

Dual Band Notched UWB Monopole Antenna Using Embedded Omega Slot and Fractal Shaped Ground Plane

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Abstract—This paper presents the development of an Ultra Wide Band (UWB) monopole antenna with dual band notch characteristics. Modified crown-square shaped fractal slots in the ground-plane are implemented to enhance the impedance bandwidth to around 58% as compared to conventional square monopole antenna without slots. Impedance bandwidth of the proposed antenna is approximately 114% with Voltage standing wave ratio (VSWR) < 2 . In addition to this, two omega-shaped (Ω) slots have been incorporated in the radiating patch to render band-notch characteristics centered at 5.5 GHz band assigned to IEEE802.11a and HIPERLAN/2 as well as X-band for satellite communication centered at 7.5 GHz band. Measured antenna gain is stable over the entire UWB region except at the notch bands. Radiation pattern of the antenna show that the proposed antenna exhibits nearly monopole like E plane radiation patterns and omni-directional H plane radiation patterns throughout the band. A fabricated prototype is developed with close agreement between simulated and measured results.

1. INTRODUCTION

Over a decade ago, the Federal Communication Commission (FCC) allocated the frequency range of 3.1 GHz to 10.6 GHz [1] for Ultra Wide Band (UWB) communication with the power spectral density of -41.3 dBm/MHz. Design of components for ultra-wide band system remains a challenging task owing to the wide impedance bandwidth. Maintaining the 7500 MHz bandwidth with compact circuit size is the main aim of the research work in this field. Several methods have been implemented to improve the bandwidth of planar monopoles based antenna topology [2,3]. However, there exist some different narrowband systems in the UWB frequency range such as 5.15–5.825 GHz band assigned for IEEE802.11a and HIPERLAN/2 and 7.35–7.75 GHz band for downlink of X-band satellite communication systems. In recent years different strategies have been investigated to reject such frequencies [4–6]. Introducing slot is a very popular and easy approach in this regard. Various shaped slots have been implemented in the ground plane [7], and radiating patches [8–12], to filter out the unwanted frequency bands. Beside this slot geometry, use of split ring resonator [13], Hilbert curves [14], are another way to notch out the desired frequency band. Stretching the impedance bandwidth beyond the specified UWB range is another challenging task for any RF engineer. Numerous topologies have been reported for this purpose, like M-shaped strip on conductor backed plane [15], inverted L-shaped slit and inverted U ring shaped slot in the ground plane [16], by using modified penta gasket Koch (PGK) in [17], T-shaped notch [18] in the ground plane and in [19] by cutting a rotated T-shaped notch in the ground plane and also by inserting rotated T-shaped parasitic structure, again in [20] by cutting a pair of L-shaped slots on the radiating patch and also by adding a pair of L-shaped conductor backed plane in the ground plane the bandwidth is increased.

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Fractal geometry has been often used to design multiband miniaturized antenna [21]. The self similar and space filling property of fractal geometry [22–26] increases effective electrical length to reduce the size of the antenna and enhances bandwidth by bringing multiple resonances closer.

This paper mainly focuses on two different issues in the development of UWB antenna. One is to design a compact UWB monopole antenna with improved impedance bandwidth and the other is to have band notch characteristic at two different frequencies. The novelty lies in the application of fractal geometry in ground plane to increase the bandwidth and reduce the size of antenna. This stems from the fact that resonance characteristics of coplanar waveguide based UWB monopole antenna depends also on the shape and size of ground plane. However, this has not been investigated extensively. This paper attempts to explore this issue by using crown-square fractal technology in the ground plane by which 114% impedance bandwidth is achieved. Omega-slots have been introduced in the patch close to each other to bring about band notch characteristics. In comparison to [17] the proposed antenna provides better bandwidth performance and in comparison to [4–6], it provides strong radiation attenuation at the notch bands. However, in literature various antennas are reported with dual, multiple band notch characteristic [9, 13] but their gain performances are low with respect to the proposed antenna. To comprehend the operation of the band rejection mechanism of the slots and radiating element a lumped element equivalent circuit model is developed. Rest of the paper is arranged as follows. Detailed antenna design with parametric studies is given in Section 2, followed by results and discussion in Section 3 with conclusion in Section 4.

2. ANTENNA DESIGN AND PARAMETRIC STUDY

2.1. Basic Antenna Design

The proposed antenna is fed by a Co Planar Waveguide (CPW) transmission line of 50 Ohm as shown in Figure 1(a). It is realized on an FR4 substrate of $\epsilon_r = 4.4$, $h = 1.6$ mm having loss tangent of $\tan \delta = 0.02$. The basic antenna structure consists of a square patch whose dimension is determined by quarter wavelength at lowest resonant frequency as given by (1).

$$f_L = \frac{c}{2(L_p + W_p)\sqrt{\epsilon_{eff}}} \quad (1)$$

where f_L is the lowest resonant frequency at 3.6 GHz, c the speed of light in free space, ϵ_{eff} the effective dielectric constant, and L_p and W_p are the length and width of the patch, respectively. The square patch has a very compact dimension of $(L_p \times W_p)$ 13×13 mm² and upon including substrate the overall antenna dimension is $L_{sub} \times W_{sub}$ 36.7×28 mm².

2.2. Fractal Slot Design on Ground

It is a common observation in CPW-fed monopole antenna that the radiation and resonance characteristics depend on the size of the ground plane. Therefore, to explore the effect by using fractal shapes in the ground plane a crown-square fractal was introduced [27, 28] in each of the coplanar ground planes which resulted in altered current path and subsequently a change in resonance characteristics. This led to 58% bandwidth enhancement in comparison to conventional square monopole antenna with no such fractal slots. Figure 1(a) shows the antenna structure followed by ground fractal in Figure 1(c).

The modified crown square fractal is created by etching a rectangle and there after adding a square from the middle portion of rectangle in an iterative manner as shown in Figure 1(b). In the first iteration of fractal, $a = 10.7$ mm, $b = 8$ mm and $a/b = 1.33$ where a and b are the sides of rectangle which is etched from the ground plane, and a square with a side length b is inscribed into the middle portion of the rectangle. This is called first iteration. In the 2nd iteration, another rectangle of 0.6 times of a and 0.6 times of b is etched in the middle of the 1st iterative square. The sides of rectangle are indicated by c and d , where $c = 6.44$ mm and $d = 4.8$ mm and maintain the ratio of $c/d = 1.33$. Again, the square of same side length as d is inscribed in the middle of rectangular slot. Same type iterative procedure for fractal has been taken for further stage. In this paper up to 4th iteration is shown as the design becomes quite complicated, and fabrication is no longer feasible with higher iterations.

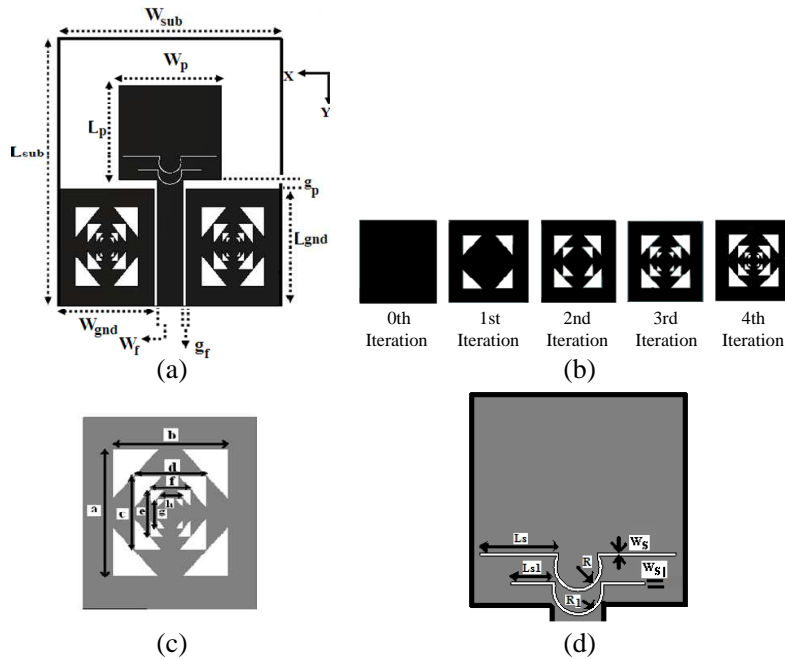


Figure 1. Proposed Antenna geometry. (a) Antenna structure. (b) Fractal algorithm. (c) Fractal on the ground plane. (d) Omega shaped notch on the radiator.

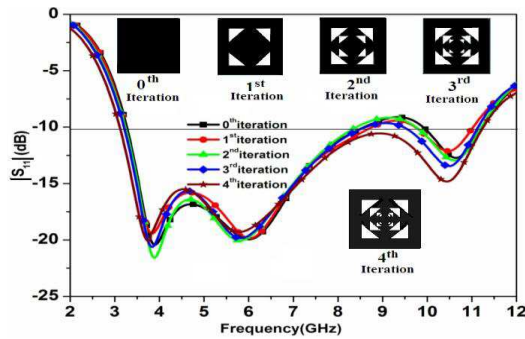


Figure 2. Simulated results of proposed antenna with respect to each iteration.

Table 1. Bandwidth improvement in different iteration.

Iteration no	Lower cut off Frequency (GHz)	Upper cut off Frequency (GHz)	Bandwidth (GHz)	% of B.W improvement with respect to the zeroth iteration
Zero	3.30	8.50	5.20	-
1st	3.25	8.70	5.45	4.80%
2nd	3.21	8.90	5.69	9.40%
3rd	3.15	9.0	5.85	12.50%
4th	3.10	11.30	8.20	57.69%

As the iteration of fractal slot on the ground plane increases, matching improves at the upper band edge, and as a result, the overall bandwidth increases by 58% in comparison to the simple square monopole antenna. The comparison is shown in Figure 2 and presented in Table 1.

2.3. Omega Type Slots with Equivalent Circuit Model

Two omega shaped slots at the radiating patch are introduced to notch out the frequency bands of 5.15–5.825 GHz and 7.25–7.75 GHz effectively. The slots provide strong radiation attenuation at the corresponding notch frequency. When the proposed antenna operates at 5.5 GHz and 7.5 GHz frequency, the electromagnetic energy is coupled strongly to omega shaped slots which prevent the energy to radiate out at those frequencies. These surfaces current are mainly responsible for providing radiation attenuation at the corresponding notch frequency.

The length of desired notches can be calculated by (2) and (3) which are of length $\lambda_g/2$ at the corresponding notch frequencies.

$$L_{\text{notch}5.5} = \pi (R + W_S) + 2L_S + 2W_S \quad (2)$$

$$L_{\text{notch}7.5} = \pi (R_1 + W_S) + 2L_{S1} + 2W_S \quad (3)$$

Here R and R_1 are radii of semicircles, and L_S and L_{S1} are the lengths of omega-shaped slots at 5.5 GHz and 7.5 GHz notch bands, respectively, as indicated in Figure 1(d). Width of slot is W_S . Basically, it acts as a band-stop filter, which filters out the desired notch frequency. Optimized slots are placed at the vicinity of the feed line, where electric field or magnetic current vectors are densely populated. Slot creates a standing wave at its design frequency and thereby minimizes radiation at that frequency. Apart from slot size, position is also critical to its effective usage. Changing the relative position of slots is an effective method to adjust each peak rejection and rejection Bandwidth. Because the surface currents are strong near the feed line, the desired current corresponding to the notch frequency is perturbed more at that position.

The antenna input impedance at the notch frequencies is similar to that of a lumped parallel RLC circuit. So the equivalent circuit of the slots has been proposed as two parallel RLC networks connected in cascade with a 50 Ohm load. ANSYS HF SuiteTM has been utilized for analyzing the circuit, shown in Figure 3. The first parallel RLC resonates at 5.5 GHz and second one at 7.5 GHz. To complete this equivalent circuit, the values of lumped element need to be determined. The numerical value of the circuit components are evaluated using Equations (4) to (6).

$$R = 2Z_0 \left(\frac{1}{|S_{21}|} - 1 \right)_{f=f_0} \quad (4)$$

From parallel RLC resonant circuit

$$L = \frac{R}{2\pi f_0 Q} \quad (5)$$

$$C = \frac{1}{4(\pi f_0)^2 L} \quad (6)$$

where Z_0 is the 50 Ohm characteristic impedance of transmission line, f_0 the resonance frequency, $|S_{21}|$ the simulated insertion loss of the filter, and Q the quality factor which indicates the sharpness of the

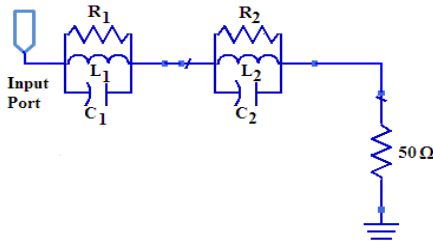


Figure 3. Equivalent lumped-element circuit model of dual band reject UWB antenna.

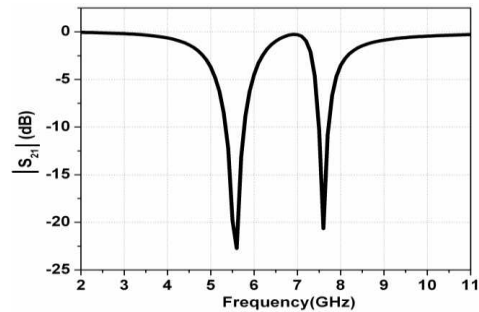


Figure 4. Insertion loss of the combined filtering network.

RLC resonators at the rejected bands and is determined by Equation (7)

$$Q = \frac{f_0}{BW} \tag{7}$$

In (7) BW is bandwidth of the rejected bands. The corresponding Q and lumped element values were calculated at 5.5 GHz and 7.5 GHz as listed in Table 2.

The width of the slot depicts the capacitance of the filter, whereas the length determines the inductance of the tank circuit. The value of ‘ R ’ can be taken as dielectric losses associated with the slot. From the insertion loss characteristics of the combined filtering network as given in Figure 4 it can be observed that at 5.5 GHz and 7.5 GHz the attenuation is more than 20 dB. So, it acts as stop band filter. The center frequency of each notch band depends on the length of the slots. Figure 5 shows that the length L_S of 5.5 GHz slot varies from 4.8 mm to 5.2 mm without changing the other slot length, the corresponding notch band shifts to lower end frequency without affecting the other one. An optimum length of 5 mm is chosen for which the desired notch at 5.5 GHz is obtained. As the slot width W_s increases, the notched band is shifted toward higher frequency as shown in Figure 6. The optimum slot width of 0.2 mm is found suitable for the notch frequency at 5.5 GHz.

Table 2. Calculated values of the equivalent circuit’s lumped element.

Notch (GHz)	BW (MHz)	Q	R (k Ω)	L (nH)	C (pF)
5.5	51	107.8	2.4	0.630	1.3
7.5	67	112.0	1.0	0.169	2.6

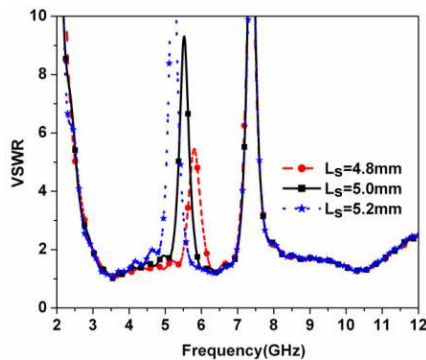


Figure 5. Simulated VSWR for different values of L_S .

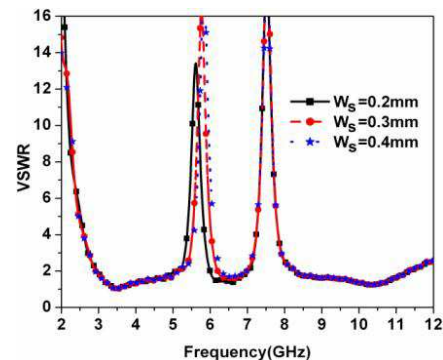


Figure 6. Simulated VSWR for different values of W_S .

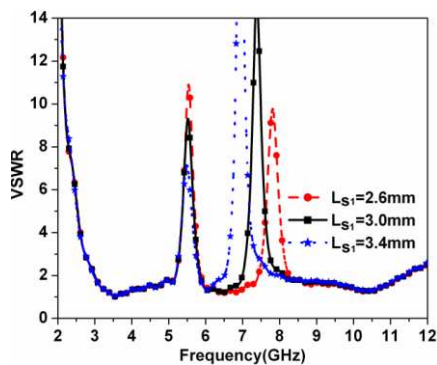


Figure 7. Simulated VSWR for different values of L_{S1} .

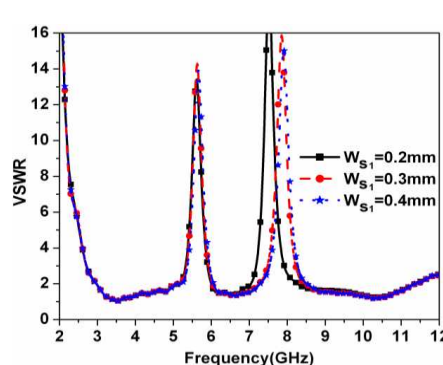


Figure 8. Simulated VSWR for different values of W_{S1} .

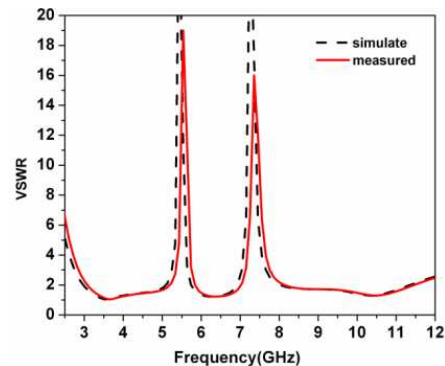
Table 3. Modified crown square fractal band notch antenna design dimensions.

Main Radiating Antenna	Value (mm)	Ground Fractal Slot	Value (mm)	Omega Slot	Value (mm)
W_{sub}	28.0	a	10.70	L_s	5.0
L_{sub}	36.7	b	8.0	L_{s1}	3.0
W_{gnd}	12.0	c	6.44	R	1.5
L_{gnd}	16.0	d	4.80	R_1	1.6
W_f	3.2	e	3.88	W_s, W_{s1}	0.2
G_f	0.4	f	2.89		
G_p	1.2	g	2.33		
W_p, L_p	13.0	h	1.74		

It is observed from parametric study that the resonant frequency of the notched band depends on the length of the slot, and notched bandwidth depends upon width of the slot. Similarly, the effects of second slot parameters are considered. The length L_{S1} is varied from 2.6 mm to 3.4 mm, the notch frequency of 7.5 GHz is shifted to lower end without affecting the first notch as shown in Figure 7. An optimum value of L_{S1} is chosen to be 3 mm for which the desired notch frequency 7.5 GHz is obtained. As the width of the notch is increased, the notch frequency shifted towards higher end frequency and an optimum value of W_{S1} is chosen to be 0.2 mm as shown in Figure 8. The optimized values of proposed antenna design parameters are shown in Table 3.

3. RESULTS AND DISCUSSION

A fabricated prototype of the proposed antenna is developed as per the design explained in previous section. A photograph is shown in Figure 9, and the measured and simulated VSWR are compared in Figure 10. For all electromagnetic analysis, CST Microwave Studio is used and for circuit simulation ANSYS HF suite™ designer software is used in this work. The S -parameter measurement is performed using Rhode and Schwarz ZVA 40 VNA, which is in good agreement with simulation results except minor discrepancies due to fabrication tolerance and SMA connector losses. The proposed antenna covers 3.1 to 11.3 GHz bandwidth with VSWR < 2 except at the notch bands centered at 5.5 GHz and 7.5 GHz, respectively. The surface current distribution at notch frequencies of 5.5 GHz and 7.5 GHz are illustrated in Figure 11. It is observed that current is concentrated on the upper omega slot and lower omega slot at 5.5 GHz and 7.5 GHz, respectively. This indicates that current is very high at the top of both omega slots and that impedance will be very low and assumed to be nearly zero (short circuit) at those frequencies.

**Figure 9.** Fabricated prototype of antenna.**Figure 10.** Comparison of simulated and measured VSWR.

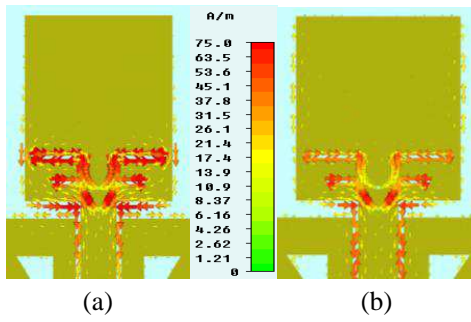


Figure 11. Current distribution over slot at centre frequency of notched bands. (a) 5.5 GHz. (b) 7.5 GHz.

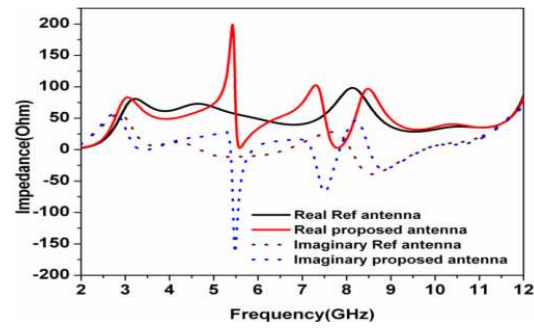


Figure 12. Real and imaginary part of input impedance of antenna.

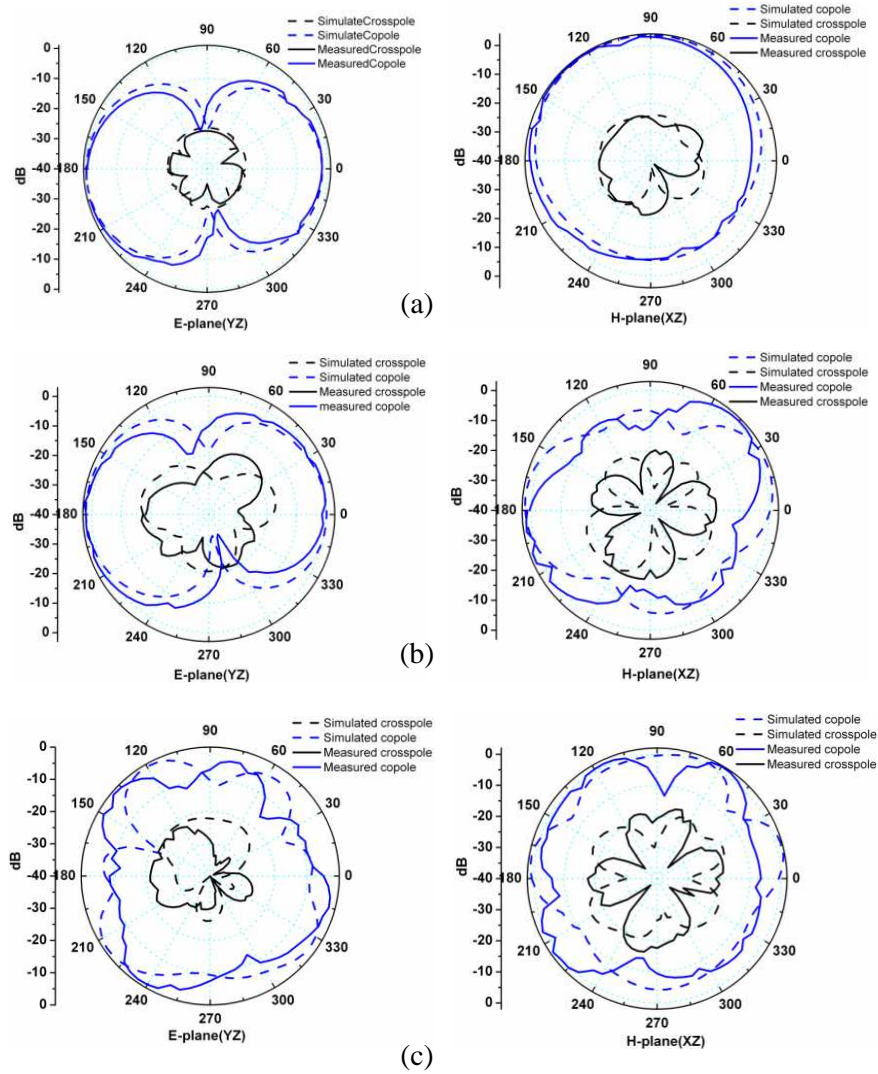


Figure 13. Measured *E* and *H* plane patterns of the proposed antenna at (a) 3.1 GHz, (b) 7 GHz, and (c) 11 GHz.

As the antenna acts as a transmission line, the impedance at the bottom or near feeding point of both omega slots will be very high and assume to be infinite (Open circuit). This infinite impedance near the feeding point is mainly responsible for these two notches. The impedance curve of Figure 12 also supports the above explanation. Imaginary part of input impedance shows a parallel resonance characteristic at 5.5 GHz and real part has a peak at 200Ω in comparison to the reference antenna. Similarly at 7.5 GHz the imaginary part of input impedance shows a parallel resonance characteristic and real part has a peak at 100Ω .

The radiation patterns of the proposed antenna resemble conventional monopole as at lower frequencies which is nearly omnidirectional in H -plane and figure of eight in E -plane. The measured and simulated co-polar and cross-polar patterns at 3.1, 7 and 11 GHz are depicted in Figure 13. From the radiation patterns, it is observed that the proposed antenna behaves satisfactorily at lower frequencies but some discrepancy is observed at higher frequency end.

The antenna shows a nearly flat group delay response in 3.1 GHz to 11.3 GHz UWB band and the variation of group delay is less than 1 ns except the notch bands as shown in Figure 14. It depicts that the antenna provides distortion less transmission throughout the operating band.

The variation of measured antenna gain and efficiency from 3 to 11 GHz is shown in Figure 15. It indicates that the antenna exhibits satisfactory gain flatness except the two notch bands where gain falls sharply to -5 dBi. The radiation efficiency varies 85–90% in the desired frequency range and sharply decreases to 20% at two notched freq indicating that the antenna has good band notched characteristic. In addition, the comparisons for different existing literature are illustrated in Table 4. The proposed antenna is better in bandwidth performances, gain and band notch characteristics than [6, 9, 13, 17]. To determine the ‘correlation coefficient’ between the signal of receiving antenna $S_2(t)$ terminal and the input antenna $S_1(t)$ terminal, the following equation is used.

$$\rho = \max_{\tau} \left[\frac{\int S_1(t) S_2(t - \tau) dt}{\sqrt{\int S_1^2(t) dt} \sqrt{\int S_2^2(t) dt}} \right] \quad (8)$$

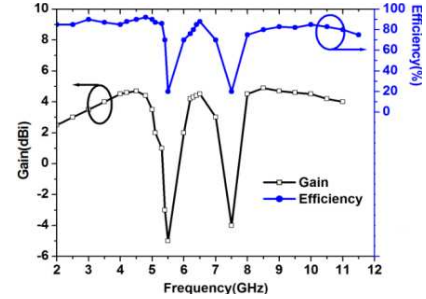
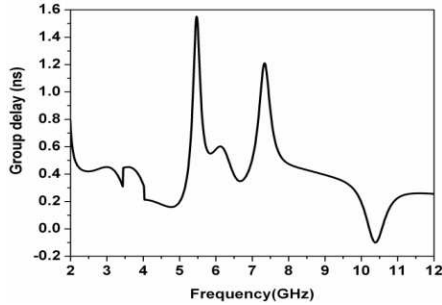


Figure 14. Variation of measured group delay (ns) of the antenna. **Figure 15.** Measured gain of proposed antenna.

Table 4. Comparisons of the proposed UWB antenna to other UWB antennas.

Performance compare	Ref. [6]	Ref. [9]	Ref. [13]	Ref. [17]	Proposed work
Dielectric constant	2.2	3.38	4.4	3.38	4.4
Thickness	17 μm	1/32 in	1.5 mm	1.5 mm	1.6 mm
Size (mm^2)	34 \times 27	39 \times 35	30 \times 25	18 \times 18	36.7 \times 28
Bandwidth (GHz)	3.1–10.6	3.1–10.6	3.03–11.4	4.25–11	3.1–11.3
1st notched VSWR value	11	14	8	-	19
2nd notch VSWR value	7	14	6	-	15
Gain (dBi)	5.5	4.0	5.0	5.0	5.8

where τ is delay, and input signal is the fifth derivative of Gaussian pulse. The ‘correlation coefficient’ indicates the similarity between source pulse and received pulse. When they are identical, ‘ ρ ’ shows maximum value which is ideally ‘1’, which represents when antenna system does not distort the input signal. The distortion is expected due to notch bands in the frequency response of the antenna. The ‘correlation coefficient’ obtained from this fractal shaped slot based ground plane antenna without notch is 0.853 and with notch bands is 0.785.

4. CONCLUSION

In this paper, a compact CPW-fed modified crown-square ground fractal antenna with dual band notched characteristic is presented. Due to introduction of modified crown-square fractal at the ground plane, 58% bandwidth is improved with respect to conventional square monopole antenna without slot. Again by using two omega-shaped slots in the radiating patch, the interference for IEEE802.11a and HIPERLAN/2 and downlink of satellite communication are reduced. Stable radiation patterns and constant gain in the UWB band are obtained. The group delay variation is less than 1 ns over the entire frequency band. The simulation and measurement results of the proposed antenna show a good agreement. The proposed antenna is expected to be a good candidate in various UWB systems.

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