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Positional Error Compensation and SLL Control of Miniature Deep Space Probe Based Antenna Arrays

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Abstract—As the boundary of the universe which is explored by human expanded, antenna used in deep space exploration (DSE) could become too large to carry and deploy, miniature deep space probe based antenna arrays (MDSPBAA) provide a novel solution for the problem. This kind of antenna array may lower the difficulty of sending antenna to the area where is tend to be detected and may also monitor cost effectively in the work of deep space detect. However, turbulence and positional errors provide a challenging operational environment when it comes to the implementation of these systems. Turbulence will deteriorate SLL badly. In some cases, the level could be changed by almost 10 dB. Therefore, a SLL control algorithm is presented, which could well compensate the SLL which is caused by positional error.

1. INTRODUCTION

Deep space exploration (DSE) means the detection activities of celestials other than earth such as planets and their moons, asteroids, comets, etc. Due to the extreme long distance for signal transmit in the DSE, the signal will become very weak over the distance of dozens millions kilometers [1]. Therefore, remote telemetry, tracking and communication has become one of the key technologies for DSE. Antenna is made to achieve the function of receiving weak signal and transmit high-power signal, and it plays an important role in the deep space communication system.

In the Mars mission, by equipping with high-gain antenna whose diameter is 3 m, Mars Reconnaissance Orbiter (MRO) can transmit signals in the Ka wave band. As the boundary of space exploration expanded, antennas with higher gain, larger aperture are needed in the DSE in the near future [2]. However, larger aperture and bigger antenna bring greater difficulty in launch and carry; the cost will rise as well, and it will become the bottleneck in the DSE [3–5].

In order to solve the problem, we propose a methodology which is to use the miniature deep space probe antenna array (MDSPAA), then we can replace the expensive, heavy and huge reflector antenna by using a swam of miniature probe to compose a large-aperture, high-gain antenna array in the DSE mission [6–8].

However, miniature deep space probes are floating in space, and turbulence and positional errors will limit the aperture's ability to transmit signal stably [9–11]. So we have to solve two problems before using the methodology: first, we have to maintain the direction of the main beam; second, the SLL should be kept steady to maintain the performance of the antenna.

The expression of the normalized array factor for an arbitrary, N-element antenna array with arbitrary excitation has been given by [12]. Literature [13–15] help us to better understand the impact that positional tolerance errors has on an antenna array system. Those papers also work to develop a methodology that can be used to reduce the impact this positional noise has on the performance of

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this antenna system. The noise models introduced in that paper show that sparse arrays are no more susceptible to noise than standard arrays as long as the tolerances are electrically similar. In addition, the papers show that arrays with more elements are more robust to positional errors than smaller sized arrays. Literature [16–18] have shown that the phase correction algorithm can be applied very effectively to aperiodic arrays to eliminate the errors around the main beam while still maintaining their grating lobe suppression and low sidelobe properties. They have also demonstrated that when applied to periodic arrays, the phase correction algorithm is not effective in eliminating grating lobes. In [19–22], they have proposed that by combining phase compensation with optimized sparse aircraft formations, one can achieve high radiation pattern resolution in a micro-UAV based radar imaging application. However, none of these literatures shows how to maintain SLL to keep the performance of antenna arrays. Therefore, this article focuses on the compensation algorithm which could well compensate the changing level of the side lobes caused by positional error while rarely affects the main beam.

2. LOBE EXPRESSION

In this section, we start with the expression of the array factor for an arbitrary, M-element antenna array with arbitrary excitation to begin the discussion.

$$S(n) = \sum_{m=1}^{M} I_m \exp j\left(kn_n r_m + \beta_m\right) \tag{1}$$

where r_m is the vector pointing from the origin to elements m and n the normal unit vector pointing from the origin to the far-field observation point at (θ, φ). The subscript n in n_n means the normal unit vector pointing to a sidelobe. I_m and β_m represent the amplitude and phase of the current excitation on element m, respectively. All illustration of this coordinate system is shown in Figure 1.

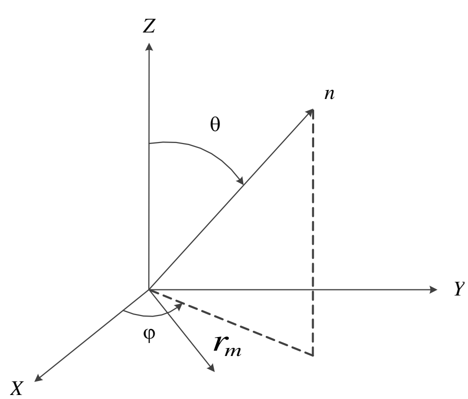

Figure 1. Antenna array coordinate systems.

The radiation pattern which is presented by (1) usually has a main beam in the position in which $\theta = 0$ and has some sidelobe with the level of ε_n in the position in which $n = n_n$, $n = 1, 2, \ldots, N$. If we make $n_i = (\pi/2, \varphi_i)$, then we will get some sidelobes pointing to the normal direction of Z axis $(\theta = \pi/2)$. These characters can be presented by the mathematical expression as follow:

$$\frac{\partial S(n)}{\partial n}|_{n=n_n} = 0, \qquad n = 1, 2, \ldots, N - i \tag{2}$$

$$S(n_n) = \varepsilon_n, \qquad n = 1, 2, \ldots, N \tag{3}$$

Since the derivative of $S(n)$ is generally not equal to zero in the normal direction of Z axis. In (2), we know that $n \neq N$. Apparently, (2) and (3) contain $2N - i$ equations with $M + N - i$ unknown parameters, such as $r_1, r_2, \ldots, r_M$; $n_1, n_2, \ldots n_{N-i}$.

3. ERROR ESTIMATION

In this section, we derive a closed-form estimate of the radiation pattern error $\Delta\varepsilon_n$ resulting from the positional turbulence of individual antenna elements of an arbitrary array. We chose $r_m = r_m^0$ to calculate the radiation pattern $S_0(n)$; the position of sidelobes can be worked out from the equation:

$$\sum_{m=1}^{M} jkr_m^0 I_m^0 \exp jkn_n^0 r_m^0 = 0, \qquad n = 1, 2, \dots, N - i \tag{4}$$

With the sidelobes n_i^0 in the position where $n_i = (\pi/2, \varphi_i)$, all of the SLL ε_n can be worked out.

$$\varepsilon_n^0 = \sum_{m=1}^{M} I_m \exp jkn_n^0 r_m^0, \qquad n = 1, 2, \dots, N \tag{5}$$

To the antenna array that has been disposed, r_m^0, n_n^0, ε_n^0 in (4) and (5) are known.

Assuming that the array elements receive a slight disturbance or have a slight displacement Δr_m, the new position vector r_m^1 is:

$$r_m^1 = r_m^0 + \Delta r_m \tag{6}$$

As a result, the new positions n_n^1 and the new levels of the sidelobes ε_n^1 become (maybe there is an exception in the position where $n_i = (\pi/2, \varphi_i)$):

$$n_n^1 = n_n^0 + \Delta n_n^1 \tag{7}$$

$$\varepsilon_n^1 = \varepsilon_n^0 + \Delta\varepsilon_n \tag{8}$$

Since the new r_m^1 and ε_n^1 should meet (2) and (3),

$$\varepsilon_n^1 = \sum_{m=1}^{M} I_m \exp jk\left(n_n^0 + \Delta n_n^1\right)\left(r_m^0 + \Delta r_m\right) = \sum_{m=1}^{M} I_m \exp jkn_n^0 r_m^0 \exp jk\vartheta \tag{9}$$

In the expression

$$\vartheta = n_n^0 \Delta r_m + \Delta n_n^1 r_m^0 + \Delta n_n^1 \Delta r_m \tag{10}$$

If Δr_m is very small, we can take approximate such as:

$$\exp jk\vartheta \approx 1 + jk\vartheta \tag{11}$$

Ignore second-order small term $\Delta n_n \Delta r_m$, consider (4) and (5), then (9) will turn into:

$$\Delta\varepsilon_n = \varepsilon_n^1 - \varepsilon_n^0 = n_n^0 \sum_{m=1}^{M} I_m \Delta r_m \exp jkn_n^0 r_m^0 \tag{12}$$

4. LOBE LEVEL COMPENSATION

In this section, we analyze the method which could compensate the error $\Delta\varepsilon_n$ by controlling the amplitude of the current in each array element. The expressions of position and level of lobes are similar to what is presented in Section 3, except that the initial lobe level should be ε_n^1, and the final lobe level should be ε_n^0, by considering (2), (3)

$$\sum_{m=1}^{M} jkr_m^1 I_m^0 \exp jkn_n^1 r_m^1 = 0, \qquad n = 1, 2, \dots, N - i \tag{13}$$

$$\varepsilon_n^1 = \sum_{m=1}^{M} I_m^0 \exp jkn_n^1 r_m^1, \qquad n = 1, 2, \dots, N \tag{14}$$

The excitation amplitude I_m should be considered as change parameter, so

$$I_m^1 = I_m^0 + \Delta I_m, \qquad m = 1, 2, \dots, M \tag{15}$$

In this expression, each I_m^0 is the initial current excitation amplitude when the compensation process begins. If we assume that the new displacement variations of the sidelobes are Δn_n^2, by putting (15) into (3), we will have:

$$\varepsilon_n^0 = \sum_{m=1}^{M} \left(I_m^0 + \Delta I_m\right) \exp jk\left(n_n^1 + \Delta n_n^2\right) r_m^1 = \sum_{m=1}^{M} \left(I_m^0 + \Delta I_m\right) \exp jkn_n^1 r_m^1 \exp jk\Delta n_n^2 r_m^1 \tag{16}$$

If $\Delta n_n^2 \to 0$

$$\exp jk\Delta n_n^2 r_m^1 \approx 1 + jk\Delta n_n^2 r_m^1 \tag{17}$$

Put (17) into (16)

$$\varepsilon_n^0 = \sum_{m=1}^{M} \exp jkn_n^1 r_m^1 \left(I_m^0 + \rho\right) \tag{18}$$

In the expression

$$\rho = I_m^0 jk\Delta n_n^2 r_m^1 + \Delta I_n + \Delta I_m \Delta n_n^2 jkr_m^1 \tag{19}$$

Ignoring second-order small term $\Delta I_m \Delta n_n^2$ and considering expression (13) and (14), the governing equation of each ΔI_m can be presented as:

$$\Delta\varepsilon_n^c = \varepsilon_n^0 - \varepsilon_n^1 = \sum_{m=1}^{M} \Delta I_m \exp jkn_n^1 r_m^1 \tag{20}$$

$\Delta\varepsilon_n^c$ is the compensated SLL, apparently:

$$\Delta\varepsilon_n = -\Delta\varepsilon_n^c \tag{21}$$

Put (12) and (20) into (21):

$$n_n^0 \sum_{m=1}^{M} I_m \Delta r_m \exp jkn_n^0 r_m^0 + \sum_{m=1}^{M} \Delta I_m \exp jkn_n^1 r_m^1 = 0 \tag{22}$$

Put (6), (7) into (22), then simplify the expression as (11), then we will get the control equation of current for compensate the positional error:

$$\sum_{m=1}^{M} \exp jkn_n^0 r_m^0 \left(I_m n_n^0 \Delta r_m + \Delta I_m\right) = 0 \tag{23}$$

When the array elements are under disturbance, we can use satellite positioning technology, INS, Star Light Navigation and so on to redefine the new location r_m^1 of the elements, which means that Δr_m in (23) can be worked out. Then by changing the circuit excitation amplitude of each element by ΔI_m which could be worked out by (23), lobe level can be controlled.

5. SIMULATED ANALYSIS

In this section, two antenna arrays are discussed, which produce array configurations suitable for MDSPAA formations. While part A focuses on the linear array, part B focuses on the rectangular planar array. We compensate both arrays with the algorithm which is presented in this article.

To begin the discussion, we propose a useful model for the positional noise which a swarm of MDSPAA is likely to encounter. Positional error Δr_m can be described by a multivariate normal random variable that varies equally in all directions. Hence, the term describing the error can be represented by:

$$n\Delta r_m \sim N\left(0, \delta\right) \tag{24}$$

Here we choose Gaussian positional noise as the turbulence model. The reasons are as follows. First, this noise model describes a characteristic: the larger the turbulence is, the less likely it is to happen, and it can well depict the suspended state of the probe with dynamic balance in different direction, while [6] used Gaussian positional noise model to describe the 3-D turbulence which the UAV is likely to encounter. Second, there are a lot of unknown variables to affect the position of a suspended probe, and under these circumstances, using Gaussian positional noise in the preliminary analysis could be an effective solution.

5.1. Unequally Spaced Symmetry Linear Array

We present an unequally spaced symmetry linear array with 2M elements (Figure 3), and the radiation pattern factor is:

$$S\left(x\right)=2\sum_{m=1}^{M}I_m\cos\left(b_m x\right) \tag{25}$$

In the expression $b_m=2\frac{d_m}{\lambda}$, $x=\pi\cos\theta$.

Assuming $b_m=b_m^0$ in the linear array, positions x_m^0 of each lobe can be worked out by Equation (26):

$$2\sum_{m=1}^{M}I_m^0 b_m^0\sin\left(b_m^0 x_n^0\right)=0 \tag{26}$$

By the above theory:

$$\Delta\varepsilon_n=-2x_n^0\sum_{m=1}^{M}I_m^0\Delta b_m\sin\left(b_m^0 x_n^0\right) \tag{27}$$

The corresponding governing equation of current can be presented as:

$$\Delta\varepsilon_n^c=2\sum_{m=1}^{M}\Delta I_m\cos\left(b_m x_n^1\right) \tag{28}$$

We propose a linear array with 8 elements, which means $M=4$. The interval is as in Table 1, and the initial lobe level is shown in Table 2.

Table 1. The intervals of linear array while $M=4$.

Factor	Element	M	d_1	d_2	d_3	d_4
Value	8	4	0.247λ	0.752λ	1.243λ	1.749λ

Table 2. The initial lobe level.

Factor	MainBeam	LobeLevel 1	LobeLevel 2	LobeLevel 3	LobeLevel 4
Value	0	−12.91 dB	−16.72 dB	−18.75 dB	−35.25 dB

We simulate different 3-D positional turbulence by adding Gaussian positional noise model with different variances to antenna array, as shown in Figure 2. The x-axis shows the value of variance while the y-axis shows the change of the lobe level. While the positional error is increasing, the level of main beam remains the same basically, but the magnitude of the SLL fluctuation is increasing. When the variance of Gaussian positional noise is close to 0.1, some side-lobe level even changes by more than 6 dB, which will deteriorate the performance of the antenna badly.

The change scope between the lobe levels adopting the compensation algorithm and the initial lobe level is illustrated in Figure 3. In order to decrease the effect of the current to the level of the main beam maximally, we add the level control equation of the main beam to the equation set. Thus, we get an overdetermined equation set with five equations to solve the change level of the current in each four array elements. We obtain the value of control current by using Least Squares (LS) and keep the main beam stable by adding proper weight to the level control equation of the main beam when we solve the equation set. We can work out the value of the lobe level which adopts the compensation algorithm by putting the value of the control current into Equation (16), and the difference between the values solved by Equation (16) and the initial lobe levels in the corresponding positions is illustrated in Figure 3. We can see that the algorithm has well compensated the changing level caused by positional error while rarely affects the main beam. Otherwise, if necessary, in order to meet the requirement of the control of SLL, we can use methods other than LS or add different weights to each equation to design the current compensation mode.

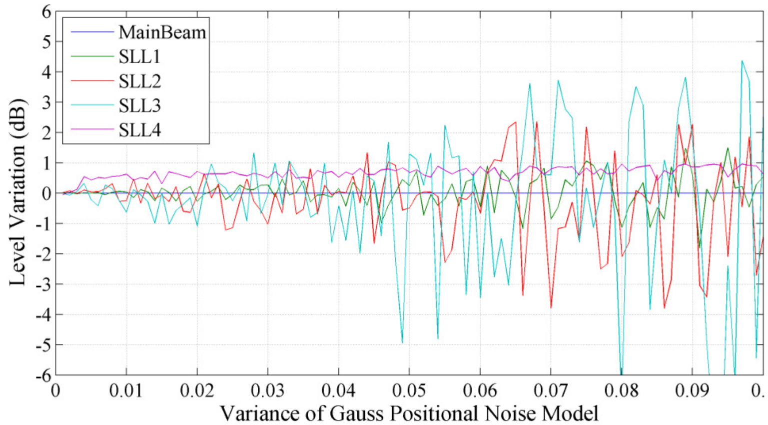

Figure 2. The level variation due to Gauss positional noise model.

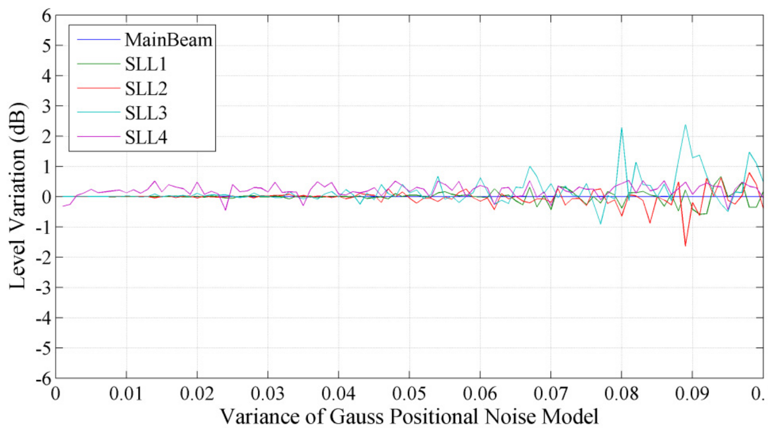

Figure 3. The level variation which is adopted the compensation algorithm.

Figure 4 and Figure 5 illustrate the degree of SLL compensation and the adjustment rate of current in each array element corresponding to different variances. It is shown that the algorithm can compensate the variation of SLL approximately 7 dB in some position in Figure 4. As the positional error is increasing, the effect of the compensation becomes even more remarkable, but the difference between the SLL with compensation algorithm and the initial SLL becomes greater. That is because we used lots of approximation in the algorithm for the reason that the assumed positional error is very small. Figure 5 shows the relationship between positional error and the adjustment rate of current. The value of the initial current is unit current I, and the x-axis shows the value of variance while the y-axis shows the adjustment rate of current. We can see that, as the positional error becomes bigger, the adjustment rate of current also needs to be increased.

In conclusion, because of the positional error, the SLL of the antenna array changes a lot, and some side lobe deteriorates badly. By using the compensation algorithm proposed in this article, we can control the SLL effectively while rarely affect the main beam.

5.2. Rectangular Planar Array

The radiation pattern factor of rectangular planar array is:

$$S(\theta,\varphi)=\sum_{m=-M}^{M}\sum_{n=-N}^{N}I_{mn}\exp\left[jk\sin\theta\left(md_x\cos\varphi+nd_y\sin\varphi\right)+j\alpha_{mn}\right] \tag{29}$$

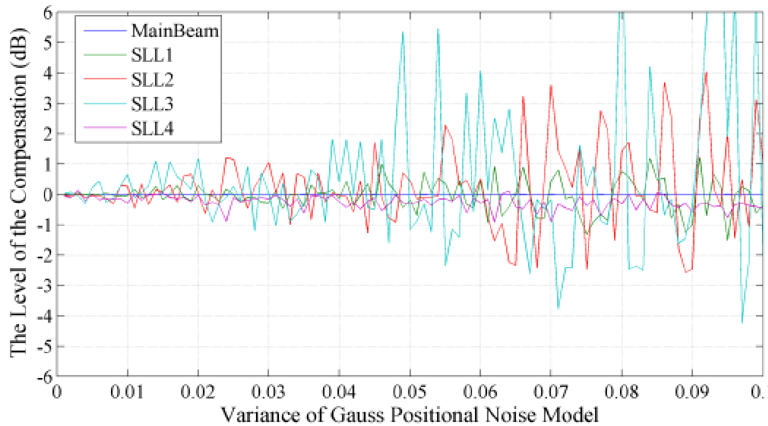

Figure 4. The level of compensation.

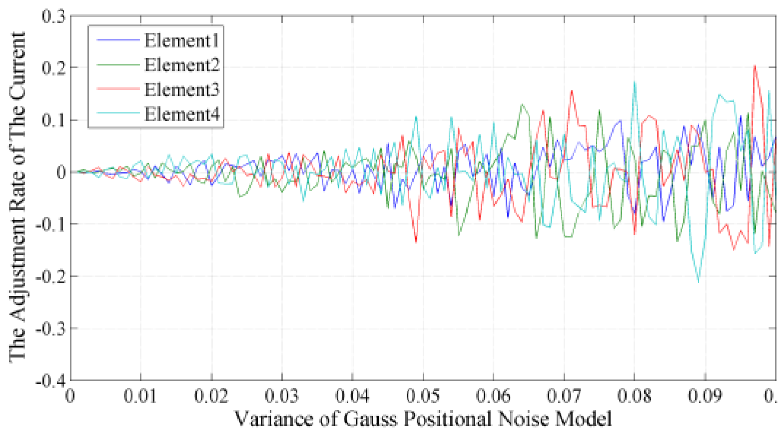

Figure 5. The adjustment rate of current.

To know the positions of the lobes, we need to work out the maximum value of the function, assuming that the phases of the current are equal and take partial of θ, φ respect to x:

$$\frac{\partial S}{\partial \theta} = \sum_{m=-M}^{M} \sum_{n=-N}^{N} I_{mn} jk \left(md_x \cos\varphi + nd_y \sin\varphi\right) \cos\theta \exp\left[jk \sin\theta \left(md_x \cos\varphi + nd_y \sin\varphi\right)\right] \quad (30)$$

$$\frac{\partial S}{\partial \varphi} = \sum_{m=-M}^{M} \sum_{n=-N}^{N} I_{mn} jk \left(nd_y \cos\varphi - md_x \sin\varphi\right) \sin\theta \exp\left[jk \sin\theta \left(md_x \cos\varphi + nd_y \sin\varphi\right)\right] \quad (31)$$

By letting (30), (31) equal to 0 and simulating the equations, we can find all the arrest points of the radiation pattern function $S(\theta, \varphi)$.

The formation of the $11 * 11$ rectangular planar array is illustrated in Figure 6. We also have ideal radiation pattern, radiation pattern with positional errors and radiation pattern with compensation algorithm, shown in Figure 7, Figure 8, and Figure 9, respectively.

In all the radiation pattern plots throughout the paper, the elevation angle (θ) is measured radially along the x and y axes whereas the azimuthal (φ) angle is measured azimuthally in the xy-plane. When the formation is corrupted by a random zero-mean Gaussian positional noise, the SLL will be changed a lot, and the position of side lobes will also be affected (see Figure 8), in some position the SLL even changes approximately by 10 dB. However, Figure 9 illustrates that when the positional error compensation algorithm is applied to the array, the SLL will be fixed effectively. Nevertheless, the

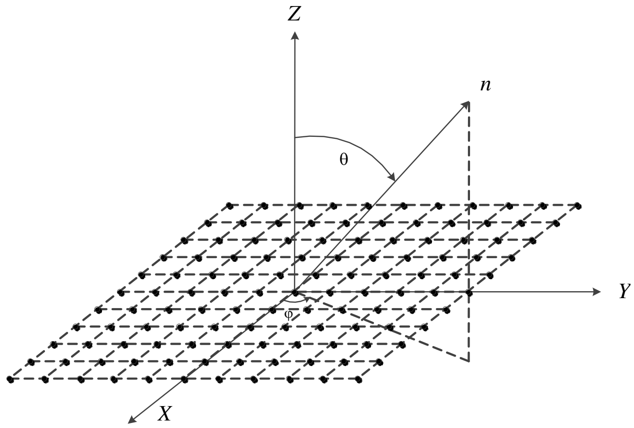

Figure 6. Rectangular planar array formation.

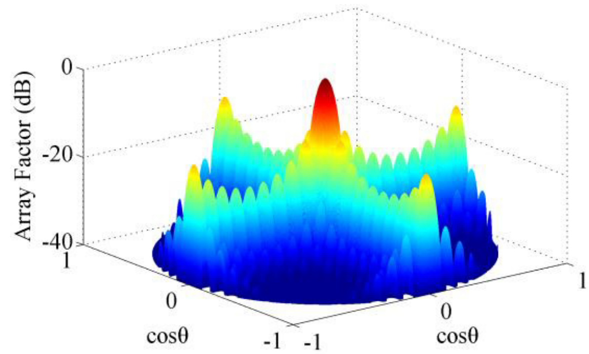

Figure 7. Ideal radiation pattern.

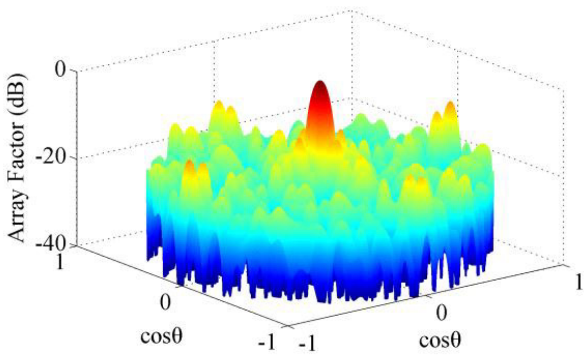

Figure 8. Pattern with positional errors.

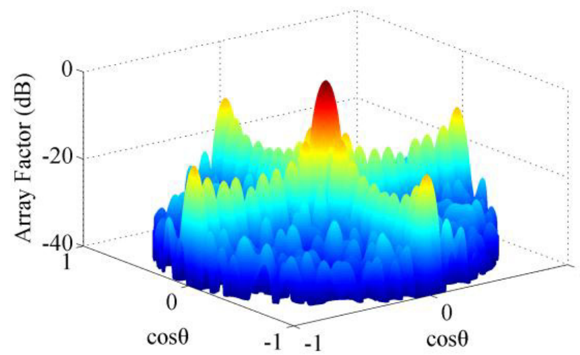

Figure 9. Pattern with compensation algorithm.

position of side lobes will still have some error, because this algorithm is insufficient for the correct of the position of side lobes. Therefore, if we combine this algorithm with the one proposed in [4], we could control the SLL effectively while maintain the stability of the main beam.

6. CONCLUSION

An error compensation algorithm has been developed to restore the SLL when the array elements of a swarm based antenna array are subject to positional noise. Moreover, the error correction algorithm has been applied to both linear array and rectangle array with positional noise errors. In some cases, the algorithm can compensate the SLL for approximately 10 dB, which can be exploited for creating robust MDSPAA. However, the Gaussian positional noise we use here is just for preliminary analysis, and we will try to get measured data instead of structured positional noise to test and improve this algorithm in the further research.

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