoidal wave. Weaver [2] gave the solution of the magnetic field induced by a harmonic surface wave, and proved that the magnitude of magnetic field is proportional to the amplitude and period of the wave. Sanford [3] showed that the conductivity weighted

Received 7 July 2014, Accepted 19 September 2014, Scheduled 26 September 2014
Invited paper for the Commemorative Collection on the 150-Year Anniversary of Maxwell's Equations.
* Corresponding author: Ming-Yao Xia (myxia@pku.edu.cn).
[1] School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China. [2] School of Electronics Engineering and Computer Science, Peking University, Beijing 100871, China.

vertically averaged velocity is important in the generation of both local and large-scale electric currents in shallow sea case. Podney [5] proved that the electromagnetic field generated by a progressive ocean wave in a stratified ocean could be regarded as a sum of a transverse electric type field, a transverse magnetic type field, and an electrostatic type field, which indicates that the vertical component of seawater velocity is important.

For motion of seawater caused by moving bodies, Tuck et al. [13, 14] has made substantial contributions to the modelling of wakes, especially for ships moving on free-surface. Zou and Arye [16] studied the detection method of wake-induced magnetic fields based on using airborne magnetic transduces. Yaakobi and Zilman [17, 18] and Madurasinghe and Tuck [19] considered detection issues of magnetic fields originated from wakes of vessels moving on ocean surface or submerged in a depth.

In the following, Section 2 outlines the modeling of seawater velocity vector for wind waves and ship wakes. Section 3 presents solution of Maxwell equations for evaluation of magnetic fields induced by moving seawater. Section 4 provides simulation results and discussions. A brief summary is given in Section 5.

2. MODELLING OF VELOCITY VECTOR OF SEAWATER

As the first step, we need to resolve the velocity vector of seawater, so that the weak electric currents in the conductive seawater can be acquired. The wind-driven ocean surface is considered as a two dimensional aperiodic rough surface as shown in Figure 1 under polar coordinate system. The wakes of a ship moving over a free surface is known to form a Kelvin track, which has a half angle of 19.5 degree and consists of a shear wave and a diffused wave, as shown in Figure 2. The seawater is taken to be incompressible and non-viscous, so that the velocity vector $\mathbf{V}$ satisfies $\nabla \cdot \mathbf{V} = 0$ and can be characterized by a potential function φ as $\mathbf{V} = -\nabla\varphi$; thus

$$\nabla^2\varphi = 0 \tag{1}$$

2.1. Velocity Vector of Wind Waves

The solution of (1) can be expressed as a Fourier integral:

$$\varphi(x, y, z, t) = \frac{1}{(2\pi)^2}\mathrm{Re}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(k_x, k_y)e^{-kz}e^{j(\omega t - k_x x - k_y y)}dk_x dk_y \tag{2}$$

in which $k = \sqrt{k_x^2 + k_y^2}$, and for an infinitely deep sea $\omega = \sqrt{kg}$ with $g = 9.81\,\mathrm{m/s^2}$ being the gravitation acceleration. $\psi(k_x, k_y)$ is a spectrum function to be determined soon. The Fourier integral of (2) can

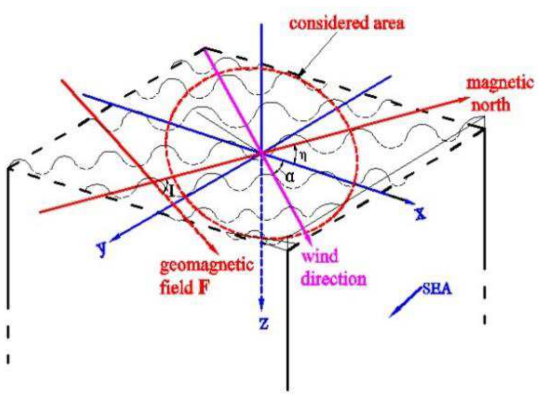


Figure 1. Configuration for modeling of seawater velocity vector and induced magnetic fields caused by wind waves.

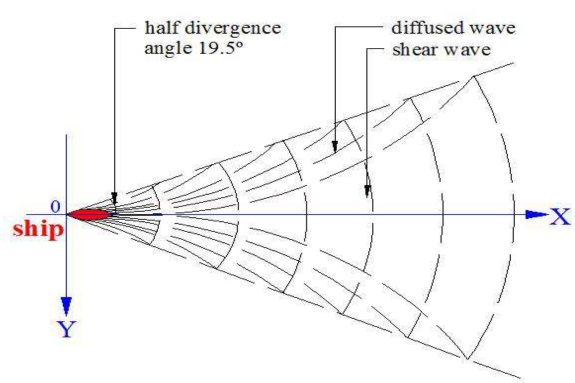


Figure 2. The Kelvin track of a moving body in wake coordinate system.

be evaluated approximately in polar coordinate system by

$$\varphi(x,y,z,t) \approx \frac{1}{LN}\mathrm{Re}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\psi(k_m,\theta_n)k_m e^{-k_m z}e^{j\omega_m t-jk_m(x\cos\theta_n+y\sin\theta_n)} \tag{3}$$

where $k_m = 2\pi m/L$, $\theta_n = 2n\pi/N$, and L is the radius of a circular region, which is divided as M sectors. The size of L should be determined by the minimum angular frequency that we want to resolve. Letting $\omega_{\min} = \sqrt{k_{\min}g} = \sqrt{(2\pi/L)g}$, we get $L = 2\pi g/\omega_{\min}^2$.

On the other hand, the elevation of a wind-driven ocean surface can be constructed by a Fourier series as [10]:

$$z(x,y,t) \approx \frac{1}{LN}\mathrm{Re}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}Z(k_m,\theta_n)k_m e^{j\omega_m t-jk_m(x\cos\theta_n+y\sin\theta_n)} \tag{4}$$

where $Z(k_m,\theta_n)$ can be related to the oceanic surface power spectrum $W(k_m,\theta_n)$ through

$$Z(k_m,\theta_n) = 2\pi\gamma_{mn}\sqrt{LNW(k_m,\theta_n)} \tag{5}$$

in which γ_{mn} is a two dimensional complex random number with both its real and imaginary parts satisfying the normal Gaussian distribution. In this work, the Pierson-Moskowitz (PM) spectrum [11, 12] is employed, which reads

$$W(k,\theta) = W(k)G(\theta) \tag{6}$$

$$W(k) = \frac{8.1\times10^{-3}}{4k^3}\exp\left(-\frac{0.74g^2}{k^2U_{19.5}^4}\right) \tag{7}$$

$$G(\theta) = \frac{8}{3\pi}\cos^4(\theta-\alpha) \tag{8}$$

where θ indicates the propagation direction of water wave, α the angle of wind direction, and $U_{19.5}$ the wind speed at 19.5 m above the mean sea surface. The z-component of the seawater velocity at the mean sea surface may be estimated by

$$V_z(x,y,t) = \frac{\partial}{\partial t}z(x,y,t) = \left[-\frac{\partial}{\partial z}\varphi(x,y,z,t)\right]_{z=0} \tag{9}$$

Substituting (3) and (4) into above, we get

$$\psi(k_m,\theta_n) = j\frac{\omega_m}{k_m}Z(k_m,\theta_n) = j2\pi\gamma_{mn}\sqrt{LN\frac{W(k_m,\theta_n)}{k_m}g} \tag{10}$$

Taking this back to (3) and using $\mathbf{V} = -\nabla\varphi$, we obtain

$$\mathbf{V}(x,y,z,t)=\frac{1}{LN}\mathrm{Re}\sum_{m=1}^{M}\sum_{n=1}^{N}\psi(k_m,\theta_n)\left(j\cos\theta_n\hat{x}+j\sin\theta_n\hat{y}+\hat{z}\right)k_m^2 e^{-k_m z}e^{j\omega_m t-jk_m(x\cos\theta_n+y\sin\theta_n)} \tag{11}$$

2.2. Velocity Vector of Ship Wakes

Modeling of seawater velocity vector due to a moving body is usually based on the Kelvin track as shown in Figure 2. The shear wave propagates in the sailing direction while the diffused wave propagates transverse to the shear wave. Assume that the ship moves opposite to the x-axis at a speed U. The wave height of the wake in the ship's coordinate system can be written as [13–15]:

$$\xi(X,Y) = \mathrm{Re}\int_{-\pi/2}^{\pi/2}\Omega(\theta)e^{-jk(X\cos\theta+Y\sin\theta)}d\theta, \quad X>0 \tag{12}$$

where $k = k_0 \sec^2\theta$, $k_0 = g/U^2$, and θ is measured from the X-axis to the wave propagation direction. According to ship body theory, the free spectral $\Omega(\theta)$ is written as:

$$\Omega(\theta) = \frac{2k\sec\theta}{\pi}\iint \frac{\partial Y(X,Z)}{\partial X} e^{-kZ} e^{-jkX\cos\theta} dXdZ \tag{13}$$

where $Y(X,Z)$ is the half width of the body at (X,Z). In order to simplify the calculation, we assume that the shape of body is a parabolic, and $Y(X,Z)$ is given by

$$Y(X,Z) = b\left[1 - \frac{(X-l)^2}{l^2}\right], \quad 0 \le X \le 2l; \quad 0 \le Z \le D \tag{14}$$

in which b is the half width of the waterline of the body, l the half length of the body, and D the submerged depth. By using (14), (13) becomes

$$\Omega(\theta) = -\frac{8b}{\pi(k_0 l)^2}(j\cos\theta)e^{-jkl\cos\theta}\left(1 - e^{-kD}\right)\left[(kl\cos\theta)\cos(kl\cos\theta) - \sin(kl\cos\theta)\right] \tag{15}$$

To eliminate the restriction of $X > 0$ in (12), we make use of

$$\int_{-\infty}^{\infty} \Phi(\lambda)e^{-j\lambda X} d\lambda = \begin{cases} 1, & X > 0 \\ 0, & X \le 0 \end{cases} \tag{16}$$

$$\Phi(\lambda) = \frac{1}{2\pi}\int_0^{\infty} e^{j\lambda X} d\lambda = \frac{1}{2}\delta(\lambda) + \frac{1}{2\pi}\frac{1}{j\lambda} \tag{17}$$

and then rewrite (12) as

$$\xi(X,Y) = \mathrm{Re}\int_{-\pi/2}^{\pi/2}\int_{-\infty}^{\infty} \Phi(\lambda)\Omega(\theta)e^{-j(\lambda+k\cos\theta)X - j(k\sin\theta)Y} d\lambda d\theta \tag{18}$$

The wake height in the ground coordinate system can be obtained by letting $X = x - Ut$ and $Y = y$, i.e., $z(x,y,t) = \xi(x - Ut, y)$. The solution of (1) for the seawater waked by the ship may be written as:

$$\varphi(x,y,z,t) = \mathrm{Re}\int_{-\pi/2}^{\pi/2}\int_{-\infty}^{\infty} \psi(\lambda,\theta)\, e^{-qz} e^{-j(\lambda+k\cos\theta)(x-Ut) - j(k\sin\theta)y} d\lambda d\theta, \quad z > 0 \tag{19}$$

$$q = \sqrt{(\lambda + k\cos\theta)^2 + (k\sin\theta)^2} \tag{20}$$

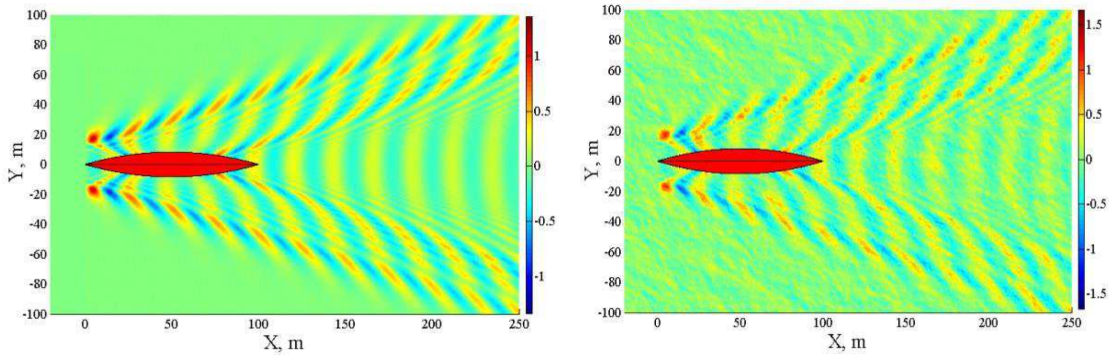
The z-component of velocity vector at the mean level $z = 0$ may be estimated in the same way as (9), so that the function $\psi(\lambda,\theta)$ is determined to be:

$$\psi(\lambda,\theta) = \frac{j\omega}{q}\Phi(\lambda)\Omega(\theta), \quad \omega = (\lambda + k\cos\theta)U \tag{21}$$

Finally, the velocity vector of the wake, according to $\mathbf{V} = -\nabla\varphi$, is written as

$$\mathbf{V}(x,y,z,t) = \mathrm{Re}\int_{-\pi/2}^{\pi/2}\int_{-\infty}^{\infty} \psi(\lambda,\theta)\left[j(\lambda + k\cos\theta)\hat{x} + j(k\sin\theta)\hat{y} + q\hat{z}\right] e^{-qz} e^{-j(\lambda+k\cos\theta)(x-Ut) - j(k\sin\theta)y} d\lambda d\theta \tag{22}$$

A typical picture of the wake due to a ship model moving on free surface or in wind waves is shown in Figure 3. The ship's speed is $U = 6\,\mathrm{m/s}$, while the speed and direction of the wind is $U_{19.5} = 6\,\mathrm{m/s}$ and $\alpha = 45°$. The size of the ship is $2l = 100\,\mathrm{m}$, $b = 8.33\,\mathrm{m}$, and $D = 6.67\,\mathrm{m}$.



(a) (b)

Figure 3. The Kelvin track of a ship model moving at $U = 6\,\mathrm{m/s}$: (a) on free surface, and (b) in wind waves with wind speed and direction $U_{19.5} = 6\,\mathrm{m/s}$ and $\alpha = 45°$.

3. MAGNETIC FIELDS INDUCED BY DYNAMIC SEAWATER

The electromagnetic field generated by the moving seawater satisfies the Maxwell equations

$$\begin{cases}\nabla\times\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t}\\ \nabla\times\mathbf{H} = \mathbf{J}+\sigma\mathbf{E}+\varepsilon\frac{\partial\mathbf{E}}{\partial t}\end{cases} \tag{23}$$

where the permeability μ is taken to be $\mu = \mu_0 = 4\pi\times10^{-7}$ in both the air and seawater, and ε is $\varepsilon = \varepsilon_0 = 10^{-9}/36\pi$ in the air and $\varepsilon = \varepsilon_1 = \varepsilon_0\varepsilon_r$ with $\varepsilon_r = 78$ in the seawater. The volume electric current density is approximated by $\mathbf{J} = \sigma[\mathbf{V}\times(\mathbf{F}+\mu\mathbf{H})] \approx \sigma(\mathbf{V}\times\mathbf{F})$ with $\mathbf{F}$ being the ambient geomagnetic field. After eliminating the electric field $\mathbf{E}$ and using $\nabla\cdot\mathbf{H} = 0$, the magnetic field $\mathbf{H}$ can be shown to satisfy

$$\nabla^2\mathbf{H}-\sigma\mu\frac{\partial\mathbf{H}}{\partial t}-\mu\varepsilon\frac{\partial^2\mathbf{H}}{\partial t^2} = \begin{cases}0, & z<0 \text{ (in the air)}\\ -\sigma(\mathbf{F}\cdot\nabla)\mathbf{V}, & z>0 \text{ (in the seawater)}\end{cases} \tag{24}$$

3.1. Magnetic Fields Generated by Wind Waves

Because the velocity vector $\mathbf{V}$ is expressed as (11), the solution of (24) may be written in similar form as

$$\mathbf{H}(x,y,z,t) = \frac{1}{LN}\mathrm{Re}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left[\hat{x}h_{x,m,n}(z)+\hat{y}h_{y,m,n}(z)+\hat{z}h_{z,m,n}(z)\right]e^{j\omega_m t-jk_m(x\cos\theta_n+y\sin\theta_n)} \tag{25}$$

By substituting (25) into (24), it can be shown that each component of the induced magnetic field satisfies:

$$\begin{cases}\left(\frac{\partial^2}{\partial z^2}-\beta_{0m}^2\right)\Psi_{u,m,n} = 0, & \text{for } z<0\\ \left(\frac{\partial^2}{\partial z^2}-\beta_{1m}^2\right)\Psi_{u,m,n} = \Gamma_{u,m,n}e^{-k_m z}, & \text{for } z>0\end{cases} \tag{26}$$

where $\Psi_{u,m,n}$ represents either $h_{x,m,n}$ or $h_{y,m,n}$, or $h_{z,m,n}$; $\beta_{0m}^2 = k_m^2-\omega_m^2\mu\varepsilon_0$, $\beta_{1m}^2 = k_m^2+j\omega_m\sigma\mu-\omega_m^2\mu\varepsilon_1$; and $\Gamma_{x,m,n} = j\Gamma_{z,m,n}\cos\theta_n$, $\Gamma_{y,m,n} = j\Gamma_{z,m,n}\sin\theta_n$, and

$$\Gamma_{z,m,n} = \sigma(j\cos\theta_n F_x + j\sin\theta_n F_y + F_z)k_m^3\psi(k_m,\theta_n) \tag{27}$$

The solution of (26) can be expressed as

$$\begin{cases}\Psi_{u,m,n}(z) = \dfrac{\Gamma_{u,m,n}}{k_m^2-\beta_{1m}^2}P_{u\cdot m\cdot n}e^{\beta_{0m}z}, & \text{for } z<0\\ \Psi_{u,m,n}(z) = \dfrac{\Gamma_{u,m,n}}{k_m^2-\beta_{1m}^2}\left(Q_{u\cdot m\cdot n}e^{-\beta_{1m}z}+e^{-k_m z}\right), & \text{for } z>0\end{cases} \tag{28}$$

The electric fields induced by the moving seawater can be obtained by applying (25) to the second equation of (23). The six constants $P_{u,m,n}$ and $Q_{u,m,n}$ $(u = x, y, z)$ are determined by: (i) the divergence of magnetic field must be zero, which contributes two equations; (ii) the tangential magnetic field must be continuous across the air-sea interface, which contributes two equations; and (iii) the tangential electric field must be continuous across the air-sea interface, which contributes two equations. The six equations are:

$$\begin{cases} k_m\cos^2\theta_n P_{x,m,n}+k_m\sin^2\theta_n P_{y,m,n}+\beta_{0m}P_{z,m,n}=0 \\ k_m\cos^2\theta_n Q_{x,m,n}+k_m\sin^2\theta_n Q_{y,m,n}-\beta_{1m}Q_{z,m,n}=0 \\ P_{x,m,n}=Q_{x,m,n}+1 \\ P_{y,m,n}=Q_{y,m,n}+1 \\ \dfrac{\sigma+j\omega_m\varepsilon_1}{j\omega_m\varepsilon_0}\sin\theta_n(\beta_{0m}P_{y,m,n}+k_mP_{z,m,n})=\sin\theta_n(\beta_{0m}Q_{y,m,n}+k_mQ_{z,m,n}+k_m+\beta_{0m})+j(ja_y+a_z\sin\theta_n) \\ \dfrac{\sigma+j\omega_m\varepsilon_1}{j\omega_m\varepsilon_0}\cos\theta_n(\beta_{0m}P_{x,m,n}+k_mP_{z,m,n})=\cos\theta_n(\beta_{0m}Q_{x,m,n}+k_mQ_{z,m,n}+k_m+\beta_{0m})+j(ja_x+a_z\cos\theta_n) \end{cases} \tag{29}$$

where

$$a_{x,y,z} = -j\sigma\psi(k_m,\theta_n)k_m^2\frac{k_m^2-\beta_{1m}^2}{\Gamma_{z,m,n}}F_{x,y,z} \tag{30}$$

Analytical solution of (29) for the six undetermined constants is very complex, but it can be solved numerically when other quantities are provided.

3.2. Magnetic Fields Generated by Wakes of a Moving Ship

In the same way, the magnetic field $\mathbf{H}$ can be written as

$$\mathbf{H}(x,y,z,t)=\mathrm{Re}\int_{-\pi/2}^{\pi/2}\int_{-\infty}^{\infty}[h_x(\lambda,\theta;z)\hat{x}+h_y(\lambda,\theta;z)\hat{y}+h_z(\lambda,\theta;z)\hat{z}]\,e^{-j(\lambda+k\cos\theta)(x-Ut)-j(k\sin\theta)y}d\lambda d\theta \tag{31}$$

Each component of the induced magnetic field satisfies:

$$\begin{cases} \left(\frac{\partial^2}{\partial z^2}-\beta_0^2\right)\Psi_u=0, & z<0 \\ \left(\frac{\partial^2}{\partial z^2}-\beta_1^2\right)\Psi_u=\Upsilon_u e^{-qz}, & z>0 \end{cases} \tag{32}$$

where $\Psi_u(\lambda,\theta;z)$ represents either h_x or h_y, or h_z; $\beta_0^2 = q^2-\omega^2\mu_0\varepsilon_0$, $\beta_1^2 = q^2+j\omega\mu_0\sigma-\omega^2\mu_0\varepsilon_1$; $\Upsilon_x = j\Upsilon_z(\eta+k\cos\theta)/q$, $\Upsilon_y = j\Upsilon_z(k\sin\theta)/q$, and

$$\Upsilon_z = \sigma\left[j(\lambda+k\cos\theta)F_x+j(k\sin\theta)F_y+qF_z\right]\psi(\lambda,\theta) \tag{33}$$

The solution of (32) can be written as

$$\begin{cases} \Psi_u(z)=\frac{\Upsilon_u}{k_n^2-\beta_1^2}P_u e^{\beta_0 z}, & \text{for } z<0 \\ \Psi_u(z)=\frac{\Upsilon_u}{k_n^2-\beta_1^2}\left(Q_u e^{-\beta_1 z}+e^{-qz}\right), & \text{for } z>0 \end{cases} \tag{34}$$

The six constants P_u and Q_u $(u = x, y, z)$ are determined in the same way as before, and the resulting equation set is

$$\begin{cases} -j(\lambda+k\cos\theta)P_x-j(k\sin\theta)P_y+\beta_0P_z=0 \\ -j(\lambda+k\cos\theta)Q_x-j(k\sin\theta)Q_y-\beta_1Q_z=0 \\ qP_x=qQ_x+j(\lambda+k\cos\theta) \\ qP_y=qQ_y+j(k\sin\theta) \\ \frac{\sigma+j\omega\varepsilon_1}{j\omega\varepsilon_0}\left[\beta_0P_y+j(k\sin\theta)P_z\right]=j(k\sin\theta)Q_z-\beta_1Q_y-c_y+j(k\sin\theta)c_z/q \\ \frac{\sigma+j\omega\varepsilon_1}{j\omega\varepsilon_0}[\beta_0P_x+j(\lambda+k\cos\theta)P_z]=j(\lambda+k\cos\theta)Q_z-\beta_1Q_x-c_x+j(\lambda+k\cos\theta)c_z/q \end{cases} \tag{35}$$

with $c_{x,y,z} = -j\sigma\psi(\lambda,\theta)q\frac{q^2-\beta_1^2}{\gamma_z}F_{x,y,z}$. Again, this equation set is solved numerically when other quantities are provided.

4. NUMERICAL EXPERIMENTS

First, we calculate the magnetic field induced by wind waves. Suppose that the frequency range we want to observe is 0.02∼1 Hz. The size of the surface would be set to be $L = 2\pi g/\omega_{\min}^2 \approx 3900\,\mathrm{m}$, and the M and N in (25) are taken to be $M = k_{\max}/k_{\min} = 2500$ and $N \approx M = 2500$. The geomagnetic field is taken to be $F = 5 \times 10^4\,\mathrm{nT}$, with dip angle $I = \pi/4$ and the angle from the magnetic north to the x-axis is $\eta = \pi/18$. The conductivity and relative permittivity of the seawater are taken to be $\sigma = 3.3\,\mathrm{S/m}$ and $\varepsilon_r = 78$, respectively.

A scalar magnetometer senses the disturbance or magnetic anomaly of the induced magnetic fields to the ambient geomagnetic field, which is calculated by

$$B(x,y,z,t) = \mu_0 \mathbf{H}(x,y,z,t) \cdot \hat{F} \tag{36}$$

where $\hat{F} = \hat{x}\cos I\cos\eta - \hat{y}\cos I\sin\eta + \hat{z}\sin I$ is the unit direction vector of the geomagnetic field. For this example, the speed and direction of wind is assumed to be $U_{19.5} = 6\,\mathrm{m/s}$ and $\alpha = 0$ (in x-axis direction). Figure 4 shows the magnetic anomaly distributions at the instant $t = 0$ along the three axis directions raised by wind waves, as well as the wave number spectrum along the x-axis and y-axis directions. It is seen that the magnetic anomalies in the x-axis direction (downwind) and in the y-axis direction (crosswind) are propagative and have a slight difference that may be characterized by some statistical quantity. However, the magnetic anomaly along the z-axis attenuates at exponent rates. The attenuation in the air is faster than in the seawater. This indicates that the magnetic anomaly induced

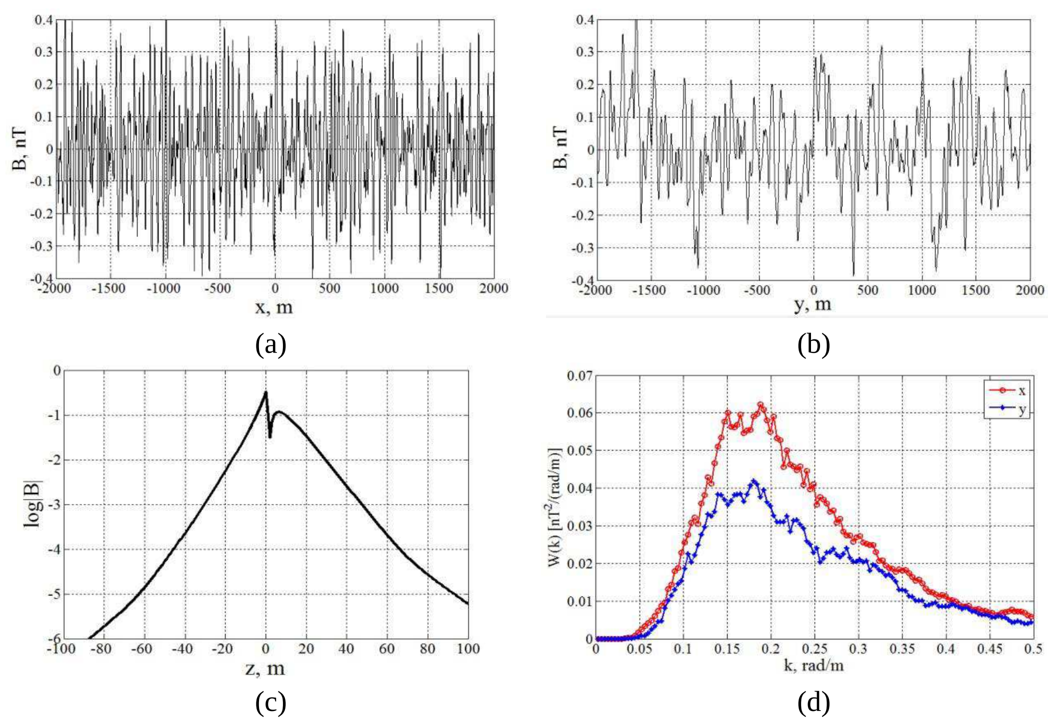


Figure 4. Magnetic anomaly induced by wind waves. The wind speed and direction is $U_{19.5} = 6\,\mathrm{m/s}$ and $\alpha = 0$ (in x-axis direction). (a) The magnetic anomaly along x-axis at 1 m above the mean sea surface, (b) the magnetic anomaly along y-axis at 1 m above the mean sea surface, (c) the attenuation property of the magnetic anomaly along z-axis, and (d) the wave number spectral along the x-axis and y-axis directions.

by wind waves is a surface phenomenon, which may be measurable at a place on land tens of kilometers away from the seacoast, but hardly detectable a few hundred meters above the sea surface.

Next, we calculate the magnetic field induced by wakes of a ship model sailing at $U = 10\,\mathrm{m/s}$ on free sea surface. The ship model is given in (14) with sizes $2l = 100\,\mathrm{m}$, $b = 8.33\,\mathrm{m}$, and $D = 6.67\,\mathrm{m}$. The magnetic anomaly induced by the wakes calculated at the instant $t = 0$ are shown in Figure 5. It is seen that the magnetic anomaly in the x-axis direction looks like a damping sinusoidal wave. The magnetic anomaly in the y-axis direction exhibits two maximums near the two crest lines of the Kelvin track. Like the case of wind waves, the magnetic anomaly in the z-axis attenuates at exponent rates. To reveal the relation of magnetic anomaly to the ship speed, we repeat the calculations by setting $U = 6\,\mathrm{m/s}$ and $U = 16\,\mathrm{m/s}$. The results are plotted in Figure 6 and Figure 7. It can be seen that the

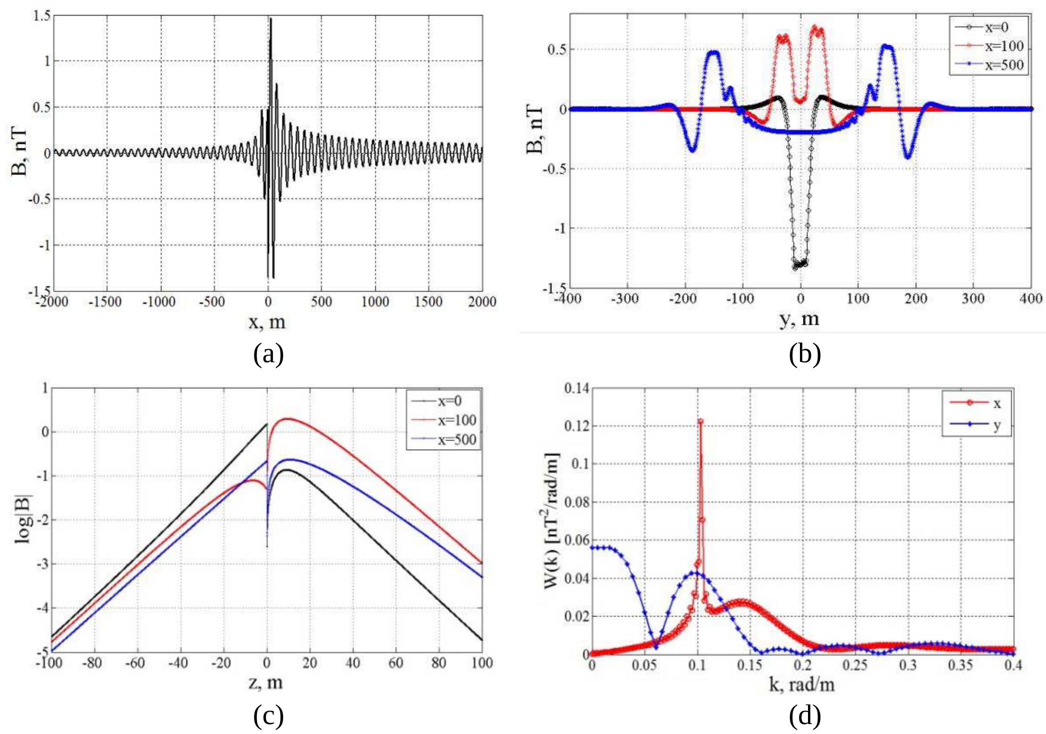


Figure 5. Magnetic anomaly induced by the wake of a ship model moving at $U = 10\,\mathrm{m/s}$. (a) The magnetic anomaly along x-axis at 1 m above the mean sea surface, (b) the magnetic anomalies at 1 m above the mean sea level along three lines: $x = 0\,\mathrm{m}$; $x = 100\,\mathrm{m}$; $x = 500\,\mathrm{m}$, (c) the attenuation property of the magnetic anomaly along z-axis, and (d) the wave number spectral along the x-axis and y-axis directions.

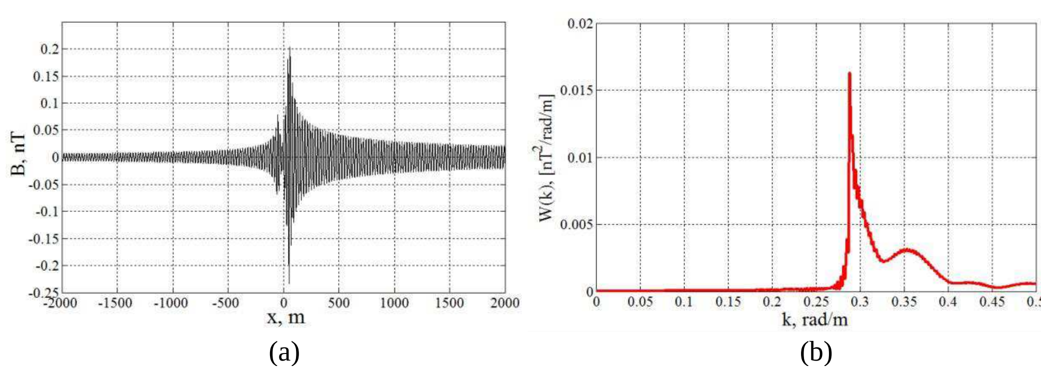
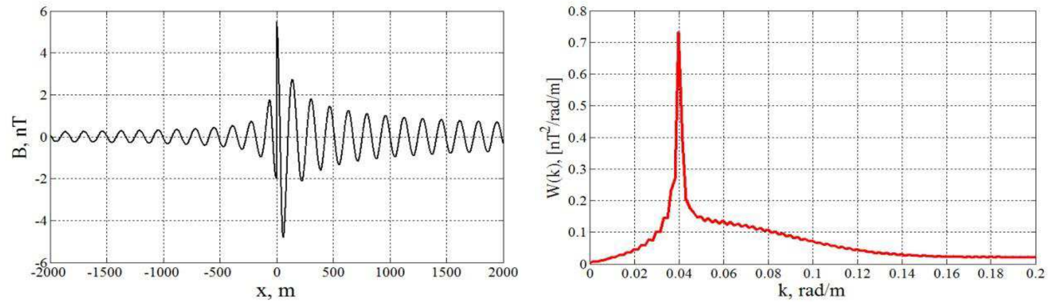


Figure 6. Magnetic anomaly induced by the wake of a ship model moving at $U = 6\,\mathrm{m/s}$. (a) The magnetic anomaly along x-axis at 1 m above the mean sea surface, and (b) the wave number spectral along the x-axis.



(a) (b)

Figure 7. Magnetic anomaly induced by the wake of a ship model moving at $U = 16\,\mathrm{m/s}$. (a) The magnetic anomaly along x-axis at 1 m above the mean sea surface, and (b) the wave number spectral along the x-axis.

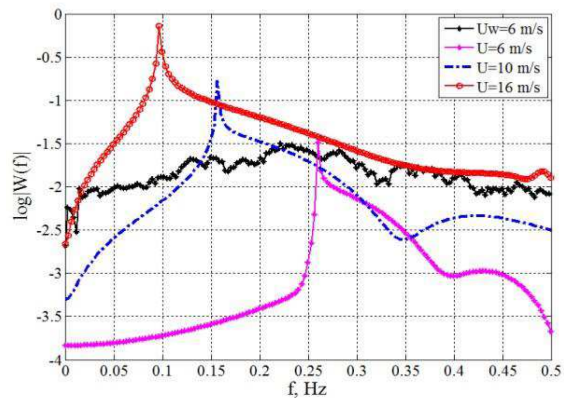


Figure 8. The frequency spectrum characteristics of magnetic anomalies induced by wake of a ship model moving at different speeds in wind waves. The sensor is stationary ($v = 0$) at $x = 0$ and $z = -1\,\mathrm{m}$ (1 m above the sea surface).

magnitude of the magnetic anomaly increase radically as the speed increases. The period also increase with the speed, or the central wave number decrease rapidly with the ship speed.

Finally, we assume that a scalar magnetometer moves at a speed v in x direction above the sea surface to acquire the time domain magnetic anomalies. The recorded data would look like Figure 5(a) with the x-axis replaced by t-axis with $t = x/(U + v)$. Of course, one may consider a general scenario that the sensor moves in any direction and both the speeds of ship and sensor, as well as the speed and direction of the wind, are changing. By Fourier transforming the time domain data, we obtain the frequency spectrum characteristics as shown in Figure 8, which are the redraws of Figure 5(d), Figure 6(b) and Figure 7(b), with the κ-axis replaced by f-axis, using the transform $\kappa x = \kappa(U + v)t = 2\pi ft$ that gives $f = \kappa(U + v)/2\pi$. It shows that magnetic anomaly induced by the wake acts as a narrowband signal while that by the wind waves behaves as a wideband noise.

5. SUMMARY

Moving conductive seawater can produce a distribution of weak electric currents in the medium by cutting the ambient geomagnetic fields. These electric currents will generate magnetic anomalies to the geomagnetic field that may be sensed by a high sensitivity magnetometer. The velocity field of wind waves can be found by using the ocean surface power spectrum. The velocity field of ship wakes can be resolved by resorting to ship wave spectrum. The induced magnetic fields may be expressed as a superposition of damping sinusoidal waves. The magnitude of induced magnetic fields near the sea surface produced by a typical vessel moving at moderate speed under moderate sea state is on the order of a few hundred pico-Tesla. The magnetic anomalies are essentially surface phenomena, which are evanescent or attenuates seriously in vertical direction. The magnetic anomaly frequency spectral of wind waves and ship wakes can be significantly different. The former behaves as a wideband noise, while the latter may act as a narrowband damping sinusoidal signal.

ACKNOWLEDGMENT

This work is supported by NSFC Project 61271032.

REFERENCES

1. Longuet-Higgins, M. S., M. E. Stern, and H. Stommel, "The electrical field induced by ocean currents and waves, with applications to the method of towed electrodes," *Pap. Phys. Oceanog. Meteorol.*, Vol. 8, No. 1, 1–37, 1954.
2. Weaver, J. T., "Magnetic variations associated with ocean waves and swell," *Journal of Geophysical Research*, Vol. 70, No. 8, 1921–1929, 1965.
3. Sanford, T. B., "Motionally induced electric and magnetic fields in the sea," *Journal of Geophysical Research*, Vol. 76, No. 15, 3476–3492, 1971.
4. Cox, C., "Electromagnetic fluctuations induced by wind waves on the deep-sea floor," *Journal of Geophysical Research*, Vol. 83, No. C1, 637–693, 1978.
5. Podney, W., "Electromagnetic fields generated by ocean waves," *Journal of Geophysical Research*, Vol. 80, No. 21, 2977–2990, 1975.
6. Chaillout, J. J., "Electromagnetic fields due to moving sea water by finite element calculation," *IEEE Transactions on Magnetics*, Vol. 31, No. 3, 554–559, 1995.
7. Bhatt, K. M. and P. Weidelt, "Motionally induced electromagnetic field within the ocean," *Inst. f. Geophys. U. Extraerrestrische Phys.*, 46–59, 2009.
8. Beal, H. T. and J. T. Weaver, "Calculations of magnetic variations induced by internal ocean waves," *Journal of Geophysical Research*, Vol. 75, No. 33, 6846–6852, 1970.
9. Wang, B. L. and H. Liu, "Space-time behaviour of magnetic anomalies induced by tsunami waves in open ocean," *Proceedings of the Royal Society A*, Vol. 469, 20130038, May 2013.
10. Qiu, D. H., *Wave Theory and Its Application in Engineering*, 5–210, High Education Presses, 1986 (in Chinese).
11. Guo, L. X., L. Wang, and Z. X. Wu, *Basic Theory and Methods for Rough Surface Scattering*, 1–30, Science Presses, 2010 (in Chinese).
12. Chang, S. W., "Effect of water depth on wind-wave frequency spectrum I. Spectral form," *China J. Oceanol. Limnol.*, Vol. 14, 97–105, 1995.
13. Tuck, E. O., J. I. Colins, and W. H. Wells, "On ship wave patterns and their spectra," *J. Ship Res.*, Vol. 15, 11–21, 1971.
14. Tuck, E. O., "Analytic aspects of slender body theory," *Wave Asymptotics*, Chapter 10, P. A. Martin, G. R. Wickham, Eds., Cambridge University Press, 1993.
15. Wu, Z. J., *On the Estimation of a Moving Ship's Velocity and Hull Geometry Information from Its Wave Spectra,* No. 91, 14–36, Department of Atmospheric, Oceanic and Space Sciences, 1991.
16. Zou, N. and N. Arye, "Detection of ship wakes using an airborne magnetic transducer," *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 38, No. 1, 532–539, 2000.
17. Yaakobi, O. and G. Zilman, "Detection of the electromagnetic field induced by the wake of a ship moving in a moderate sea state of finite depth," *J. Eng. Math.*, Vol. 70, 17–27, Spring Science and Business Media, 2010.
18. Yaakobi, O., "The induced electromagnetic field associated with submerged moving body in a stratified sea," M.Sc. Thesis, Tel-Aviv University, Israel, 2005.
19. Madurasinghe, D. and E. O. Tuck, "The induced electromagnetic field associated with submerged moving bodies in an unstratified conducting fluid," *IEEE Journal on Ocean Engineering*, Vol. 19, No. 2, 193–199, 1994.