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Analysis of Planar Circuits Using an Efficient Laguerre-Based FDTD Method

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Abstract—In this paper, an efficient three-dimensional Laguerre-based finite-difference time-domain (FDTD) method is used to analyze planar circuits. An iterative procedure is introduced to improve the accuracy. Both the time-domain waveforms and the S-parameters are presented. The numerical results show that at the comparable accuracy, the efficiency of the Laguerre-based FDTD method with an iterative procedure is superior to the FDTD method and alternating-direction implicit (ADI) FDTD method.

1. INTRODUCTION

The finite-difference time-domain (FDTD) method has been successfully applied in simulating various modern microwave and millimeter-wave planar circuits [1–8]. It possesses the advantages of simple and accurate implementation for relatively complex problems. However, one of the major disadvantages of this method is that the time step is constrained by the Courant-Friedrich-Levy condition [9]. To overcome the Courant limitation, an unconditionally stable scheme with weighted Laguerre polynomials for the FDTD method was introduced [10]. This marching-on-in-order scheme uses the weighted Laguerre polynomials as the temporal basis functions and the Galerkin's method as the temporal testing procedure to eliminate the time variable. In this way, the stability condition is no longer affected by the time step. The update equations [10] are the Laguerre-domain difference equations, and each update equation involves the expansion coefficients of the field components from order 0 to q. At present, the marching-on-in-order scheme has been widely used by many researchers [11–14].

The Laguerre-based FDTD method produces a huge sparse matrix equation for the 3-D cases, which is very challenging to solve. In [15, 16], an efficient algorithm for implementing the Laguerre-based FDTD method was introduced. The huge sparse matrix equation is solved with a factorization-splitting scheme. This paper furthers the work of [15, 16] by applying an iterative procedure to the efficient 3-D Laguerre-based FDTD method to analyze the 3-D planar circuits. It leads to an accuracy-improved unconditionally stable Laguerre-based FDTD method. In this paper, in terms of accuracy and computational efficiency, the numerical performances of different numbers of iterations were given in the numerical examples. The numerical simulations demonstrate the validity of this Laguerre-based FDTD method. The timedomain and frequencydomain simulation results indicate that, at the comparable accuracy, the efficiency of the proposed method with an iterative procedure is superior to the FDTD method and the alternating-direction implicit (ADI) FDTD method [17–20].

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2. FORMULATIONS

For simplicity, a simple and lossless medium is considered. The 3-D differential Maxwell's equations are stated by

$$\varepsilon\frac{\partial E_x}{\partial t} = (D_y H_z - D_z H_y) - J_x \tag{1}$$

$$\varepsilon\frac{\partial E_y}{\partial t} = (D_z H_x - D_x H_z) - J_y \tag{2}$$

$$\varepsilon\frac{\partial E_z}{\partial t} = (D_x H_y - D_y H_x) - J_z \tag{3}$$

$$\mu\frac{\partial H_x}{\partial t} = D_z E_y - D_y E_z \tag{4}$$

$$\mu\frac{\partial H_y}{\partial t} = D_x E_z - D_z E_x \tag{5}$$

$$\mu\frac{\partial H_z}{\partial t} = D_y E_x - D_x E_y \tag{6}$$

where D_x, D_y, and D_z are the first-order central difference operators along x, y, and z axes, respectively. Using a set of orthogonal basis functions $\varphi_p(s,t) = e^{-st/2} L_p(st)$ [10], we can expand the electric and magnetic fields in (1)–(6) as

$$\{E_x, E_y, E_z, H_x, H_y, H_z\} = \sum_{p=0}^{\infty} \left\{E_x^p, E_y^p, E_z^p, H_x^p, H_y^p, H_z^p\right\} \varphi_p(st) \tag{7}$$

where $L_p(st)$ is the Laguerre polynomials of order p, $s > 0$ is a time-scale factor, and E_x^p, E_y^p, E_z^p, H_x^p, H_y^p, and H_z^p are the coefficients of the Laguerre basis functions for E_x, E_y, E_z, H_x, H_y, and H_z, respectively.

The first derivative of field components, taking E_x for example, with respect to t is [10]

$$\frac{\partial E_x}{\partial t} = s\sum_{p=0}^{\infty}\left(0.5E_x^p + \sum_{k=0,p>0}^{p-1} E_x^k\right)\varphi_p(st) \tag{8}$$

Inserting (7)–(8) into (1)–(6) and using a temporal Galerkin's testing procedure to eliminate the time variable, we get

$$s\left(0.5E_x^q + \sum_{k=0,q>0}^{q-1} E_x^k\right) = \frac{1}{\varepsilon}D_y H_z^q - \frac{1}{\varepsilon}D_z H_y^q - \frac{J_x^q}{\varepsilon} \tag{9}$$

$$s\left(0.5E_y^q + \sum_{k=0,q>0}^{q-1} E_y^k\right) = \frac{1}{\varepsilon}D_z H_x^q - \frac{1}{\varepsilon}D_x H_z^q - \frac{J_y^q}{\varepsilon} \tag{10}$$

$$s\left(0.5E_z^q + \sum_{k=0,q>0}^{q-1} E_z^k\right) = \frac{1}{\varepsilon}D_x H_y^q - \frac{1}{\varepsilon}D_y H_x^q - \frac{J_z^q}{\varepsilon} \tag{11}$$

$$s\left(0.5H_x^q + \sum_{k=0,q>0}^{q-1} H_x^k\right) = \frac{1}{\mu}\left(D_z E_y^q - D_y E_z^q\right) \tag{12}$$

$$s\left(0.5H_y^q + \sum_{k=0,q>0}^{q-1} H_y^k\right) = \frac{1}{\mu}\left(D_x E_z^q - D_z E_x^q\right) \tag{13}$$

$$s\left(0.5H_z^q + \sum_{k=0,q>0}^{q-1} H_z^k\right) = \frac{1}{\mu}\left(D_y E_x^q - D_x E_y^q\right) \tag{14}$$

where

$$J_i^q = \int_0^\infty J_i \varphi_q(st)\, d(st), \quad i = x, y, z \tag{15}$$

With some manipulations, we can write (9)–(14) in the following matrix forms:

$$W_E^q = aD_H W_H^q + V_E^{q-1} + aJ_E^q \tag{16}$$

$$W_H^q = bD_E W_E^q + V_H^{q-1} \tag{17}$$

where $a = 2/(s\varepsilon)$, $b = 2/(s\mu)$, and a set of auxiliary matrices are defined as

$$W_E^q = \begin{pmatrix} E_x^q & E_y^q & E_z^q \end{pmatrix}^T \tag{18a}$$

$$W_H^q = \begin{pmatrix} H_x^q & H_y^q & H_z^q \end{pmatrix}^T \tag{18b}$$

$$D_E = (D_H)^T = \begin{pmatrix} 0 & D_z & -D_y \\ -D_z & 0 & D_x \\ D_y & -D_x & 0 \end{pmatrix} \tag{18c}$$

$$J_E^q = \begin{pmatrix} -J_x^q & -J_y^q & -J_z^q \end{pmatrix}^T \tag{18d}$$

$$V_E^{q-1} = \begin{pmatrix} -2\sum_{k=0}^{q-1} E_x^k & -2\sum_{k=0}^{q-1} E_y^k & -2\sum_{k=0}^{q-1} E_z^k \end{pmatrix}^T \tag{18e}$$

$$V_H^{q-1} = \begin{pmatrix} -2\sum_{k=0}^{q-1} H_x^k & -2\sum_{k=0}^{q-1} H_y^k & -2\sum_{k=0}^{q-1} H_z^k \end{pmatrix}^T \tag{18f}$$

Inserting (17) into (16), we have

$$(I - abD_H D_E)\, W_E^q = aD_H V_H^{q-1} + V_E^{q-1} + aJ_E^q \tag{19}$$

Here we decompose $abD_H D_E$ into two triangular matrices A and B, A is a lower triangular matrix, and B is an upper triangular matrix, and we have

$$(I - A - B)\, W_E^q = aD_H V_H^{q-1} + V_E^{q-1} + aJ_E^q \tag{20}$$

Adding a perturbation term $AB(W_E^q - V_E^{q-1})$ to (20), Equation (20) can be solved into two sub-steps with the following splitting scheme [15, 16]:

$$(I - A)\, W^* = (I + B)\, V_E^{q-1} + aD_H V_H^{q-1} + aJ_E^q \tag{21a}$$

$$(I - B)\, W_E^q = W^* - BV_E^{q-1} \tag{21b}$$

where $W^* = \begin{pmatrix} E_x^{*q} & E_y^{*q} & E_z^{*q} \end{pmatrix}^T$ is a nonphysical intermediate value. To solve (21), we choose the matrices A and B as [16]:

$$A = ab\begin{pmatrix} D_{2y} & 0 & 0 \\ -D_x D_y & D_{2z} & 0 \\ -D_x D_z & -D_y D_z & D_{2x} \end{pmatrix} \tag{22a}$$

$$B = ab\begin{pmatrix} D_{2z} & -D_y D_x & -D_z D_x \\ 0 & D_{2x} & -D_z D_y \\ 0 & 0 & D_{2y} \end{pmatrix} \tag{22b}$$

where D_{2x}, D_{2y}, and D_{2z} are the difference operators for the second derivatives. Using (18) and (22) to expand (21), and applying the central-difference scheme introduced by Yee, we can obtain discrete space equations for the efficient Laguerre-based FDTD method [16]. The equations are six tri-diagonal matrix equations which can be solved efficiently.

In order to reduce the error introduced by the perturbation term $AB(W_E^q - V_E^{q-1})$, we present an iterative procedure, like the iterative ADI-FDTD method [21–23] and the iterative LOD-FDTD method [24] to improved the accuracy. Because the weighted Laguerre polynomials tend to zero as time $t \to \infty$, the expanded quantities of the fields will converge to zero as time progresses. When an iteration procedure is applied to the 3-D Laguerre-based FDTD method, the obtained solutions are therefore stable. We suppose that W_0^q is the solution of (21), we can replace $AB(W_E^q - V_E^{q-1})$ with $AB(W_E^q - W_0^q)$ and add it to (20). Thus, the factorized form of (20) can be written as

$$(I - A)\,(I - B)\,W_E^q = ABW_0^q + aD_H V_H^{q-1} + V_E^{q-1} + aJ_E^q \tag{23}$$

By using the values calculated by (23) repeatedly, we can obtain an iterative procedure as follows:

$$(I - A)\,(I - B)\,W_{E,r+1}^q = ABW_{E,r}^q + aD_H V_H^{q-1} + V_E^{q-1} + aJ_E^q \tag{24}$$

where the subscript r denotes the rth iteration. Thus, we can use the initial Equation (21) and the iterative Equation (24) to simulate the numerical examples. Using the same splitting scheme to solve (24) leads to

$$(I - A)\,W^* = BW_{E,r}^q + aD_H V_H^{q-1} + V_E^{q-1} + aJ_E^q \tag{25a}$$

$$(I - B)\,W_{E,r+1}^q = W^* - BW_{E,r}^q \tag{25b}$$

Using (18) and (22) to expand (25), and applying the central-difference scheme introduced by Yee, we can obtain discrete space equations for the iterative procedure. Taking E_z^{*q} and E_z^q as example, the updating equation is

$$\begin{aligned}
&-\frac{2}{s\varepsilon\Delta x}\frac{2}{s\mu\Delta x}\,E_z^{*q}\big|_{i-1,j,k} + \left(1 + 2\frac{2}{s\varepsilon\Delta x}\frac{2}{s\mu\Delta x}\right)E_z^{*q}\big|_{i,j,k} - \frac{2}{s\varepsilon\Delta x}\frac{2}{s\mu\Delta x}\,E_z^{*q}\big|_{i+1,j,k} \\
&= -\frac{2}{s\varepsilon\Delta x}\frac{2}{s\mu\Delta z}\left(E_x^{*q}\big|_{i,j,k+1} - E_x^{*q}\big|_{i-1,j,k+1} - E_x^{*q}\big|_{i,j,k} + E_x^{*q}\big|_{i-1,j,k}\right) - 2\sum_{k=0,q>0}^{q-1} E_z^k\Big|_{i,j,k} \\
&\quad -\frac{2}{s\varepsilon\Delta y}\frac{2}{s\mu\Delta z}\left(E_y^{*q}\big|_{i,j,k+1} - E_y^{*q}\big|_{i,j-1,k+1} - E_y^{*q}\big|_{i,j,k} + E_y^{*q}\big|_{i,j-1,k}\right) - \frac{2}{s\varepsilon}\,J_z^q\big|_{i,j,k} \\
&\quad +2\frac{2}{s\varepsilon\Delta y}\sum_{k=0,q>0}^{q-1}\left(H_x^k\Big|_{i,j,k} - H_x^k\Big|_{i,j-1,k}\right) - 2\frac{2}{s\varepsilon\Delta x}\sum_{k=0,q>0}^{q-1}\left(H_y^k\Big|_{i,j,k} - H_y^k\Big|_{i-1,j,k}\right) \\
&\quad +\frac{2}{s\varepsilon\Delta y}\frac{2}{s\mu\Delta y}\left(E_{z,r}^q\big|_{i,j+1,k} + E_{z,r}^q\big|_{i,j-1,k} - 2\,E_{z,r}^q\big|_{i,j,k}\right)
\end{aligned} \tag{26a}$$

$$\begin{aligned}
&-\frac{2}{s\varepsilon\Delta y}\frac{2}{s\mu\Delta y}\,E_{z,r+1}^q\Big|_{i,j-1,k} + \left(1 + 2\frac{2}{s\varepsilon\Delta y}\frac{2}{s\mu\Delta y}\right)E_{z,r+1}^q\Big|_{i,j,k} - \frac{2}{s\varepsilon\Delta y}\frac{2}{s\mu\Delta y}\,E_{z,r+1}^q\Big|_{i,j+1,k} \\
&= E_z^{*q}\big|_{i,j,k} - \frac{2}{s\varepsilon\Delta y}\frac{2}{s\mu\Delta y}\left(E_{z,r}^q\big|_{i,j+1,k} + E_{z,r}^q\big|_{i,j-1,k} - 2\,E_{z,r}^q\big|_{i,j,k}\right)
\end{aligned} \tag{26b}$$

3. NUMERICAL RESULTS

In order to verify the proposed formulations above, numerical examples involving two typical planar circuits are taken into consideration. The numerical simulations are carried out using the proposed method, the ADI-FDTD method, and the conventional FDTD method for comparison.

The first example is related to a coplanar stripline structure [25], as shown in Figure 1. This structure is a class of printed microwave circuits, which can be fabricated in a uniplanar manner. The microstrip line has a lossless isotropic dielectric substrate with permittivity $\varepsilon_r = 10.2$ and thickness $d = 762\,\mu\text{m}$. The cell size is chosen to be $d_x = 95.25\,\mu\text{m}$, $d_y = 25.4\,\mu\text{m}$, and $d_z = 45.075\,\mu\text{m}$. A baseband Gaussian pulse with $\tau = 20\,\text{ps}$ is used to excite the horizontal electric-field component in the aperture between the two strips. The Mur's first-order absorbing boundary condition [26] is implemented to terminate the outer surfaces including the ground plane on the bottom side. In the Laguerre-based FDTD method, we choose $q = 80$, $s = 1.6 \times 10^{12}$ [27, 28].

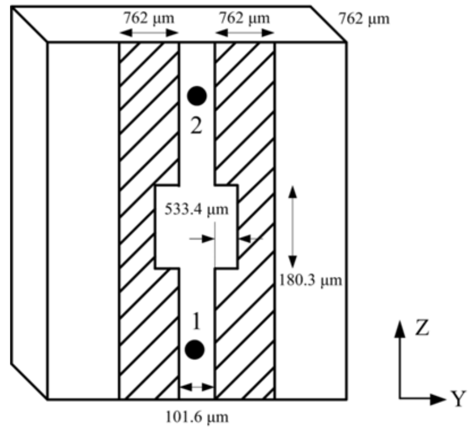

Figure 1. The geometry and dimensions of a coplanar stripline structure.

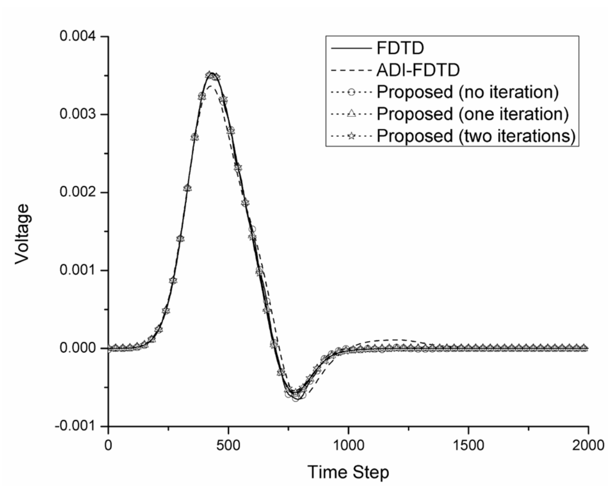

Figure 2. Comparison of time-domain waveforms for voltages at observation point 1.

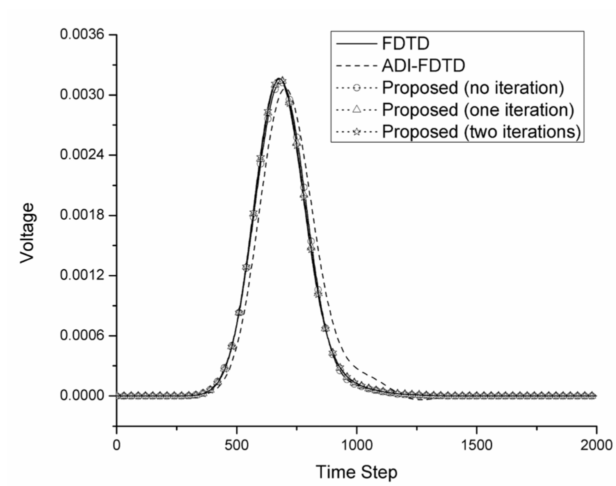

Figure 3. Comparison of time-domain waveforms for voltages at observation point 2.

Figure 2 and Figure 3 graph the time-domain voltage waveforms at the observation points 1 and 2. It is clear that the proposed formalism is more accurate, and lager errors occur for the ADI-FDTD method. Additionally, S-parameters between the two observation points are calculated, as shown in Figure 4. The result obtained using the proposed method is compared with those of the conventional FDTD and good agreement is observed. The comparison of the numerical performance versus the iteration number is shown in Table 1. The computation time for the ADI-FDTD method is 69 s, while the proposed formalism used 30 s without iteration. With the increase of the iteration number, the CPU time is increased.

The second example is related to the microstrip T-junction of [29], as shown in Figure 5. The substrate is 254-µm-thick and has a relative dielectric constant of 9.9. The microstrip is 230-µm-wide and is assumed to have zero thickness. The microstrip stub is 510-µm-wide and 1380-µm-long. The cell size is chosen to be $d_x = 63.5\,\mu\text{m}$, $d_y = 57.5\,\mu\text{m}$, and $d_z = 102\,\mu\text{m}$. A baseband Gaussian pulse $e^{-((t-t_c)/\tau)^2}$ with $\tau = 15\,\text{ps}$ and $T_c = 3\tau$ is used as the excitation to obtain the S-parameters between DC and the 30-GHz band. The Mur's first-order absorbing boundary condition [26] is implemented to terminate the outer surfaces except for the ground plane on the bottom side. For the conventional FDTD, we choose the time step $\Delta t_{\text{FDTD}} = 0.2\,\text{ps}$. For the ADI-FDTD method, the time step is $\Delta t_{\text{ADI}} = 10\ \Delta t_{\text{FDTD}}$. In the Laguerre-based FDTD method, we choose $q = 60$, $s = 1.2 \times 10^{12}$ [27, 28], and we set $\Delta t = 0.4\,\text{ps}$ to calculate the Laguerre coefficient of the excitation pulse.

Table 1. Simulation results for a coplanar stripline structure.

Scheme	Δt	Marching-on steps	CPU time (s)
FDTD	0.15 ps	3000	98
ADI-FDTD	1.5 ps	300	69
Laguerre-based FDTD (without iteration)	-	81	30
Laguerre-based FDTD (with one iteration)	-	81	54
Laguerre-based FDTD (with two iterations)	-	81	78

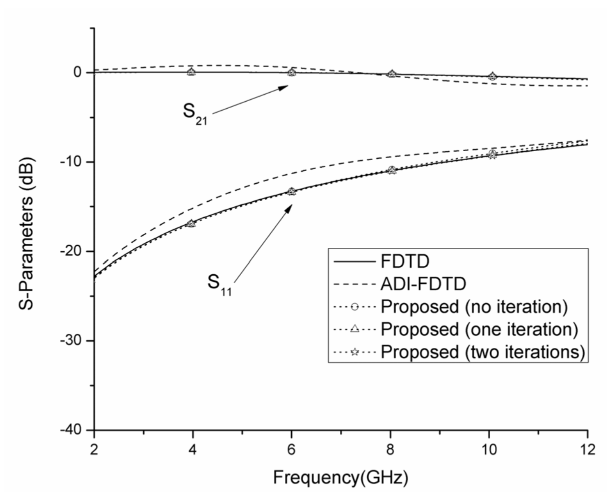

Figure 4. Comparison of the S-parameters.

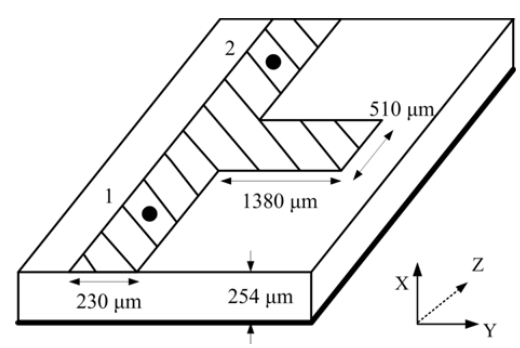

Figure 5. The geometry and dimensions of the microstrip T-junction.

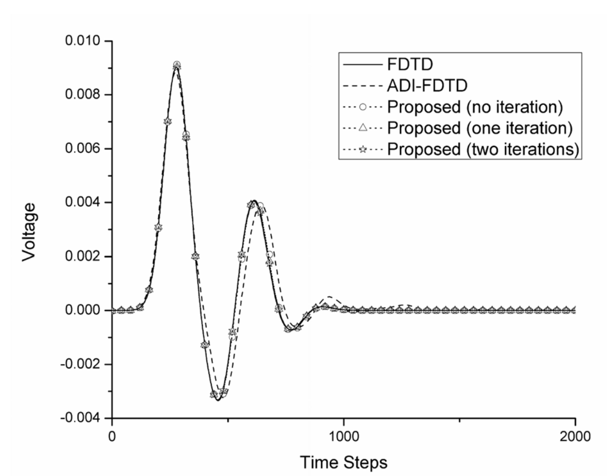

Figure 6. Comparison of time-domain waveforms for voltages at observation point 1.

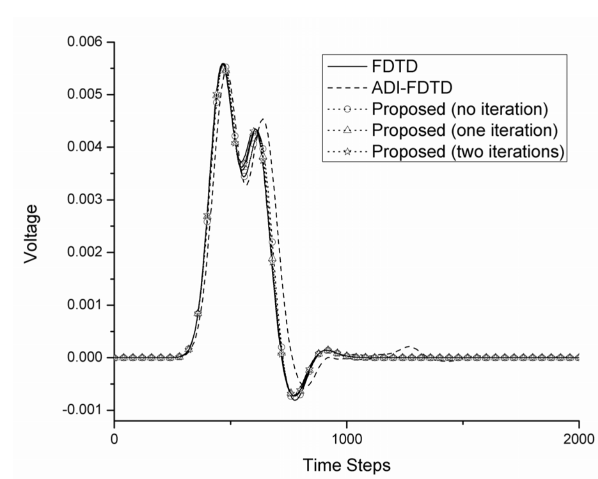

Figure 7. Comparison of time-domain waveforms for voltages at observation point 2.

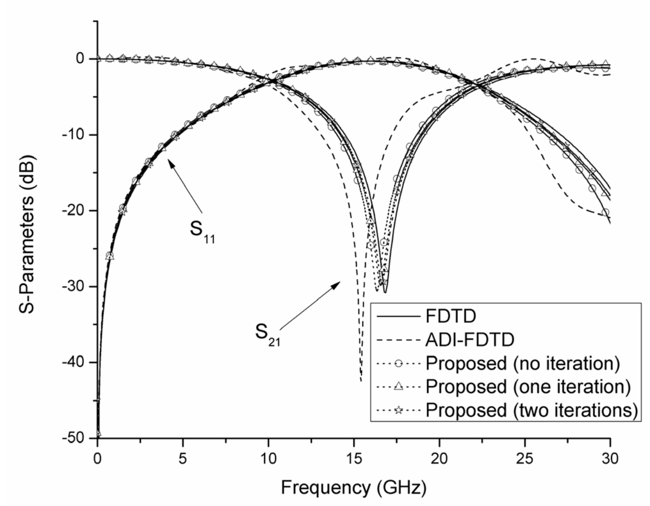

Figure 8. Comparison of the S-parameters.

Table 2. Simulation results for the microstrip T-junction.

Scheme	Δt	Marching-on steps	CPU time (s)	Resonant frequency (GHz)
FDTD	0.2 ps	4000	63	1.686
ADI-FDTD	2 ps	400	40	1.541
Laguerre-based FDTD (without iteration)	-	61	9	1.637
Laguerre-based FDTD (with one iteration)	-	61	17	1.656
Laguerre-based FDTD (with two iterations)	-	61	25	1.663

Figure 6 and Figure 7 plot the time-domain voltage waveforms at the observation points 1 and 2. The S-parameters between the two observation points are also extracted from the time-domain data, as shown in Figure 8. It can be seen that the agreement between the conventional FDTD and the Laguerre-based FDTD method is very good. However, the ADI-FDTD method shows large errors when the time step is ten times larger than that used in the conventional FDTD.

The comparison of the computational efficiency and accuracy versus the iteration number is shown in Table 2, the resonant frequency is also considered. It can be see that, with the increase of the numbers of iterations, the accuracy can be improved at the cost of additional CPU time. With two additional iterations, the Laguerre-based method not only achieves about 1.6 times the saving in CPU time in comparison with the ADI-FDTD method but also gives better results than the ADI-FDTD method. In addition, we can see that the simulation takes 63 s for the FDTD method, and 25 s for the proposed method with two additional iterations. The CPU time for the proposed method is reduced to about 39.7% of the FDTD method.

From both the time-domain and the frequency-domain analysis above, it can be seen that the Laguerre-based method gives a more accurate result than the ADI-FDTD method in analyzing the planar circuits. Additionally, the CPU time usage are saved when the present method are used compared with the other two schemes.

4. CONCLUSION

An efficient Laguerre-based FDTD method is used in this work for the analysis of the planar circuits. The accuracy of the proposed method is improved by using an iterative procedure. To verify the validity of the proposed method, two examples are included. The accuracy of the proposed method has been proved from the comparison of both time domain and frequency domain results. It is demonstrated that the Laguerre-based FDTD method is highly efficient while timesaving compared with the ADI-FDTD method.

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