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Azimuthally Non-Symmetric Surface Waves Propagating in Metal Waveguides Filled with Isotropic Plasma

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Abstract—The paper is devoted to the theory of eigen electromagnetic waves propagating across the axis of symmetry in waveguides with a non-circular cross-section. The case of waveguides filled with isotropic cold plasma is studied theoretically. Plasma particles motion is described in fluid approximation; expressions for the waves' fields are derived from Maxwell equations. Cross-section of the studied waveguide is modeled by Fourier series with coefficients, which values are less than unity. This allows one to apply method of successive approximations for analytical research of this problem. Boundary conditions, which are formulated in non-linear form over the small parameters of the problem have been applied for derivation the dispersion equations, which determine frequency spectrum of these surface waves for waveguides of different constructions. Studied eigen electromagnetic waves propagate in the form of wave packets, which are approximately described by the main azimuthal harmonic and two nearest satellite spatial harmonics. Represented results have been obtained both analytically and numerically. Possible spheres of the studied eigen waves are discussed.

1. INTRODUCTION

The most effective way to excite waveguide structures (including such of them, which are either corrugated or contain someone slowing-down elements like corrugated walls, disks, etc.) can be realized in the range of their eigen frequencies [1]. This circumstance supports the interest to studying frequency spectra of multi-component plasma filled waveguides supplied with different type slowing-down elements. Multi-component waveguide structures are widely used [2, 3] in modern devices of plasma electronics. Presence of different semiconductor, dielectric and/or plasma insertions in a metal waveguide increases the number of their eigen modes and power of electromagnetic emission that can be obtained from generators built up on the basis of these type waveguides. It also leads to appearance of some specific features in frequency spectra of the waves propagating in such waveguides, in spatial distribution of their fields as compared with the case of uniform waveguides application. In particular, different types of surface waves (SWs) can propagate there [4]. Properties of the SW, which propagate along a planar boundary between two different media, and properties of the SW propagating along a boundary, which has a finite value of curvature radius, differ substantially. For instance, in the case of the planar form of the plasma boundary, most of SWs are slow waves and are either potential or at least can be considered in the potential approach, see, e.g., [5, 6] unlike the case of SW propagating along the plasma boundary with finite value of curvature radius. The frequency ranges wherein they can exist are different in these two cases [7]. Thus studying of SW propagating across axis of symmetry of the plasma filled waveguides with non-circular cross-section satisfies request of novelty and actuality.

One of the main branches for application of the theoretical result presented here is plasma radio-physics and electronics. It is known, for example, that a metal bar with a non-circular cross-section that is disposed into plasma can be utilized for excitation of electromagnetic waves. In this case the bar plays the role of a metal antenna with plasma coating. Utilization of antenna with plasma coating is known [8, 9] for a long time. It has some advantages as compared with utilization of antennas without

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such coating. For instance, such application of plasma allows one to increase the electromagnetic power radiation, to control the frequency spectrum of this radiation by the way of corrugating the surface of the metal antenna. Experiments performed by the authors of [10] have confirmed that propagation of the SW, which sustains a plasma column, can replace a metal bar as the guiding element in RF antennas. Moreover, such a plasma-antenna system is characterized by low radar detectability and negligible mutual coupling in the switched-off state. Their experiments confirm that this plasma-antenna system has high efficiency and that its noise level is approximately the same as for a corresponding metal antenna. These results are obtained for plasma-antennas, which operate in pulsed regimes and can transmit and receive signals in noncurrent-carrying plasmas. Thus, the investigation of the SWs' properties is very prospective for plasma electronics.

Using an expansion into Fourier series one can simulate any real shape of the waveguide's cross-section. If coefficients in this series are small values then one can apply the method of successive approximations for studying the problem on the SW propagation in such corrugated waveguides. The theory of azimuthal surface waves (ASW) can be applied as zero approximation while carrying out this investigation. Spatial periodical non-uniformity of the considered waveguides is the reason that waves propagate in the form of wave packets there. Existence of the small parameter of the problem allows one: first, to assume that considered wave packets can be represented in the form of superposition of the main harmonic and two nearest satellite harmonics; second, to formulate non-linear (over the small parameter of the considered problem) boundary conditions for the considered SW fields in the form of Taylor series over this parameter. So the carried out investigation allows one: to calculate the correction to ASW frequency, to analyze spatial distribution of the transverse SWs fields in the cases of waveguides of different construction.

The paper is organized as follows. Section 2 is devoted to the formulation of the problem, description of geometry of the studied waveguides and boundary conditions applied to derivation of the dispersion relations. Influence of the waveguide's wall corrugation on the SW properties is under the consideration in Section 3. Problems of the resonant influence of the waveguide's wall corrugation on the SW dispersion properties are studied in Section 4. Section 5 is devoted to the investigation of the SWs propagating along azimuthally corrugated plasma column immersed into cylindrical metal waveguide of circular cross-section. Section 6 summaries the obtained results.

2. FORMULATION OF THE PROBLEM

In cylindrical metal waveguides with non-circular cross-section, which is partially filled by cold isotropic plasma SWs with zero axial wave number, propagate both along the azimuthal and radial directions. Problem of these transverse surface waves (TSWs) propagation in corrugated waveguides is solved here by the method of successive approximations, applying the theory of azimuthal surface waves (ASWs) as a zero approximation. Let's start our consideration from description of general geometrical features of the waveguides, where the TSWs can propagate, and formulation of the boundary conditions, which are applied to solving the problem.

In general case, radius of the waveguide's metal chamber with arbitrary smooth shape of its cross-section can be determined by the following Fourier series:

$$R_2 = b \cdot \left[1 + \sum_{n=1}^{\infty} h_n \sin\left(n\varphi - \varphi_{2n}\right) \right], \tag{1}$$

here b is a mean value of radius of the considered waveguide's metal chamber and $h_n \ll 1$ a relative depth of azimuthal corrugation of the chamber wall. Expression (1) is a sample of Fourier series that can describe any shape of waveguide's cross-section. Such a corrugated waveguide, generally speaking, can be filled by cylindrical plasma column with radius R_1, whose value is also described by Fourier series as expression (1):

$$R_1 = a \cdot \left[1 + \sum_{n=1}^{\infty} \delta_n \cos(n\varphi + \varphi_{1n}) \right], \tag{2}$$

here a is a mean value of the radius of the plasma column; $\varphi_{1n,2n}$ are the phases of corrugation of the inner surface of the metal wall and plasma interface, correspondingly; $\delta_n \ll 1$ characterizes maximum dimensionless deviation of the plasma columns' surface from its mean value a. Term smooth is applied in expressions (1) and (2) in the sense of limiting values of both the goffers (or in other words corrugation parameters δ_n and h_n) as compared with averaged dimensionless thickness of the dielectric gap, which separates plasma column and waveguide's metal wall, so that the inequality $|\delta_n| + |h_n| < (b/a - 1) < 1$ is satisfied.

Let's assume that plasma, which is located in this waveguide, has a uniform density profile, and it is also non-magnetized and cold. It is assumed that plasma column is separated by thin ($\Delta = R_2 R_1^{-1} - 1 < 1$) dielectric layer with permittivity constant ε_d from the wall of the waveguide's chamber. Permittivity of the plasma is as follows: $\varepsilon = \varepsilon_0 - \sum_\alpha \Omega_\alpha^2 \omega^{-2}$, where ε_0 is the dielectric constant of a meta-material or crystal lattice of semiconductor (in the case of semiconductor plasma), $\varepsilon_0 = 1$ in the case of gaseous plasma, Ω_α the plasma frequency, and subscript α specifies the type of particles: ions, electrons, holes, etc. [4]. In general case, scheme of the studied plasma waveguide is presented in Fig. 1 where one can see both corrugated interfaces of plasma column and inner wall of the waveguide, as well as possible axial slot in the metal screen of the waveguide, which could be utilized for emission of the TSW power from the waveguide.

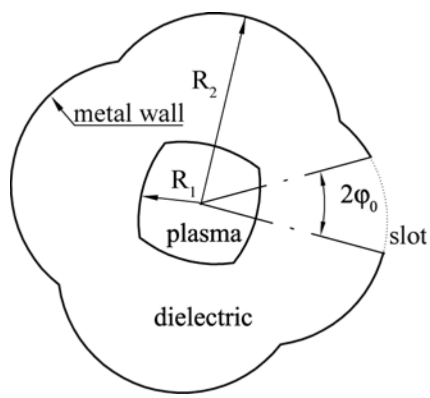

Figure 1. Scheme of the studied corrugated waveguide with possible narrow axial slot.

To derive the dispersion relation for TSWs propagating in such waveguide, one can apply the followings non-linear (over the small parameters δ_n and h_n, which determine the waveguide corrugation) boundary conditions:

$$E_{r,\varphi}(r = 0) < \infty, \quad H_z(r = 0) < \infty, \tag{3}$$

(these conditions mean that the TSW fields are of finite values in the volume of plasma column);

$$\begin{aligned} &E_\tau(R_1 + 0) = E_\tau(R_1 - 0), \quad E_\tau(R_1) \propto R_1(\varphi) E_\varphi(R_1) + E_r(R_1) \frac{dR_1}{d\varphi}, \\ &H_z(R_1 + 0) = H_z(R_1 - 0), \end{aligned} \tag{4}$$

(these conditions mean continuity of the waves' tangential electric and magnetic fields on the plasma-dielectric interface);

$$E_\tau(R_2) \propto R_2(\varphi) E_\varphi(R_2) + E_r(R_2) \frac{dR_2}{d\varphi} = 0, \tag{5}$$

(this condition means equality to the zero of the tangential electric field on the inner metal wall of the waveguide chamber).

3. INFLUENCE OF WAVEGUIDE'S WALL CROSS-SECTION NON-CIRCULARITY ON THE TSW PROPAGATION

In this section a corrugated waveguide, which is partially filled by cylindrical plasma column with radius a is under consideration. Surface of the waveguide's metal wall is assumed to be described by expression

(1), and plasma column with circular cross-section is located co-axially in the metal waveguide and separated from it by a dielectric layer, whose permittivity is ε_d. The frequency range, within which ASW can propagate in a metal waveguide with circular cross-section filled with uniform cold plasma, is determined by the following inequality: $\varepsilon_0\omega^2 < \Omega_e^2 + \Omega_i^2$ [11]. Solving the Maxwell set of equations allows one to obtain the expressions, which describe the spatial distribution of the waves' fields in the plasma column region and dielectric region, correspondingly. TSW magnetic field in plasma region is described by modified Bessel equation, thus it is determined by superposition of modified Bessel functions $I_n(z)$ and MacDonald functions $K_n(z)$ [12]:

$$H_z = \sum_m H_z^{(m)}(r)\exp(im\varphi - i\omega t), \quad H_z^{(m)} = C_1^{(m)} I_m(r\kappa_p) + C_2^{(m)} K_m(r\kappa_p), \tag{6}$$

here $\kappa_p = k\sqrt{|\varepsilon|}$, $kc = \omega$. Electric components of the waves can be derived using expressions (6) in the following way:

$$E_r = \frac{-ik}{\kappa_p^2 r}\frac{\partial}{\partial\varphi}H_z, \quad E_\varphi = \frac{-ik}{\kappa_p^2}\frac{\partial}{\partial r}H_z. \tag{7}$$

In the dielectric region ($a \le r \le R_2$), the TSW magnetic field is described by Neumann functions $N_n(z)$ and Bessel functions of the first kind $J_n(z)$ [12]:

$$H_z^{(m)} = C_3^{(m)} J_m(r\kappa_d) + C_4^{(m)} N_m(r\kappa_d), \tag{8}$$

here $\kappa_d = k\sqrt{\varepsilon_d}$, $C_{1,2,3,4}^{(m)}$ in expressions (6) and (8) are integration constants, electric fields are expressed by the aid of expression (8) in the form, which is similar to expressions (7), where one can change κ_p by k. Application of expressions to tangential components of TSW fields allows one to calculate impedances of plasma and dielectric on the plasma-dielectric interface $r = R_1$. Then by the aid of boundary conditions (3)–(5) in zero approximation over corrugation parameters, i.e., assuming $h_n = \delta_n = 0$, one can derive the following dispersion equation for TSW propagating in the plasma filled waveguide with circular cross-sections of the both screen and plasma column:

$$\frac{J_m'(R_1\kappa_d)\,N_m'(R_2\kappa_d) - J_m'(R_2\kappa_d)\,N_m'(R_1\kappa_d)}{J_m'(R_2\kappa_d)\,N_m(R_1\kappa_d) - J_m(R_1\kappa_d)\,N_m'(R_2\kappa_d)} = \sqrt{\frac{\varepsilon_0}{|\varepsilon_p|}}\frac{I_m'(R_1\kappa_p)}{I_m(R_1\kappa_p)}. \tag{9}$$

In the limiting case of a narrow dielectric layer $\Delta < 1$ one can find following analytical solutions of the dispersion Equation (9):

$$\omega \approx \begin{cases} \Omega_e m\sqrt{\dfrac{\Delta}{\zeta\sqrt{\varepsilon_0}}}, & \zeta \gg m, \quad \Delta\zeta < 1, \\ \Omega_e\left(1 + \zeta^2\dfrac{\varepsilon_0}{m^2} + \dfrac{\zeta\sqrt{\varepsilon_0}}{\Delta m^2}\right)^{-1/2}, & m \gg 1, \end{cases} \tag{10}$$

here $\zeta = R_1\Omega_e/c$ is dimensionless radius of the plasma column. It should be underlined that the upper limiting value of the TSW frequency is Langmuir frequency Ω_e, unlike the commonly known [13] cut-off frequency $\Omega_e/\sqrt{2}$ obtained for surface waves propagating along flat plasma boundary.

Let's consider now the case of the waveguide with the cross-section, whose shape is described by expression (1), where instead sum over n we shall apply only one term with $n = N$. After that the obtained result can be generalized for the case of arbitrary cross-section (1). This choice of the cross-section shape has a self-contained sense as well. For instance, the case $N = 1$ describes the case of violating the coaxiality between plasma column and waveguide's metal chamber; the case $N = 2$ describes the ellipsoid shape of the chamber; cases with $N \ge 3$ describe slowing-down waveguide structures with the angular period $2\pi/N$. Application of the boundary conditions (3)–(5) allows one to derive the dispersion relation for TSW in a metal waveguide with arbitrary non-circular cross-section.

In the zero approximation ($h_N = 0$) the TSWs with different azimuthal mode numbers m propagate independently from each other. Magnetic field of the waves in dielectric region can be found in this approximation in the following form:

$$H_z^{(m)} = 0,5\pi\kappa_d a C_1 L_m(\kappa_d r), \tag{11}$$

here

$$L_m(\kappa_d\, r) = G_1 J_m(\kappa_d\, r) - G_2 N_m(\kappa_d\, r)\,, \tag{12}$$

$$G_1 = \sqrt{\varepsilon_d/|\varepsilon|} I'_m(\kappa_p a)\, N_m(\kappa_d a) + I_m(\kappa_p a)\; N'_m(\kappa_d a)\,, \tag{13}$$

$$G_2 = \sqrt{\varepsilon_d/|\varepsilon|} I'_m(\kappa_p a)\, J_m(\kappa_d a) + I_m(\kappa_p a)\; J'_m(\kappa_d a)\,. \tag{14}$$

Here and in the following expressions, superscribe "prime" denotes the derivative of the corresponding cylindrical function with respect to its argument.

In the first approximation over the small parameter of a waveguide corrugation h_N, the TSW magnetic field in the dielectric region can be written as the following sum:

$$\begin{aligned} H_z \;=\; & 0,5\pi\kappa_d a\, C_1 \left\{ L_m(\kappa_d r)\exp(im\varphi) + H_+ h_N L_{m+N}(\kappa_d r)\exp\left[i(m+N)\varphi\right] \right. \\ & \left. + H_- h_N L_{m-N}(\kappa_d r)\exp\left[i(m-N)\varphi\right] \right\} \exp(-i\omega t)\,. \end{aligned} \tag{15}$$

To satisfy the boundary condition (4), the expression for the TSW magnetic field in the form (15) should contain the factors $H_\pm$ multiplied by the h_N, which are determined by the following expressions:

$$H_\pm = \pm \frac{iL_m(\kappa_d a)\;\left(m^2 \pm mN - \kappa_d^2 a^2\right)}{2\kappa_d\, a\, L'_{m\pm N}(\kappa_d a)}. \tag{16}$$

The results of numerical analysis of the radial distribution of the TSW fields in the corrugated waveguide are represented in Fig. 2 for the following values of the waveguide parameters: $h_N = 0.1$, $m = 2$, $\varepsilon_d = 1$. The TSW fields' amplitudes are calculated in relative units, and radial co-ordinate is normalized by the plasma column radius. The spatial distribution of the TSW magnetic field along azimuthal angle is presented in Fig. 3. There one can see that parameter of the corrugation was assumed to be equal to $N = 3$, and influence of the corrugation on TSW with wavenumbers $m = 2$ and $m = 4$ was studied numerically. Unfortunately increasing azimuthal mode number leads to strong decreasing of the field amplitude, so amplitude of TSW with $m = 4$ is shown as multiplied on 50 as compared with the case $m = 2$. Thus one can clearly see three perturbations connected with the plasma column corrugation on the background of the sinusoid corresponding to the case $m = 2$. These disturbances are located nearby azimuthal angle $\varphi = 2\pi j/3$, here integer number $j = 1, 2, 3$. Thus in the case $m = 2$, one can see a broadening of the region of maximum value of the angular distribution of the STW magnetic field $H_z(\varphi)$ nearby $\varphi \approx 2$, narrow disturbance of the distribution nearby $\varphi \approx 4$ and constriction of the region of minimum value of the $H_z(\varphi)$ nearby $\varphi \approx 6$, as it is predicted by presented analytical studying. And in the case $m = 4$ one can see influence of the corrugation as slight modulation of the quasi-regular sinusoid.

Solving the dispersion relation in the second approximation over the h_N one can find the TSW frequency as a sum of ASW eigen frequency ω_0 (it is calculated in [11] for the case of $h_N = 0$) and frequency correction $\Delta\omega_N$ caused by the non-circularity of the waveguide cross-section. The expression for the $\Delta\omega_N$ has the following relatively complex form:

$$\begin{aligned} \Delta\omega_N \;=\; & \frac{h_N^2\left(m^2 + a^2\kappa_d^2\right)}{4\kappa_d a}\; \frac{L_m(\kappa_d a)}{\dfrac{dL'_m(\kappa_d a)}{d\omega_0}} \left[1 + \frac{L_{m+N}(\kappa_d a)}{L'_{m+N}(\kappa_d a)}\; \frac{\left(m^2 + mN - \kappa_d^2 a^2\right)^2}{m^2 + \kappa_d^2 a^2} \right. \\ & \left. + \frac{L_{m-N}(\kappa_d a)}{L'_{m-N}(\kappa_d a)}\; \frac{\left(m^2 - mN - \kappa_d^2 a^2\right)^2}{m^2 + \kappa_d^2 a^2} \right]. \end{aligned} \tag{17}$$

Analysis of expression (17) allows one to make the conclusion that frequency correction does not depend on the sign of azimuthal wave number m, which is generally typical for SW propagating in isotropic plasma waveguides [4]. Using the asymptotic expressions and recurrent correlations for Bessel functions [12], it is possible to find the approximate formulas for the frequency corrections in the limiting cases of wide $\kappa_{d,p} a \gg |m|$ and narrow $\kappa_{d,p} a \ll 1$ waveguides.

In the case of narrow waveguides: $\Delta\omega_N \propto -h_N^2 \Delta/(\kappa_d a)^{2+N}$. The analysis of expression (17) testifies that in the case of wide waveguides, taking into account the value of derivative $(dL'_m(\kappa_d a)/d\omega_0)$, there can be an effect of changing the sign of the frequency correction $\Delta\omega_N$. It takes place if the following inequality is valid:

$$b\Omega_e c^{-1}\Delta > 1 > kb\varepsilon_d(|\varepsilon|)^{-1/2}\Delta. \tag{18}$$

If inequality (18) is not valid (in the case of waveguides with large radii: $1 < k\varepsilon_d\, b\Delta/\sqrt{|\varepsilon|}$, $ka \gg 1$), then one can write the following expression for $\Delta\omega_N$:

$$\Delta\omega_N \approx -0.25\omega_0 h_N^2 \varepsilon_d (1+\Delta)\kappa_d^2 a^2. \tag{19}$$

It should be emphasized the substantial difference between the cases of TSWs propagation in wide and narrow waveguides. In narrow waveguides, absolute value of the correction $\Delta\omega_N$ diminishes with increasing ε_d and angular period of the corrugation $2\pi/N$. In the case of wide waveguides, one can observe that the rate of the TSWs deceleration does not depend on the parameters m and N, but their frequency correction value is strongly depended on the value of dielectric constant: $|\Delta\omega_N| \propto \omega_0\varepsilon_d^2$.

One can see that in this approximation value of the frequency correction $|\Delta\omega_N|$ does not depend on the presence of other small terms $\propto \sin(n\varphi)$, here $n \neq N$ in the expression for the waveguide's surface (1). Therefore, in the case of arbitrary shape of the waveguide chamber cross-section, the TSW eigen frequency can be found within accuracy up to small terms $\propto h_n^2$ as the following sum:

$$\omega = \omega_0 + \sum_{n \neq 2m} \Delta\omega_n. \tag{20}$$

But it should be underlined here that influence of the terms $\propto bh_{2m}\sin(2m\varphi - \varphi_{2m})$ in expression (1) on the TSW frequency is not described by the formula (17). This case needs additional research (see the next section).

The results of numerical research of the $\Delta\omega_n/\Omega_e$ magnitude versus the effective dimensionless wave number $k_{ef} = m\delta/a$ are presented in Fig. 4 and Fig. 5 in the cases $N = 1$ and $N = 4$, correspondingly ($\delta = c/\Omega_e$ is the plasma skin-depth). Numerals in these figures indicate the meanings of azimuthal mode numbers m, and the parameters of corrugation are chosen as follows: $h_1 = 0.7$ and $h_4 = 0.25$. Numerical analysis proves that the main influence on the $\Delta\omega_n/\Omega_e$ magnitude is affected by the thickness of dielectric layer, parameter Δ. Increasing dielectric constant ε_d value from 1 to 5, ceteris paribus, leads to approximately twice diminishing both ω_0 and $\Delta\omega_n$. Let's underline as well that the point, where $\Delta\omega_n = 0$, moves toward the greater values of effective wave-number k_{ef} under these conditions. Diminishing the value of Δ from 0.3 to 0.1, ceteris paribus, leads to insufficiently decreasing the TSW

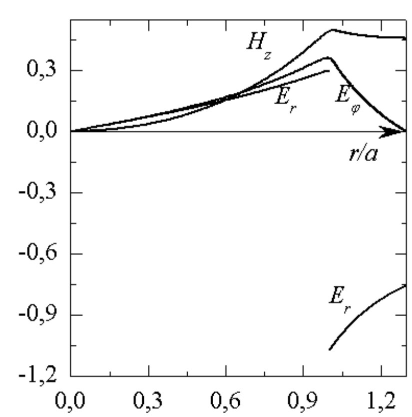

Figure 2. Radial distribution of TSW with $m = 2$ in corrugated metal waveguide, where $a = 2\delta$, $b = 1.3 \cdot a$, $\varepsilon_d = 1$. Normal electric field has discontinuity on the plasma interface.

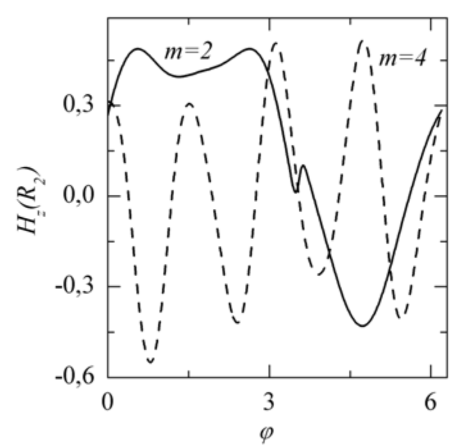

Figure 3. Angular distribution of the TSW magnetic field in the case of $N = 3$. Solid and dashed lines relate to the TSW with azimuthal mode numbers $m = 2$ and $m = 4$, correspondingly.

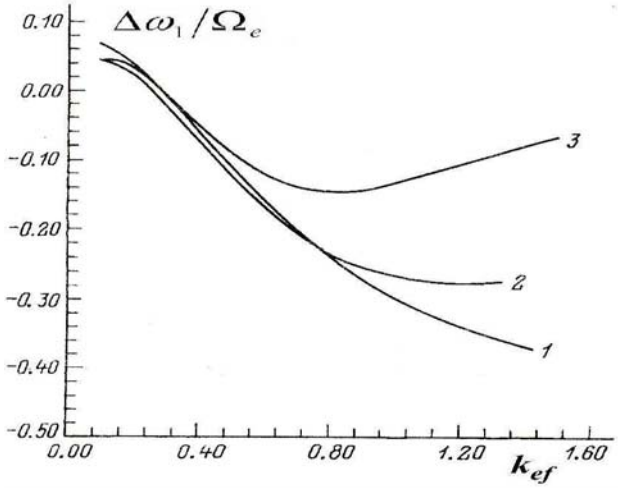

Figure 4. Correction to the TSW eigen frequency versus effective wave number, $N = 1$. Azimuthal mode numbers is indicated by numerals 1, 2 and 3.

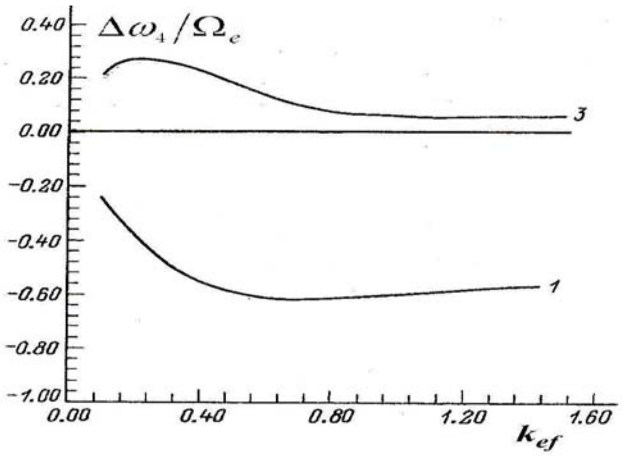

Figure 5. Correction to the TSW eigen frequency versus effective wave number, $N = 4$. Azimuthal mode numbers is indicated by numerals 1 and 3.

eigen frequency, and at the same time it increases $\Delta\omega_n$ approximately sevenfold for TSW with $m = 1$, and twice for TSW with $m = 2$.

4. RESONANT INFLUENCE OF THE DEVIATION OF A METAL CHAMBER CROSS-SECTION SHAPE FROM CIRCULAR ONE ON THE TSW PROPAGATION

Let's study here that special case, which is not described by the formula (17). It is known [11] that ASW frequency in waveguides with circular cross-section filled by isotropic plasma does not depend on sign of azimuthal mode number m, and consequently their frequency spectrum in such waveguides is degenerated in the respect of the m sign. Therefore, the case when angular period of variation of the dielectric layers' thickness is twice less than the azimuthal period of the TSW can be studied separately by special method of the general theory of perturbations [14].

As far as ASW with different azimuthal mode numbers m propagate independently in waveguides with circular cross-section, then let's consider zero approximation that there are only two waves with azimuthal wave numbers $\pm M$. It should be emphasized that both the harmonics propagate at identical frequencies $\omega_M^{(0)}$, which are the solutions of the ASW dispersion relation that describes the case of isotropic waveguide [11]. For this approximation, the ASW dispersion curves are presented in Fig. 6 and Fig. 7 by solid lines.

In the waveguides with non-circular cross-section, frequencies of TSWs with the angular period $2\pi/M$ split if the equation of metal chamber surface (1) contains the small term with $n = 2M$. In the case of degenerated spectrum, according to the special case of general theory of perturbations [14], one can find the solutions to the Maxwell equations for the magnetic field of the TSWs in the region of dielectric layer in the form of standing waves, taking into account the terms of the first order of smallness:

$$\begin{aligned} H_z(r,\varphi,t) \;=\; & \kappa_d a \exp(-i\omega t)\{C_{11} L_M(\kappa_d r)[\exp(iM\varphi) \pm \exp(-iM\varphi)] \\ & + C_r L_{3M}(\kappa_d r)[\exp(i3M\varphi) \pm \exp(-i3M\varphi)]\}\pi/2. \end{aligned} \tag{21}$$

Here $L_n(k_d r)$ is determined by expressions (12); constant C_{11} determines the amplitude value of the TSW main harmonic; small coefficient C_r can be expressed as follows [15]:

$$C_r = \frac{-ih_{2M} L_M(\kappa_d a)}{2\kappa_d a L'_{3M}(\kappa_d a)} \left(\kappa_d^2 a^2 - 3M^2\right) C_{11}. \tag{22}$$

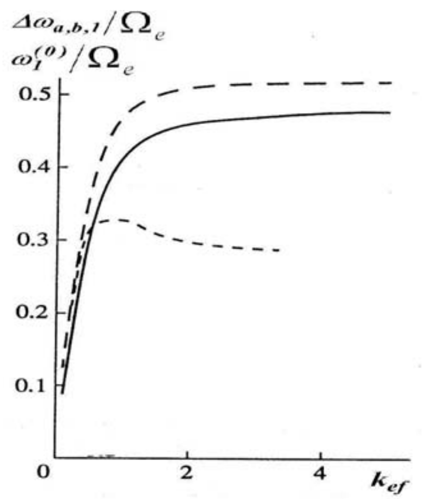

Figure 6. TSW eigen frequency with $M = 1$ (solid line) and values of its splitting caused by the deviation from the circular shape of the cross-section of a metal wall (short dashed line) and that of a plasma column (long dashed line).

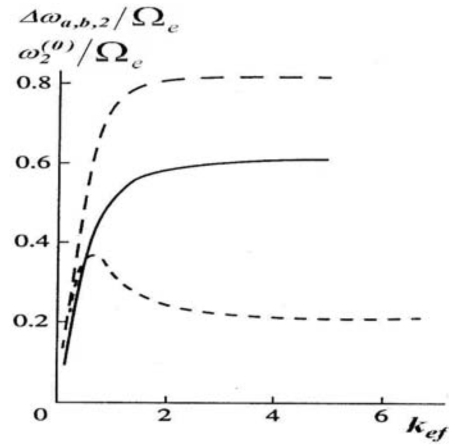

Figure 7. The same as in the Fig. 4, but for TSW with azimuthal mode number of the main harmonic $M = 2$.

Expression (22) is obtained as the result of application of the boundary condition (5), i.e., by equating the amplitudes of satellite harmonics $\propto \exp(\pm i3M\varphi)$ of the tangential component of the TSW electric field to zero on the metal surface.

Boundary condition that means equality of amplitudes of main harmonics of the TSW tangential electric field to zero on the metal surface of the waveguide allows one to derive the dispersion relation. It has a form of secular equation: $(D_M^{(0)})^2 - (D_M^{(1)})^2 = 0$, where $D_M^{(0)}(\omega_M^{(0)}) = 0$ is dispersion relation of the ASW (zero approximation) and $D_M^{(1)}$ the correction of the first order of smallness (see for details [15]). One can find the solution of this secular equation in the form $\omega = \omega_M^{(0)} \pm \Delta\omega_{b,M}$, where the frequency correction is equal to:

$$\Delta\omega_{b,M} = D_M^{(1)} / \frac{\partial D_M^{(0)}}{\partial \omega}\Big|_{\omega=\omega_M^{(0)}} = \frac{h_{2M} L_M(\kappa_d a)}{2\kappa_d a}(M^2 + \kappa_d^2 a^2)\left(\frac{\partial L'_M(\kappa_d a)}{\partial \omega}\right)^{-1}_{\omega=\omega_M^{(0)}}. \tag{23}$$

Expression (23) can be simplified in the case of wide plasma cylinder ($\kappa_p a \gg 1$):

$$\Delta\omega_{b,M} = -h_{2M} M \left(1 + \Lambda a \sqrt{\varepsilon_d}\,\delta^{-1}\right) \sqrt{\delta/(a\Lambda)} \cdot \omega_M^{(0)} / (4\sqrt[4]{\varepsilon_d}). \tag{24}$$

It should be indicated that TSW frequency correction in this case is proportional to the first power of the small parameter ($\Delta\omega_{b,M} \propto h_{2M}$), while presence in equation for the metal surface (1) of small terms with $n \neq 2M$ leads to weaker ($\propto h_n^2$) changes in the value of TSW frequency. In this case, curves, which illustrate the dependence of TSW frequency corrections on effective wave number k_{ef}, are represented in Fig. 6 and Fig. 7 by short dashed lines. Value of the corrugation parameter is chosen as follows: $h_{2M} = 1$ in order to make plots more demonstrable ones for readers.

Concerning practical application of the TSWs considered in Sections 3 and 4, one can indicate the following: they can be utilized as operating modes for antenna system. Antenna systems based on propagation of surface waves are actively developed now due to a lot of their advances as compared with conventional metal antennas [16]. Let's make brief analysis of possibility to radiate their power from corrugated metal waveguide into outer space. To radiate TSWs power one can apply narrow slot in the metal wall of the waveguide, and its angular sizes $\varphi_0/\pi \ll 1$ (see Fig. 1). If the inequality $h_n \ll \varphi_0$ is valid then one can neglect the influence of corrugation of the waveguide's wall on the SW frequency. In this case, application of the indicated above boundary conditions (in approach $h_n = 0$) can be added by the condition that there is no wave propagating from outer space into the waveguide in the angular range $-\varphi_0 < \varphi < +\varphi_0$. With the accuracy up to terms of first order smallness over the parameter φ_0 one can derive the equation, which describes radiation of the TSW power through the narrow axial slot in the waveguide's wall. It has the following form:

$$\frac{I'_m(\kappa_p a)}{I_m(\kappa_p a)\sqrt{|\varepsilon|}} + \frac{k^2a^2 - m^2}{ka}\Delta + D_1 = 0, \tag{25}$$

here the value of D_1 is a small quantity of first order smallness over the parameter φ_0 and describes influence of a narrow slot in the waveguide's wall on the SW propagation. Its expression is as follows:

$$D_1 = \frac{\varphi_0[iN'_m(kR_2) + J'_m(kR_2)]N_m(ka)/\pi}{J_m(ka)\,N'_m(kR_2) - J'_m(kR_2)\,N_m(ka)}\left[\frac{kI'_m(\kappa_p a)}{\kappa_p I_m(\kappa_p a)} + \frac{N'_m(ka)}{N_m(ka)}\right]. \tag{26}$$

Then in the limiting case of a narrow vacuum layer $\Delta \ll 1$, for TSWs with large values of azimuthal mode numbers propagating in wide plasma waveguides $1 \ll |m| \ll \kappa_p a$, it is easy to derive an expression of their damping rate γ_{rad} caused by radiation of their power from the waveguide into the outer space:

$$\gamma_{rad} \approx \Omega_e\varphi_0\,(m!)^2\left(4\delta/\left(m^2a\Delta\right)\right)^{m+0.5}/(4\pi^2). \tag{27}$$

Numerical analysis of Equation (26) proves that damping rate of the TSWs connected with radiation of their power through narrow axial slot increases with increasing plasma density, size of the slot and k_{ef}. It should be underlined that one cannot decrease thickness of the dielectric layer Δ sufficiently, because it leads to decreasing both values of the TSWs frequency and their field in such waveguides. Thus applying plasma with density $n_{pl} \geq 10^{14}\,\text{cm}^{-3}$ allows one to obtain radiation in millimeter range, because wavelength of the radiation is approximately determined by the following relation: $\lambda \approx 6\cdot 10^6/\sqrt{n_{pl}}$ (cm). Antenna system based on emission of these SWs will be controlled smoothly by changing plasma density (it will allow one to change frequency of the radiation) and by changing value of the slot φ_0 (it will allow one to change power of the radiation). A similar task, but for the case of magnetoactive plasma column located in a metal waveguide, has been considered in monograph [17]. Dependence of the damping rate caused by radiation of the eigen modes power upon the plasma density, radius of the plasma column and width of the slot is the same in both cases, namely magnetized and non-magnetized plasmas. So it confirms correctness of the obtained results.

5. INFLUENCE OF A PLASMA COLUMN CROSS-SECTION'S NON-CIRCULARITY ON THE TSW PROPAGATION

Now let's investigate the propagation of TSW along isotropic plasma column with noncircular cross-section, whose shape is determined by expression (2) and which is located in cylindrical metal waveguide with circular cross-section of radius b. Splitting of the frequency of the TSW propagating with an angular period $(2\pi/M)$ takes place in this case as it was in the previous case under the condition of presence of the small term with $n = 2M$ in expression (2) for plasma column radius $R_1(\varphi)$.

In this case, one can find the solution of Maxwell equations in the form of superposition of the main and satellite harmonics, which are proportional to $\exp(\pm iM\varphi)$ and $\exp(\pm i3M\varphi)$, correspondingly. It is convenient to write the TSW axial magnetic field for the plasma region ($r \leq R_1(\varphi)$) in the following form (taking into account the terms of the first order of smallness [15]):

$$\begin{aligned} H_z^{(p)}(r,\varphi,t) = \exp(-i\omega\,t)\Big\{&I_M(\kappa_p r)\left[A_0^{(+)}\exp(iM\varphi) + A_0^{(-)}\exp(-iM\varphi)\right] \\ &+I_{3M}(\kappa_p r)\left[A_1^{(+)}\exp(i3M\varphi) + A_1^{(-)}\exp(-i3M\varphi)\right]\Big\}, \end{aligned} \tag{28}$$

here $A_{0,1}^{(\pm)}$ are the constants of integration.

In the first approximation, spatial distribution of the TSW axial magnetic field in the dielectric layer, which satisfies the boundary condition (5) on the circular metal surface ($r = b$), can be represented in the following form:

$$\begin{aligned} H_z^{(d)} \;=\; \exp(-i\omega\,t)\Big\{&L_M(\kappa_d r)\left[F_0^{(+)}\exp(iM\varphi) + F_0^{(-)}\exp(-iM\varphi)\right] \\ &+L_{3M}(\kappa_d r)\left[F_1^{(+)}\exp(i3M\varphi) + F_1^{(-)}\exp(-i3M\varphi)\right]\Big\}, \end{aligned} \tag{29}$$

here $F_{0,1}^{(\pm)}$ are constants of integration, and functions $L_M(\kappa_d r)$ are determined by expression (12), in which one has to apply the following notations to the present case instead of expressions (13) and (14):

$$G_1 = N'_M(\kappa_d b), \quad G_2 = J'_M(\kappa_d b). \tag{30}$$

The boundary conditions (4), which mean the continuity of amplitudes of satellite harmonics of the TSW tangential fields H_z and E_τ on the plasma-dielectric interface $R_1(\varphi)$, allow one to determine the constants of integration $A_1^{(\pm)}$ and $F_1^{(\pm)}$, which determine the amplitudes of the indicated above satellite harmonics in the different regions of the waveguide:

$$A_1^{(\pm)} = \frac{-\delta_{2M} A_0^{(\pm)}}{2D_{3M}^{(0)}} \frac{\kappa_p^2 + \kappa_d^2}{\kappa_p \kappa_d} [I'_M(\kappa_p a) L'_{3M}(\kappa_d a) a^2 + 3M^2 I_M(\kappa_p a) L_{3M}(\kappa_d a)], \tag{31}$$

$$F_1^{(\pm)} = \frac{\delta_{2M} A_0^{(\pm)}}{2D_{3M}^{(0)}} \frac{\kappa_p^2 + \kappa_d^2}{\kappa_p \kappa_d} \left[I'_M(\kappa_p a) L'_{3M}(\kappa_d a) a^2 - 3M^2 I_M(\kappa_p a) L_{3M}(\kappa_d a)\right]. \tag{32}$$

These boundary conditions applied to the main harmonics of the TSW allow one to determine the relation between amplitudes of the main harmonics in the plasma and the dielectric regions:

$$F_0^{(\pm)} = A_0^{(\pm)} I_m(\kappa_p a) / L_M(\kappa_g a). \tag{33}$$

Then in its turn, application of relation (33) makes it possible to derive the secular dispersion relation for TSW, which takes into account spatial non-uniformity (or in other words, corrugation) of plasma-dielectric interface (see expression (2)). From mathematical point of view this equation is similar to the one considered in the previous section. Thus the correction $\Delta\omega_{a,M}$ to TSW frequency in the present case can be calculated in the same way as it has been done in the previous case of corrugated metal wall of the waveguide. Explicit expression for the $\Delta\omega_{a,M}$ can be written in the following form:

$$\Delta\omega_{a,M} = \frac{\delta_{2M}\left(\kappa_p^2 + \kappa_d^2\right)}{2\kappa_p^2 \kappa_d^2 \left(\partial D_M^{(0)} / \partial \omega_M^{(0)}\right)} \left[I_M(\kappa_p a) L_{3M}(\kappa_d a) - a^2 \kappa_p \kappa_d I'_M(\kappa_p a) L'_M(\kappa_d a)\right]. \tag{34}$$

Expression (34) can be simplified in the limiting case of wide waveguides ($k_{ef} \ll 1$):

$$\Delta\omega_{a,M} \approx 0.25 M \delta_{2M} \Omega_e \sqrt{\delta/(a\Delta)} \varepsilon_d^{-1/4}, \tag{35}$$

and in the limiting case of narrow waveguides ($k_{ef} \gg 1$):

$$\Delta\omega_{a,M} \approx 0.25 \delta_{2M} \Omega_e \sqrt{M/\Delta}. \tag{36}$$

If an angular period of the considered plasma column corrugation is twice less than angular period of basic harmonic of the TSWs propagating in it, then such electromagnetic waves exist in the form of standing waves ($A_0^{(-)} = \pm A_0^{(+)}$), whose frequencies $\omega = \omega_M^{(0)} \pm \Delta\omega_{a,M}$ are approximately equal to each other. The curves of dependence of the TSWs frequency correction on the k_{ef} value are represented in Fig. 6 and Fig. 7 by dashed lines with long strokes. To make the comparison of these results with those obtained in the case of noncircular cross-section of the waveguide metal wall more easy-to-use, a small parameter of corrugation is put here as equal to unit, $\delta_{2M} = 1$.

Numerical analysis confirms that value of TSW frequency correction increases with diminishing of thickness of dielectric layer or/and permittivity of the dielectric layer. Correction $\Delta\omega_{a,M}$ caused by the deviation of the plasma-dielectric interfaces' cross-section shape from circular one is larger than the correction $\Delta\omega_{b,M}$ caused by the deviation of the waveguide metal walls' cross-section, ceteris paribus. This is explained by the peculiarity of the spatial distribution of TSW energy, see Fig. 2. One can see that most part of the TSW energy is concentrated just near the plasma-dielectric interface. Consequently, their dispersion properties are more perceptible to spatial non-uniformity (corrugation) just of this surface.

If the waveguide structure under the consideration is immersed into an external axial magnetic field $\vec{B}_0 \| \vec{z}$, then splitting of the studied SWs spectrum takes place [11] (in other words, waves with different sign of azimuthal mode number will propagate with different frequencies in this case). Let's compare values of the frequency corrections determined by corrugation of the waveguide where these waves can propagate without utilization of an external magnetic field and in the case of magnetized waveguides with circular cross-sections. Correction to the ASW frequency propagating in the waveguide with circular cross-section caused by a small value of the applied external axial magnetic field is of the following order:

$$\Delta\omega_B \sim -0.5m\,|\omega_e|\left(m^2 + a^2\delta^{-2}\right)^{-0.5}. \tag{37}$$

Comparison of expressions (34) and (37) allows one to conclude that influence of the weak axial magnetic field on the dispersion properties of ASW as compared with influence of longitudinal corrugation of plasma surface on the TSW propagating with an angular period that is twice larger than the angular period of the plasma column surface non-uniformity (corrugation) can be neglected, if electron cyclotron frequency $|\omega_e|$ is sufficiently small:

$$|\omega_e|\,\Omega_e^{-1} \ll 0.5\delta_{2M}\sqrt{M/\Delta}\left[1 + \sqrt{a/(M\delta)}\varepsilon_d^{-1/4}\left(1 + a\Delta/\delta\right)\right]. \tag{38}$$

As far as condition (38) is fulfilled for the majority of tokamaks quite well, splitting of the TSW eigen frequency, studied in this section, can be observed in such type of fusion devices, where it is caused by D-shape of the poloidal cross-sections of the both tokamak's metal chamber and plasma column confined in the device, for the waves with poloidal mode number of the main harmonic $M = 1$. It should be added that influence of the plasma column cross-section shape on sawtooth oscillations experimentally observed in DIII-D tokamak plasma has been studied in [18].

Another wide branch of application of SW propagation is gas discharges intended for production of uniform and high density plasma. Concerning that we should like to refer the paper [19]. It is one of the first reviewing articles devoted to the development of large-size, high-density $n_e \geq 10^{11}\,\mathrm{cm}^{-3}$ microwave plasma production, which can be performed at the regime of gas low pressures (less than 20 mTorr), without utilization of an external magnetic field. The authors can produce a large-diameter plasma column sustained by azimuthally non-symmetric SWs. They are sure that after improvement of the utilized antenna system this technique will be commercially available for processing of large scale flat panel displays and solar cells with diameter larger than one meter.

6. CONCLUSIONS

The paper is devoted to the theory of transverse surface waves propagating in waveguides with non-circular cross-section, which are partially filled by cold isotropic plasma. Periodic change of curvature of plasma column interface with a dielectric layer and/or a metal chamber along the azimuth angle is shown to cause the propagation of these electromagnetic surface waves in the form of the wave packet. In general case (arbitrary shape of the cross-section is modeled by Fourier series), this packet contains infinite number of spatial harmonics, whose mode numbers depend on the mode number of the main harmonic and angular period of the waveguide non-uniformity. In the most practically applicable cases, it is enough to take into account in this packet along with the main harmonic only two nearest satellite harmonics, whose amplitudes are found to be the values of the first order in the respect of small parameter that characterizes the deviation of the waveguide cross-section from circular shape (small parameter of the problem).

As a rule, the TSW eigen frequency correction caused by the deviation of a waveguide cross-section from circular shape is found to be a small value of the second order over a small parameter of the problem. In the special (resonant) cases associated with degeneration of the frequency spectrum of surface waves, value of the frequency correction can be proportional to the small parameter of the problem. Correction to surface waves eigen frequency, caused by non-circularity of the waveguide cross-section, is more sensitive to the corrugation of just plasma-dielectric layer interface than that of a metal chamber. Utilization of corrugated waveguides allows one to choose suitable operating mode, to supply stable regime of generation of electromagnetic radiation.

Since studied TSWs can radiate their power through a narrow slot into outer space, they can be used as operating modes in plasma-antenna systems. Designing the waveguide with special shape of the cross-section can be used for the control over the eigen frequency and spectral contents of the eigen electromagnetic waves propagating in it. Thus application of TSWs seems prospective for development of plasma-antenna systems, which are widely used at present time for various civil purposes [16]. It is also shown that the phenomenon of splitting TSW frequency spectrum can be observed experimentally in modern fusion devices, which have non-circular cross-section and confine plasma with a high density $\Omega_e^2 > \omega_e^2$. TSWs propagation can be applied to sustaining gas discharges, which will be able to produce uniform plasma with large diameter. Taking into account conclusions of [19] it is a very prospective branch of their possible application.

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