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A Modified Coupled-Line Schiffman Phase Shifter with Short Reference Line

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Abstract—A coupled-line circuit structure is proposed in this paper to design a modified Schiffman phase shifter with short reference line. Based on the traditional transmission line theory and ABCD parameters, closed-form mathematical equations for electrical and scattering parameters are obtained. Obviously, this proposed coupled-line phase shifter has several advantages such as arbitrary phase difference, easy implementation, and analytical design method. Finally, two examples of microstrip coupled-line phase shifter, which have fractional bandwidth over 45%, are fabricated and measured. Good agreements between the simulated and measured results verify our design.

1. INTRODUCTION

Phase shifter, as an important microwave component, has been widely used in the design of wideband phase-array antennas, Butler matrices [1, 2] which consist of 90° hybrids and phase shifters, couplers [3] which are a combination of coupled-line power divider and improved coupled-line phase shifter, etc. Phase shifters are mainly categorized into three types: mechanical phase shifters, ferrite phase shifters and semiconductor device phase shifters. In this paper only some planar phase shifters are discussed.

One important type is load-line phase shifter [4–7]. The digital loaded-line phase-shift network [4], which uses the loads to change the electrical length of a fixed transmission line, is used to design two types of microwave hybrid thin-film phase shifters. Digital diode-loaded-line phase shifters is described in [5]. It shows that a 75° separation between the loading susceptances has the same or better performance than a 90° separation. In addition, analytical equations for a loaded-line phase shifter are derived, and it is shown that the maximum bandwidth is obtained when the spacing between the switched susceptances is 90° in [6]. By utilizing a phasor diagram representation of the line, the circuit design procedure for the loaded-line phase shifter is presented in [7].

Another important type of phase shifter structure is firstly described by Schiffman in [8]. By proper selection of the length of these lines and coupling coefficients, the phase difference can be made to be almost constant over a broad bandwidth. However, the Schiffman phase shifter requires a tight coupling in the coupled line sections. Therefore, the improved Schiffman phase shifters [9] shows that different configurations can be used to design differential phase shifters with loose coupled lines and the same performance as for the standard case. The analysis of a phase-shift network is given in [10]. This network consists of a cascade of N pairs of coupled transmission lines connected together at their far end with different electrical lengths. General synthesis procedures are given to determine its phase deviation and maximum bandwidth when the coupling coefficients are known, or to determine the coupling coefficients for a desired bandwidth or maximum phase deviation [11]. Moreover, [12] shows significant differences in theoretical performance by using an exact analysis in terms of odd and even mode propagation velocities. In order to cancel the influence of the even-odd phase velocity, a novel method is put forward

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in [13] by choosing the characteristic impedance and electrical length of the linking line between the coupled lines. In addition, there are many methods to improve the bandwidth of the Schiffman phase shifter, such as multi-section cascading connection [14, 15], utilizing the unique frequency-independent quadrature property of symmetric couplers [16], and using two 90° parallel coupled lines [17]. Besides, there are other phase shifter structures, such as a broadband phase shifter which employs loaded $\lambda/4$ transmission line [18], single-layer ultra wideband 90° differential phase shifter whose main line consists of a multi-section radial TL in the middle [19], etc. However, these circuit structures of the phase shifters are complex and very large.

Especially, the conventional Schiffman phase shifter [8, 9] can be used to achieve flat relative-phase differences between output ports. However, these circuits require long reference lines and tight coupled sections which are difficult to realize. Compared with the circuits in [8, 9], this proposed modified Schiffman phase shifter, composed of only two different coupled lines with weak coupling coefficients, can realize arbitrary phase difference by changing the reference line when the main line is fixed. This novel phase shifter offers several advantages, including: 1) arbitrary phase difference; 2) short reference line; 3) weak coupled sections; 4) all ports matched; 5) analytical design equations for circuit electrical parameters and scattering parameters; and 6) easy realization in the common microstrip technology. In Section 2, closed-form design equations and scattering parameters are derived using conventional transmission line theory and $ABCD$ parameters. For observation of the phase difference at the operating frequency and the influence of electrical parameters, six examples will be discussed in the Section 3. Two experimental microstrip circuits are fabricated and measured in Section 4. The consistency between the calculated and simulated results validates this proposed phase shifter.

2. CIRCUIT STRUCTURE AND DESIGN THEORY

Figure 1 shows the proposed circuit structure of the modified Schiffman phase shifter, which is composed of two different coupled lines. The coupled-line parameters include even-mode characteristic impedances (Z_{e1}, Z_{e2}), and odd-mode characteristic impedances (Z_{o1}, Z_{o2}). For simplification, the even and odd electrical lengths are assumed to be the same. In addition, each port impedance is denoted by Z_0.

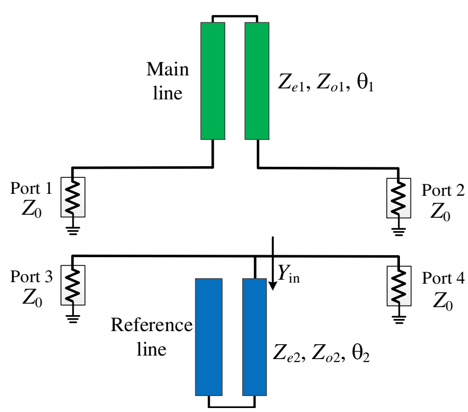

Figure 1. The proposed modified coupled-line Schiffman phase shifter.

2.1. Analytical Design Equations of the Phase Shifter

According to [20], $ABCD$ parameter of the coupled line connected at one end is:

$$\begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} = \begin{bmatrix} \dfrac{Z_{ei}\cot\theta_i - Z_{oi}\tan\theta_i}{Z_{ei}\cot\theta_i + Z_{oi}\tan\theta_i} & \dfrac{2jZ_{ei}Z_{oi}}{Z_{ei}\cot\theta_i + Z_{oi}\tan\theta_i} \\ \dfrac{2j}{Z_{ei}\cot\theta_i + Z_{oi}\tan\theta_i} & \dfrac{Z_{ei}\cot\theta_i - Z_{oi}\tan\theta_i}{Z_{ei}\cot\theta_i + Z_{oi}\tan\theta_i} \end{bmatrix} \quad (i = 1, 2). \tag{1}$$

The ideal matching at the operation frequency is necessary, and the corresponding scattering parameters can be expressed as follows:

$$S_{ii} = 0, \quad (i = 1, 2, 3, 4) \tag{2}$$

The scattering parameters of the main line can be obtained in terms of $ABCD$ parameters in [21]:

$$S_{11} = \frac{A_1 + B_1/Z_0 - C_1 Z_0 - D_1}{A_1 + B_1/Z_0 + C_1 Z_0 + D_1}, \tag{3a}$$

$$S_{12} = S_{21} = \frac{2}{A_1 + B_1/Z_0 + C_1 Z_0 + D_1}, \tag{3b}$$

$$S_{22} = \frac{-A_1 + B_1/Z_0 - C_1 Z_0 + D_1}{A_1 + B_1/Z_0 + C_1 Z_0 + D_1}. \tag{3c}$$

After substituting (1), (3) into (2) and some simple linear mathematical operation, the following equation can be obtained:

$$Z_{e1} Z_{o1} = Z_0^2 \tag{4}$$

Then the scattering parameters of the main line at the operating frequency in Equation (3) can be simplified:

$$S_{11} = S_{22} = 0, \tag{5a}$$

$$S_{21} = S_{12} = \frac{Z_{e1} \cot\theta_1 + Z_{o1} \tan\theta_1}{Z_{e1} \cot\theta_1 - Z_{o1} \tan\theta_1 + j2Z_0} = e^{j\psi_1}, \tag{5b}$$

where the phase shifter Ψ_1 is defined by:

$$\cos\psi_1 = \frac{g_1 - \tan^2\theta_1}{g_1 + \tan^2\theta_1}, \quad \left(g_1 = \frac{Z_{e1}}{Z_{o1}}\right). \tag{6}$$

For the reference line in Figure 1, the scattering parameters can also be obtained in terms of $ABCD$ parameters in [21]:

$$S_{33} = \frac{A_3 + B_3/Z_0 - C_3 Z_0 - D_3}{A_3 + B_3/Z_0 + C_3 Z_0 + D_3}, \tag{7a}$$

$$S_{34} = S_{43} = \frac{2}{A_3 + B_3/Z_0 + C_3 Z_0 + D_3}, \tag{7b}$$

$$S_{44} = \frac{-A_3 + B_3/Z_0 - C_3 Z_0 + D_3}{A_3 + B_3/Z_0 + C_3 Z_0 + D_3}. \tag{7c}$$

where

$$\begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y_{in} & 1 \end{bmatrix}. \tag{8}$$

According to the transmission line theory, we can obtain the following equation:

$$Y_{in} = \left.\frac{C_2 Z_L + D_2}{A_2 Z_L + B_2}\right|_{Z_L = \infty} = \frac{2j}{Z_{e2} \cot\theta_2 - Z_{o2} \tan\theta_2} \tag{9}$$

After substituting Equations (1), (7), (8), and (9) into (2), the following equation can be obtained:

$$\theta_2 = n\frac{\pi}{2} (n = 1, 2, 3, \ldots) \tag{10}$$

Then the scattering parameters of the reference line at the operating frequency in Equation (7) can be simplified:

$$S_{33} = S_{44} = 0, \tag{11a}$$

$$S_{43} = S_{34} = 1. \tag{11b}$$

Equation (11) implies that the power from port 3 (4) is transmitted to port 4 (3) totally.

2.2. Scattering Parameters and the Phase Difference

From Section 2.1, this proposed circuit structure can be designed analytically according to Equations (4), (10). The scattering parameters and phase difference will be given in this section.

After substituting Equations (1), (4), (8), and (9) into Equations (3) and (7), the scattering parameters will be obtained as follows:

$$S_{11} = S_{22} = 0, \tag{12a}$$

$$S_{21} = S_{12} = \frac{Z_{e1}\cot\theta_1 + Z_{o1}\tan\theta_1}{Z_{e1}\cot\theta_1 - Z_{o1}\tan\theta_1 + j2Z_0}, \tag{12b}$$

$$S_{33} = S_{44} = \frac{-jZ_0}{(Z_{e2}\cot\theta_2 - Z_{o2}\tan\theta_2) + jZ_0}, \tag{12c}$$

$$S_{43} = S_{34} = \frac{Z_{e2}\cot\theta_2 - Z_{o2}\tan\theta_2}{(Z_{e2}\cot\theta_2 - Z_{o2}\tan\theta_2) + jZ_0}. \tag{12d}$$

From Equations (12), a conclusion that S_{11}, S_{22} are independent of the frequency can be obtained. The phase difference at the operating frequency can be calculated as follows:

$$\Delta\theta = phase(S_{21}) - phase(S_{43}) = \psi_1. \tag{13}$$

Equation (13) means that the phase difference $\Delta\theta$ at the operating frequency only depends on the main line. Nevertheless, the fractional bandwidth lies on the whole circuit.

3. ANALYSIS OF CIRCUIT ELECTRICAL PARAMETERS

Based on the previous theoretical analysis, the electrical parameters of the proposed circuit can be easily calculated according to Equations (4), (6), and (10). For observation of the phase difference at the operating frequency and influence of the electrical parameters, six examples will be discussed in this section.

3.1. The Discussion of the Phase Difference $\Delta\theta$

According to Equation (12), the phase difference at the operating frequency is only decided by the main line. From Figure 2(a), it can be observed that when θ_1 varies from 0° to 180°, the phase shift $\Delta\theta$ of the proposed circuit varies from 360° to 0°, monotonously. Though the impedance ratio g_1 of the coupled line increases from 1.2 to 1.9, the phase difference $\Delta\theta$ changes slightly. The effect of g_1 and θ_1 on the bandwidth of the phase shifter will be analyzed in Section 3.3. From Equation (12) and Figure 2(a), we can easily find appropriate g_1 and θ_1 to satisfy the desired phase difference $\Delta\theta$. Figure 2(b) shows that the circuit has the phase difference $\Delta\theta$ of 325°, 289°, 248°, 203°, 156°, 111°, 70°, 34° when θ_1 is equal to 20°, 40°, 60°, 80°, 100°, 120°, 140°, and 160°, respectively. Other circuit parameters are $g_1 = 1.4$, $Z_{e2} = 100\,\Omega$, $Z_{o2} = 70\,\Omega$, and $\theta_2 = 90°$.

3.2. The Selection of the Impedance of the Reference Line

For observing the effects of the reference coupled-line section, Table 1 lists the design parameters (Z_{e2}, Z_{o2}) of **Examples** (**1**, **2**, and **3**) with different impedances. It is assumed that these three examples operate at 1 GHz, and the parameters of the main line are fixed. The fixed parameters are $g_1 = 1.9$, $\theta_1 = 54°$. Figure 3 illustrates the calculated scattering parameters of the reference line and the phase information, which are based on the lossless coupled-line model. Since the scattering parameters of the main line are independent of the frequency, they are not discussed in this and next sections. From Table 1 and Figure 3, it can be observed that with the decrease of the characteristic impedances of the reference line, the bandwidths of S_{33}, S_{43}, and $\Delta\theta$ diminish correspondingly, indicating that the selection of the reference line impedance influences the bandwidth of the phase shifter.

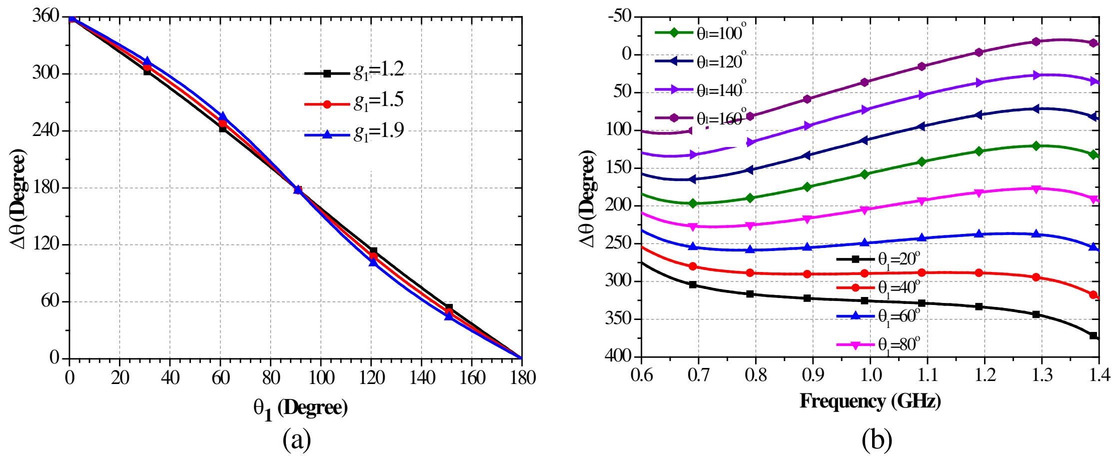

Figure 2. (a) The phase difference $\Delta\theta$ vs the electrical length θ_1 when $g_1 = 1.2, 1.5$, and 1.9. (b) The phase difference $\Delta\theta$ when θ_1 varies from 20° to 160°.

Table 1. The design parameters of **Examples 1**, **2**, and **3**.

Example	$Z_{e2}(\Omega)$	$Z_{o2}(\Omega)$	BandWidth of $S_{33}(< -10\,\text{dB})$	BandWidth of $S_{43}(> -3\,\text{dB})$	BandWidth of $\Delta\theta$ ($\pm 10°$)
Example 1	100	70	44%	72%	75%
Example 2	80	60	40%	70%	73%
Example 3	60	40	30%	60%	64%

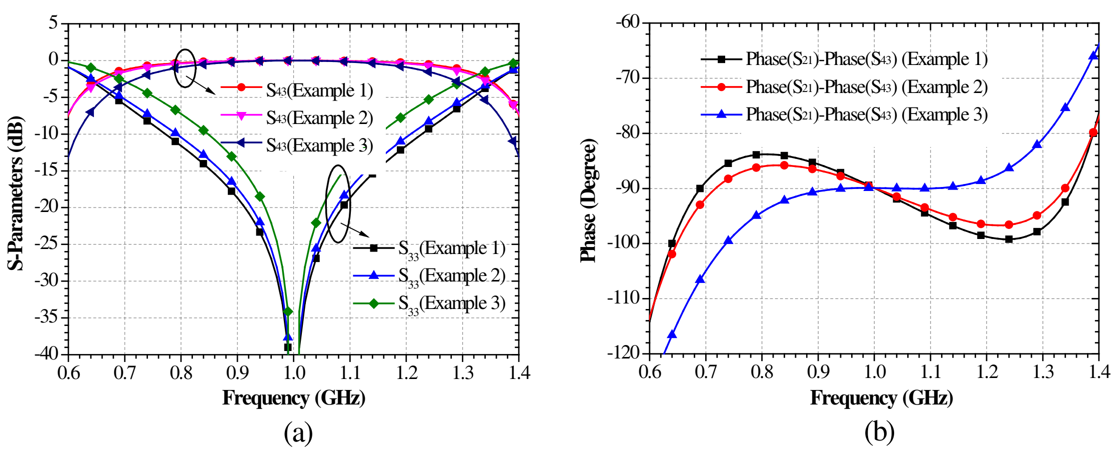

Figure 3. The calculated results of **Example 1**, **2**, and **3**: (a) Scattering Parameters; (b) Phase information.

3.3. The Selection of g_1 and θ_1

Since a fixed phase difference $\Delta\theta$ can be decided by many pairs of g_1 and θ_1, the selection will become very complicated. Table 2 lists the design parameters (g_1, and θ_1) of three **Examples** (**4**, **5**, and **6**) with different impedances, but the same phase difference $-45°$. It is assumed that these examples operate at 1 GHz, and the parameters of the reference line are fixed. The fixed parameters are $Z_{e2} = 109\,\Omega$, $Z_{o2} = 84\,\Omega$, $\theta_2 = 90°$. Figure 4 illustrates the calculated scattering parameters of the reference line, which are based on the lossless coupled-line model. Note that the circuit parameters of the reference line are the same, only scattering parameters of **Example 4** are plotted in Figure 4. From Table 2 and Figure 4, it can be observed that with different g_1 and θ_1, the bandwidth of the phase shifter is almost the same, indicating that the selection of g_1 and θ_1 has little influence on the bandwidth of the phase shifter.

Table 2. The design parameters of **Examples 4**, **5**, and **6**.

Example	g_1	$\theta_1(^\circ)$	BandWidth of $S_{33}(< -10\,\text{dB})$	BandWidth of $S_{43}(> -3\,\text{dB})$	BandWidth of $\Delta\theta$ $(\pm 10^\circ)$
Example 4	1.1	23.5	50%	76%	58%
Example 5	1.4	26	50%	76%	58%
Example 6	1.7	28.5	50%	76%	59%

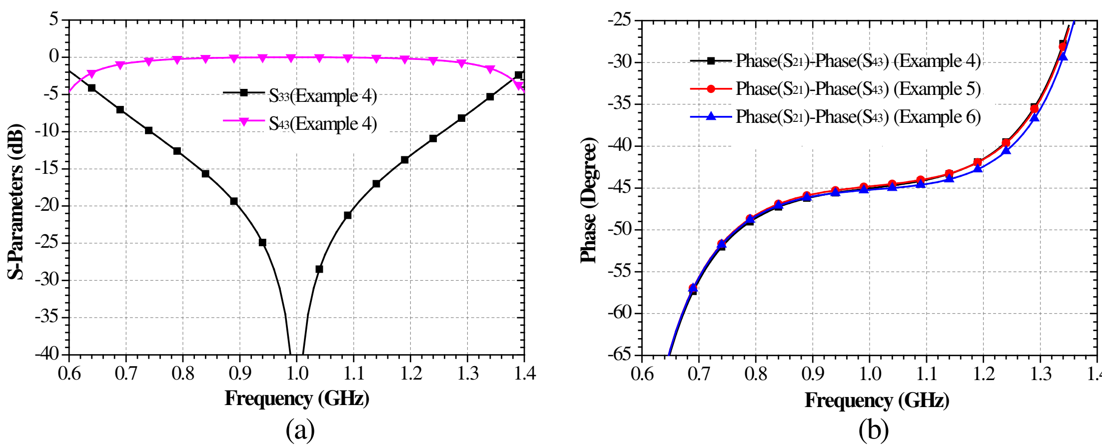

Figure 4. The calculated results of **Examples 4**, **5**, and **6**: (a) Scattering Parameters; (b) Phase information.

4. SIMULATED AND MEASURED RESULTS

The analytical solutions and parameter analyses have been theoretically explained in Sections 2 and 3. In order to verify this proposed wideband phase shifter experimentally, two examples (**Example A** and **Example B**) are fabricated on a radio frequency substrate F4B with a relatively dielectric constant of 2.65 and a thickness of 1 mm. These two examples are designed by using microstrip realization technology with complete ground plane in the bottom layer, and the operating frequency is chosen at 1 GHz. Tables 3 and 4 list the initial electrical parameters and the corresponding physical circuit parameters, respectively. The above-layer circuit trace layout of the proposed circuit with defined dimension parameters is shown in Figure 5. Note that all port impedances are 50 Ω in this paper.

Table 3. The circuit parameters of **Examples A** and **B**.

Examples	$Z_{e1}(\Omega)$	$Z_{o1}(\Omega)$	$\theta_1(^\circ)$	$Z_{e2}(\Omega)$	$Z_{o2}(\Omega)$	$\theta_2(^\circ)$
Example A	60	41.67	50	100	67	90
Example B	72	34.72	40	90	80	90

Table 4. The physical circuit parameters (unit: mm) of **Examples A** and **B**.

Examples	w_0	w_1	w_2	w_3	w_4	w_5
Example A	2.59	2.56	1.98	2.51	2.56	1.72
Example B	2.59	2.59	2.01	2.18	2.01	2.72
Examples	w_6	w_7	l_1	l_2	l_3	l_4
Example A	1.07	1.09	28.39	22.43	9.98	19.98
Example B	0.74	1.09	21.72	9.40	9.98	17.98
Examples	l_5	l_6	l_7	l_8	s_1	s_2
Example A	2.01	3.43	2.01	53.42	0.56	0.74
Example B	1.98	7.42	2.01	53.82	0.23	2.29

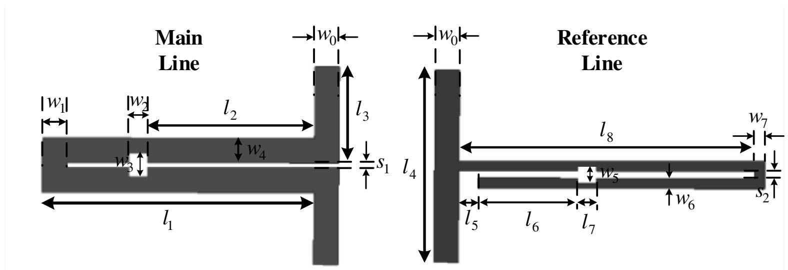

Figure 5. The layout of the proposed phase shifter.

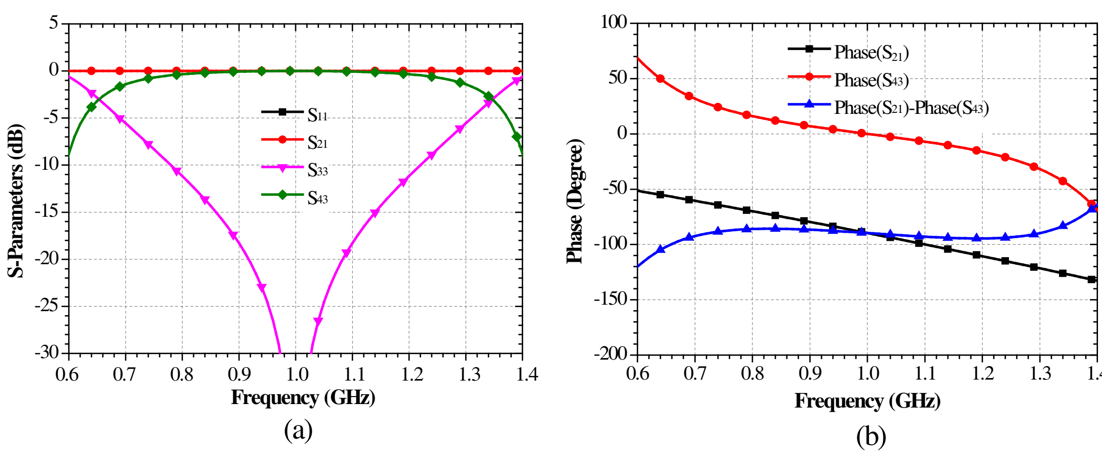

Figure 6. The calculated results of **Example A**: (a) Scattering Parameters; (b) Phase information.

The phase difference of **Example A** (**Example B**) is $-90°$ ($-60°$). The definition of the circuit parameters of **Example B** is the same as that of **Example A** listed in Table 3 and Table 4. Figure 6(a) and Figure 7(a) plot the calculated scattering parameters of the two examples, while Figure 6(b) and Figure 7(b) present the phase information. Figures 6 and 7 show that all ports are matched ideally. The 10-dB return loss fractional bandwidth is almost 44% (from 0.78 GHz to 1.22 GHz) for **Example A** and 50% (from 0.75 GHz to 1.25 GHz) for **Example B**. In addition, the calculated phase difference fractional bandwidth between the main line and the reference line is almost 71% (from 0.66 GHz to 1.35 GHz) for **Example A** and 72% (from 0.65 GHz to 1.37 GHz) for **Example B** based on the $\pm 10°$ phase error. Note that the dB values of S_{11} are all below -30 dB and invisible in Figure 6(a) and Figure 7(a).

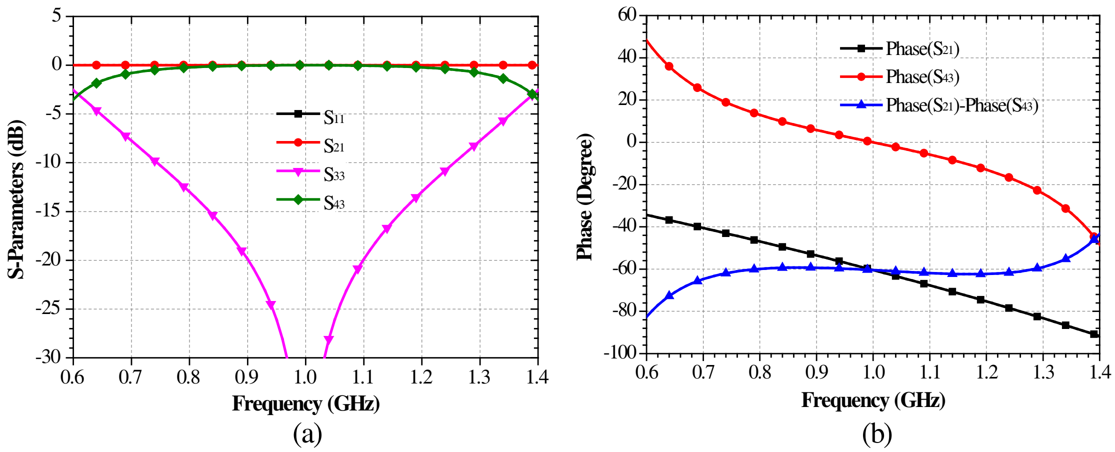

Figure 7. The calculated results of **Example B**: (a) Scattering Parameters; (b) Phase information.

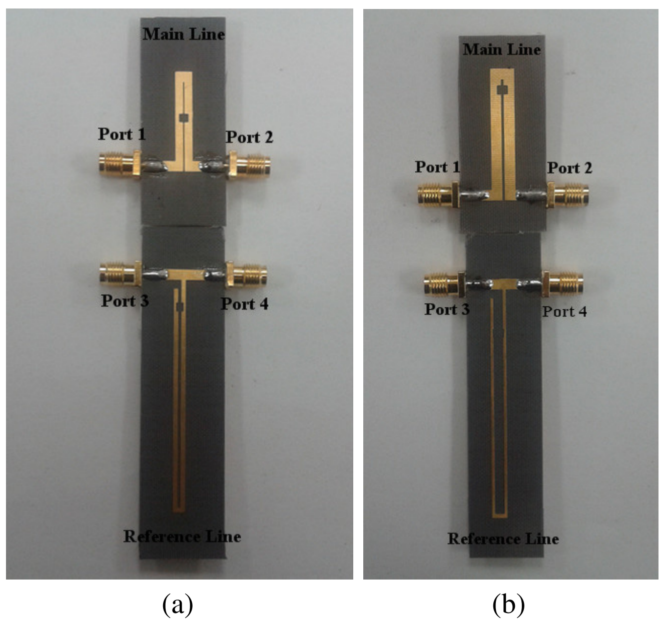

Figure 8. Top view of the fabricated microstrip phase shifter: (a) **Example A**; (b) **Example B**.

Photographs of the final fabricated circuit (**Examples A** and **B**) are shown in Figure 8. The measured results using Agilent N5230C Vector Network Analyzer are plotted in Figures 9 and 10. The 10-dB return loss fractional bandwidth is almost 46% (from 0.81 GHz to 1.27 GHz) for the **Example A** and 54% (from 0.73 GHz to 1.27 GHz) for the **Example B**. However, the calculated phase difference fractional bandwidth between the main line and the reference is almost 72% (from 0.66 GHz to 1.38 GHz) for **Example A** and 74% (from 0.65 GHz to 1.39 GHz) for **Example B** based on the $\pm 10°$ phase error. To see the measured results more clearly, Table 5 lists the bandwidths of S_{33}, S_{43}, and $\Delta\theta$. Small performance degradation can be due to the conductor loss, dielectric loss, or measurement errors and so on. In general, it can be concluded that there is a good agreement between the calculated and measured results including magnitude and phase information.

Based on the investigation on the proposed circuit, a simple design procedure can be summarized as follows.

1) Determine the center frequency f_0 and the phase difference $\Delta\theta$ according to the design requirements. Obtain the values of dielectric constant and thickness of the substrate material.

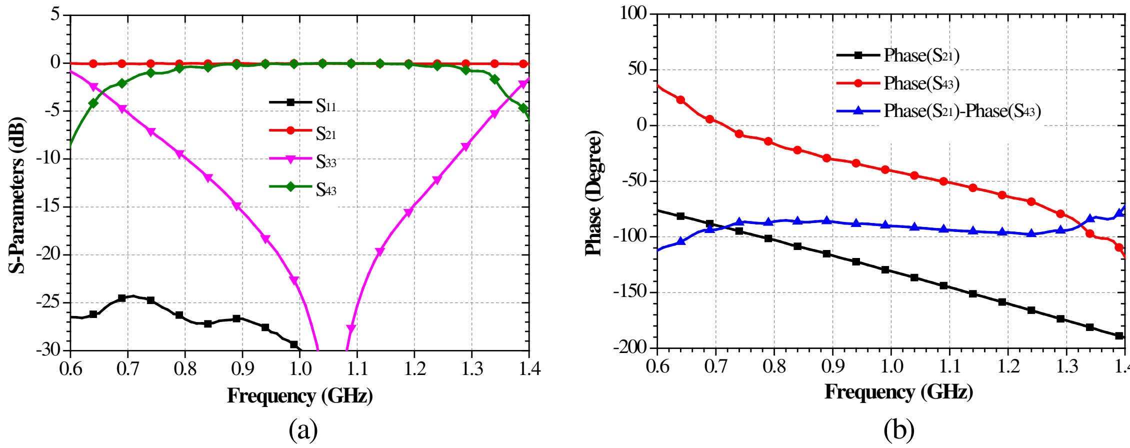

Figure 9. The measured results of **Example A**: (a) Scattering Parameters; (b) Phase information.

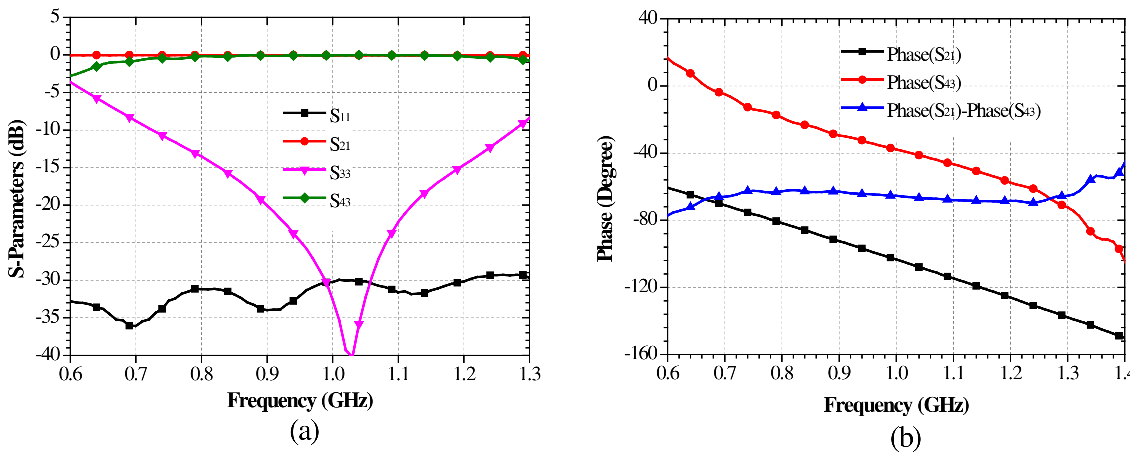

Figure 10. The measured results of **Example B**: (a) Scattering Parameters; (b) Phase information.

Table 5. The measured results of **Examples A** and **B**.

Example	$\Delta\theta$	BandWidth of $S_{33}(< -10\,\text{dB})$	BandWidth of $S_{43}(> -3\,\text{dB})$	BandWidth of $\Delta\theta$ $(\pm 10^\circ)$
Example A	-90°	46%	69%	72%
Example B	-60°	54%	66%	74%

2) Determine the proper g_1 and the electrical length θ_1 of the main line according to the Equations (6) and (12) when the phase difference is fixed. Then calculate the impedance Z_{e1}, Z_{o1}.

3) Choose the appropriate impedance of the reference line.

4) Convert all the electrical parameters to the physical dimensions, and calculate the total scattering parameters and the available bandwidth by using lossless transmission lines.

5) Tune the physical dimensions to obtain the correct phase difference and the available bandwidth. If the proper physical dimensions with the required performance cannot be realized, the design should be restarted from (2) with other potential values.

5. CONCLUSION

In this paper, a modified coupled-line Schiffman phase shifter with short reference line is proposed. The design theories, including analytical equations, scattering parameters and phase difference, are provided. For observation of the bandwidth of the scattering parameters and phase difference, six examples are discussed. For experimental demonstration, two microstrip phase shifters are designed, fabricated, and measured. Good agreements can be observed for the calculated and measured results. Due to easy realization of the proposed circuit structure, it is expected that this phase shifter will be used in various microwave components and systems.

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