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Negative Absorption Coefficient of a Weak Electromagnetic Wave Caused by Electrons Confined in Rectangular Quantum Wires in the Presence of Laser Radiation Modulated by Amplitude

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Abstract—The analytic expressions for the absorption coefficient (ACF) of a weak electromagnetic wave (EMW) by confined electrons in rectangular quantum wires (RQWs) in the presence of laser radiation modulated by amplitude are calculated by using the quantum kinetic equation for electrons with the electron-optical phonon scattering mechanism. Then, the analytic results are numerically calculated and discussed for $GaAs/GaAsAl$ RQWs. The numerical results show that the ACF of a weak EMW in a RQW can have negative values, which means that in the presence of laser radiation (non-modulated or modulated by amplitude), under proper conditions, the weak EMW is increased. This is different from the similar problem in bulk semiconductors and from the case of the absence of laser radiation. The results also show that in some conditions, when laser radiation is modulated by amplitude, ability to increase a weak EMW can be enhanced in comparison with the use of non-modulated laser radiation.

1. INTRODUCTION

Quantum wires are one-dimensional semiconductor structures. In quantum wires, many the physical properties of the material changes significantly from the properties of normal bulk semiconductors, including optical properties [1, 2]. The linear absorption of a weak EMW and the nonlinear absorption of a strong EMW in low-dimensional systems have been studied [3–6]. The influence of laser radiation on the absorption of a weak EMW in normal bulk semiconductors, quantum wells and cylindrical quantum wires have been investigated using the quantum kinetic equation method [7–10]. The influence of laser radiation (non-modulated and modulated by amplitude) on the absorption of a weak EMW in component superlattices has been investigated by using the Kubo-Mori method [11]. However, the influence of laser radiation modulated by amplitude on the absorption of a weak EMW in RQW are still unsolved. Therefore, in this paper, we theoretically calculate the ACF of a weak EMW caused by electrons confined in a RQW in the presence of laser radiation modulated by amplitude by using the quantum kinetic equation for electrons. The electron-optical phonon scattering mechanism is considered. The results are numerically calculated for the specific case of $GaAs/GaAsAl$ RQW. We show that the ACF of a weak EMW in a RQW can have negative values. That means that in the presence of laser radiation (non-modulated or modulated by amplitude), under proper conditions, the weak EMW is increased; and in some conditions, when laser radiation is modulated by amplitude, ability to increase a weak EMW can be enhanced in comparison with the use of non-modulated laser radiation.

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2. THE ABSORPTION COEFFICIENT OF A WEAK EMW IN RQWS IN THE PRESENCE OF LASER RADIATION FIELD MODULATED BY AMPLITUDE

2.1. The Laser Radiation Field Modulated by Amplitude

As in [11], here we also assume that the strong EMW (laser radiation) modulated by amplitude has the form:

$$\vec{F}(t) = \vec{F}_1(t) + \vec{F}_2(t) = \vec{F}_1 \sin(\Omega_1 t + \alpha_1) + \vec{F}_2 \sin(\Omega_2 t + \alpha_2) \tag{1}$$

where, $\vec{F}_1$ and $\vec{F}_2$ has same direction, Ω_1 and Ω_2 are a bit different from each other or $\Omega_1 \approx \Omega_2$; $|\Delta\Omega| = |\Omega_1 - \Omega_2| \ll \Omega_1, \Omega_2$.
After some transformations, we obtain:

$$\vec{F}(t) = \vec{E}_{01} \sin(\Omega t + \varphi_1) \tag{2}$$

with $E_{01} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos(\Delta\Omega t + \Delta\alpha)}$, $\Delta\Omega = \Omega_1 - \Omega_2$, $\Delta\alpha = \alpha_1 - \alpha_2$, $\Omega = \frac{\Omega_1+\Omega_2}{2}$, $\varphi_1 = \alpha + \alpha'$, $\alpha = \frac{\alpha_1+\alpha_2}{2}$, $\mathrm{tg}\alpha' = \frac{F_1-F_2}{F_1+F_2}\mathrm{tg}(\frac{\Delta\Omega}{2}t + \frac{\Delta\alpha}{2})$.

Here, Ω is the reduced frequency (or the frequency of the laser radiation modulated by amplitude), $|\Delta\Omega|$ the modulated frequency, and $\vec{E}_{01}$ the intensity of the laser radiation modulated by amplitude. In the case that $\vec{F}_1$, $\vec{F}_2$, Ω_1, Ω_2, Ω satisfy the conditions: $\frac{\vec{F}_1}{\Omega_1^2} = \frac{\vec{F}_2}{\Omega_2^2} = \frac{1}{2}\frac{\vec{F}}{\Omega^2}$, and $\Delta\alpha = 0$, the above formulas can be approximated as in [11].

When $\Delta\Omega = 0$, laser radiation modulated by amplitude becomes non-modulated laser radiation.

2.2. Calculations of the Absorption Coefficient of a Weak EMW in RQWs in the Presence of Laser Radiation Modulated by Amplitude

We consider a wire of $GaAs$ with a rectangular cross section ($L_x \times L_y$) and a length L_z, embedded in $GaAsAl$. The carriers (electron gas) are assumed to be confined by infinite potential barriers in the xOy plane and to be free along the wire's axis (the Oz-axis), where O is the origin. The EMW is assumed to be planar and monochromatic, to have a high frequency, and to propagate along the x direction. In a RQW, the state and the electron energy spectrum have the forms [12]

$$\psi_{n,\ell,\vec{p}_z} = \begin{cases} \dfrac{2e^{ip_z z}}{\sqrt{L_xL_yL_z}} \sin\dfrac{n\pi x}{L_x} \sin\dfrac{\ell\pi y}{L_y} & \begin{cases} 0 \le x \le L_x \\ 0 \le y \le L_y \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$\varepsilon_{n,\ell}(\vec{p}_z) = \frac{\hbar^2 p_z^2}{2m^*} + \frac{\hbar^2\pi^2}{2m^*}\left(\frac{n^2}{L_x^2} + \frac{\ell^2}{L_y^2}\right), \tag{4}$$

where n and ℓ (n, $\ell = 1, 2, 3, \dots$) denote the quantization of the energy spectrum in the x and the y directions, respectively. $\vec{p}_z = (0, 0, p_z)$ is the wave vector of an electron along the wire's z axis, and m^* is the effective mass of an electron.

We consider a field of three EMWs: two laser radiations as two strong EMWs (that creates laser radiation modulated by amplitude) with the intensities $\vec{F}_1$, $\vec{F}_2$ and the frequencies Ω_1, Ω_2 ($\Omega_1 \approx \Omega_2$); and a weak EMW with an intensity $\vec{E}_{02}$ and a frequency ω:

$$\vec{E}(t) = \vec{F}(t) + \vec{E}_2(t) = \vec{F}_1 \sin(\Omega_1 t + \alpha_1) + \vec{F}_2 \sin(\Omega_2 t + \alpha_2) + \vec{E}_{02} \sin(\omega t) \tag{5}$$

The Hamiltonian of the electron-optical phonon system in the RQW in that field of three EMWs in the second quantization representation can be written as

$$H = \sum_{n,\ell,\vec{p}_z} \varepsilon_{n,\ell}\left(\vec{p}_z - \frac{e}{\hbar c}\vec{A}_z(t)\right) a^+_{n,\ell,\vec{p}_z} a_{n,\ell,\vec{p}_z} + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}} + \sum_{n,\ell,n',\ell',\vec{p}_z,\vec{q}} C_{\vec{q}} I_{n,\ell,n',\ell'}(\vec{q}_\perp) a^+_{n',\ell',\vec{p}_z+\vec{q}_z} a_{n,\ell,\vec{p}_z}(b_{\vec{q}} + b^+_{-\vec{q}}), \tag{6}$$

where e is the elemental charge, c is the velocity of light, $\omega_{\vec{q}} \approx \omega_0$ is the frequency of an optical phonon, $(n, \ell, \vec{p}_z)$ and $(n', \ell', \vec{p}_z + \vec{q}_z)$ are the electron states before and after scattering, respectively, $a^+_{n,\ell,\vec{p}_z}$ $(a_{n,\ell,\vec{p}_z})$ is the creation (annihilation) operator of an electron, $b^+_{\vec{q}}$ $(b_{\vec{q}})$ is the creation (annihilation) operator of an phonon for a state having wave vector $\vec{q} = (q_x, q_y, q_z)$, and $\vec{q}_z = (0, 0, q_z)$. $C_{\vec{q}}$ is the electron — optical phonon interaction constant [3, 4], $\left|C_{\vec{q}}\right|^2 = \frac{e^2\hbar\omega_0}{2\varepsilon_0 V q^2}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)$, where V and ε_0 are the normalization volume and the electronic constant, χ_0 and χ_∞ are the static and the high-frequency dielectric constants, respectively. $\vec{A}(t)$ is the vector potential of field of the three EMWs. $I_{n,l,n',l'}(\vec{q}_\perp)$ is the electron form factor (which characterizes the confinement of electrons in a RQW). This form factor can be written as [12]

$$I_{n,\ell,n',\ell'}(\vec{q}_\perp) = \frac{32\pi^4(q_xL_xnn')^2\left(1-(-1)^{n+n'}\cos(q_xL_x)\right)}{\left[(q_xL_x)^4 - 2\pi^2(q_xL_x)^2(n^2+n'^2) + \pi^4(n^2-n'^2)^2\right]^2} \times \frac{32\pi^4(q_yL_y\ell\ell')^2\left(1-(-1)^{\ell+\ell'}\cos(q_yL_y)\right)}{\left[(q_yL_y)^4 - 2\pi^2(q_yL_y)^2(\ell^2+\ell'^2) + \pi^4(\ell^2-\ell'^2)^2\right]^2}. \quad (7)$$

The current density vector of electrons along the z direction in the RQW has the form:

$$\vec{j}_z(t) = \frac{e\hbar}{m^*}\sum_{n,\ell,\vec{p}_z}\left(\vec{p}_z - \frac{e}{\hbar c}\vec{A}_z(t)\right)n_{n,\ell,\vec{p}_z}(t). \quad (8)$$

The ACF of a weak EMW caused by the confined electrons in the presence of laser radiation modulated by amplitude in the RQW takes the form [7]

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty}E_{02}^2}\left\langle \vec{j}_z(t)\vec{E}_{02}\sin\omega t\right\rangle_t. \quad (9)$$

The general quantum equation for the statistical average value of the electron particle number operator (or electron distribution function) $n_{n,\ell,\vec{p}_z}(t) = \langle a^+_{n,\ell,\vec{p}_z}a_{n,\ell,\vec{p}_z}\rangle_t$ [7]:

$$i\hbar\frac{\partial n_{n,\ell,\vec{p}_z}(t)}{\partial t} = \left\langle\left[a^+_{n,\ell,\vec{p}_z}a_{n,\ell,\vec{p}_z}, H\right]\right\rangle_t. \quad (10)$$

Because the strong EMW (laser radiation) is modulated by amplitude, according to Section 2.1, it is expressed by formula (2). According to the hypothesis, due to $|\Delta\Omega| \ll \Omega$, then in a small amount of time there are about a few periods $T = \frac{2\pi}{\Omega}$, we can presume that $(\Delta\Omega t + \Delta\alpha)$ is changeless. Therefore, we let t get a certain specific value τ in such a small amount of time. Then, we have:

$$E_{01} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos(\Delta\Omega\tau + \Delta\alpha)} = \text{const}; \quad \varphi_1 = \alpha + \alpha' = \text{const}. \quad (11)$$

Using Equation (10) and the Hamiltonian in Equation (6) and Equation (11), we obtain the quantum kinetic equation for electrons in the RQW (see Appendix A) [5, 6, 8]:

$$\begin{aligned}\frac{\partial n_{n,\ell,\vec{p}_z}(t)}{\partial t} = & -\frac{1}{\hbar^2}\sum_{n',\ell',\vec{q}}\left|C_{\vec{q}}\right|^2\left|I_{n,\ell,n',\ell'}(\vec{q}_\perp)\right|^2\sum_{u,s,m,f=-\infty}^{+\infty}J_u(a_{1z}q_z)J_s(a_{1z}q_z)J_m(a_{2z}q_z)J_f(a_{2z}q_z)\\ & \times\exp\left\{i\left\{\left[(s-u)\frac{\Omega_1+\Omega_2}{2} + (m-f)\omega - i\delta\right]t + (s-u)\left(\frac{\alpha_1+\alpha_2}{2} + \alpha'\right)\right\}\right\}\\ & \times\int_{-\infty}^{t}dt_2\left\{\left[n_{n,\ell,\vec{p}_z}(t_2)N_{\vec{q}} - n_{n',\ell',\vec{p}_z+\vec{q}_z}(t_2)(N_{\vec{q}}+1)\right]\right.\\ & \times\exp\left\{\frac{i}{\hbar}\left[\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z) - \varepsilon_{n,\ell}(\vec{p}_z) - \hbar\omega_{\vec{q}} - s\hbar\frac{\Omega_1+\Omega_2}{2} - m\hbar\omega + i\hbar\delta\right](t-t_2)\right\}\\ & + \left[n_{n,\ell,\vec{p}_z}(t_2)(N_{\vec{q}}+1) - n_{n',\ell',\vec{p}_z+\vec{q}_z}(t_2)N_{\vec{q}}\right]\end{aligned}$$

$$\times\exp\left\{\frac{i}{\hbar}\left[\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z)-\varepsilon_{n,\ell}(\vec{p}_z)+\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta\right](t-t_2)\right\}$$
$$-\left[n_{n',\ell',\vec{p}_z-\vec{q}_z}(t_2)N_{\vec{q}}-n_{n,\ell,\vec{p}_z}(t_2)(N_{\vec{q}}+1)\right]$$
$$\times\exp\left\{\frac{i}{\hbar}\left[\varepsilon_{n,\ell}(\vec{p}_z)-\varepsilon_{n',\ell'}(\vec{p}_z-\vec{q}_z)-\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta\right](t-t_2)\right\}$$
$$-\left[n_{n',\ell',\vec{p}_z-\vec{q}_z}(t_2)(N_{\vec{q}}+1)-n_{n,\ell,\vec{p}_z}(t_2)N_{\vec{q}}\right]$$
$$\times\exp\left\{\frac{i}{\hbar}\left[\varepsilon_{n,\ell}(\vec{p}_z)-\varepsilon_{n',\ell'}(\vec{p}_z-\vec{q}_z)+\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta\right](t-t_2)\right\}\Bigg\},\quad(12)$$

where a_{1z} and a_{2z} are the z-components of $\vec{a}_1=\frac{4e\vec{E}_{01}}{m^*(\Omega_1+\Omega_2)^2}$ and $\vec{a}_2=\frac{e\vec{E}_{02}}{m^*\Omega_2^2}$, respectively. $N_{\vec{q}}$ is the balanced distribution function of phonons, $\alpha'=\arctan(\frac{F_1-F_2}{F_1+F_2}\tan(\frac{\Delta\Omega}{2}\tau+\frac{\Delta\alpha}{2}))=\text{const}$ and $J_k(x)$ is the Bessel function.

In Equation (12), the quantum numbers n and ℓ characterize the quantum wire. Similar equations can be found in bulk semiconductors, quantum wells and cylindrical quantum wires [5, 8–10]. The first-order tautology approximation method is used to solve this equation [7, 8]. The initial approximation of $n_{n,\ell,\vec{p}_z}(t)$ is chosen as

$$n^0_{n,\ell,\vec{p}_z}(t_2)=\bar{n}_{n,\ell,\vec{p}_z},\,n^0_{n,\ell,\vec{p}_z+\vec{q}_z}(t_2)=\bar{n}_{n,\ell,\vec{p}_z+\vec{q}_z},\,n^0_{n,\ell,\vec{p}_z-\vec{q}_z}(t_2)=\bar{n}_{n,\ell,\vec{p}_z-\vec{q}_z}.$$

The first-order tautology approximation method is implemented as follows: instead of the initial approximation of $n_{n,\ell,\vec{p}_z}(t)$ to the right hand side of the Equation (12), then integrating the Equation (12), we will receive first-order approximation of $n_{n,\ell,\vec{p}_z}(t)$ and we stop here. Then, the expression for the unbalanced electron distribution function $n_{n,\ell,\vec{p}_z}(t)$ is received as follows:

$$n_{n,\ell,\vec{p}_z}(t)=\bar{n}_{n,\ell,\vec{p}_z}-\frac{1}{\hbar}\sum_{n',\ell',\vec{q}}\left|C_{\vec{q}}\right|^2\left|I_{n,\ell,n',\ell'}(\vec{q}_\perp)\right|^2\sum_{k,s,r,m=-\infty}^{+\infty}J_s(a_{1z}q_z)J_{k+s}(a_{1z}q_z)J_m(a_{2z}q_z)J_{r+m}(a_{2z}q_z)$$
$$\times\frac{\exp\left\{-i\left\{\left[k\frac{\Omega_1+\Omega_2}{2}+r\omega+i\delta\right]t+k\left(\frac{\alpha_1+\alpha_2}{2}+\alpha'\right)\right\}\right\}}{k\frac{\Omega_1+\Omega_2}{2}+r\omega+i\delta}$$
$$\times\left\{\frac{\bar{n}_{n',\ell',\vec{p}_z-\vec{q}_z}N_{\vec{q}}-\bar{n}_{n,\ell,\vec{p}_z}(N_{\vec{q}}+1)}{\varepsilon_{n,\ell}(\vec{p}_z)-\varepsilon_{n',\ell'}(\vec{p}_z-\vec{q}_z)-\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta}\right.$$
$$+\frac{\bar{n}_{n',\ell',\vec{p}_z-\vec{q}_z}(N_{\vec{q}}+1)-\bar{n}_{n,\ell,\vec{p}_z}N_{\vec{q}}}{\varepsilon_{n,\ell}(\vec{p}_z)-\varepsilon_{n',\ell'}(\vec{p}_z-\vec{q}_z)+\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta}$$
$$-\frac{\bar{n}_{n,\ell,\vec{p}_z}N_{\vec{q}}-\bar{n}_{n',\ell',\vec{p}_z+\vec{q}_z}(N_{\vec{q}}+1)}{\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z)-\varepsilon_{n,\ell}(\vec{p}_z)-\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta}$$
$$\left.-\frac{\bar{n}_{n,\ell,\vec{p}_z}(N_{\vec{q}}+1)-\bar{n}_{n',\ell',\vec{p}_z+\vec{q}_z}N_{\vec{q}}}{\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z)-\varepsilon_{n,\ell}(\vec{p}_z)+\hbar\omega_{\vec{q}}-s\hbar\frac{\Omega_1+\Omega_2}{2}-m\hbar\omega+i\hbar\delta}\right\},\quad(13)$$

where $\bar{n}_{n,\ell,\vec{p}_z}$ is the balanced distribution function of electrons, and the quantity δ is an infinitesimal and appears due to the assumption of an adiabatic interaction of the EMW.

Substituting $n_{n,\ell,\vec{p}_z}(t)$ into the expression for $\vec{j}_z(t)$, we calculate the ACF of the weak EMW by using Equation (9). The resulting ACF of a weak EMW in the presence of laser radiation modulated by amplitude in a RQW can be written as:

$$\alpha=\frac{e^4n_0\omega_0}{2\pi\varepsilon_0c\sqrt{2\pi\chi_\infty m^*k_BT}m^*\omega^3Z_1Z_2}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right)\cos^2\beta_2\sum_{n,\ell,n',\ell'=1}^{+\infty}A_{n,\ell,n',\ell'}\times(B_1-B_{-1})\quad(14)$$

With:

$$B_1 = Q_{0,1} - \frac{1}{2}F_{0,1} + \frac{3}{32}M_{0,1} + \frac{1}{4}\left(F_{-1,1} + F_{1,1}\right) - \frac{1}{16}\left(M_{-1,1} + M_{1,1}\right) + \frac{1}{64}\left(M_{-2,1} + M_{2,1}\right)$$

Quantity B_1 includes the contributions of the absorption processes of a photon of weak EMW; the absorption processes and emission processes of none photon, a photon, two photons of strong EMW modulated by amplitude.

$$B_{-1} = Q_{0,-1} - \frac{1}{2}F_{0,-1} + \frac{3}{32}M_{0,-1} + \frac{1}{4}\left(F_{-1,-1} + F_{1,-1}\right) - \frac{1}{16}\left(M_{-1,-1} + M_{1,-1}\right) + \frac{1}{64}\left(M_{-2,-1} + M_{2,-1}\right)$$

Quantity B_{-1} includes the contributions of the emission processes of a photon of weak EMW; the absorption processes and emission processes of none photon, a photon, two photons of strong EMW modulated by amplitude.

In the above formulas:

$$Q_{s,m} = e^{-\frac{R_{s,m}}{2k_BT}} K_0\left(\frac{|R_{s,m}|}{2k_BT}\right)\left[e^{-\frac{\hbar^2\pi^2}{2m^*k_BT}\left(\frac{n^2}{L_x^2}+\frac{\ell^2}{L_y^2}\right)}\left(N_{\omega_0}+1\right) - e^{-\frac{\frac{\hbar^2\pi^2}{2m^*}\left(\frac{n'^2}{L_x^2}+\frac{\ell'^2}{L_y^2}\right)-R_{s,m}}{k_BT}} N_{\omega_0}\right]$$

$$F_{s,m} = a_1^2\cos^2\beta_1 e^{-\frac{R_{s,m}}{2k_BT}}\left(\frac{4m^{*2}R_{s,m}^2}{\hbar^4}\right)^{\frac{1}{2}} K_1\left(\frac{|R_{s,m}|}{2k_BT}\right)$$
$$\left[e^{-\frac{\hbar^2\pi^2}{2m^*k_BT}\left(\frac{n^2}{L_x^2}+\frac{\ell^2}{L_y^2}\right)}\left(N_{\omega_0}+1\right) - e^{-\frac{\frac{\hbar^2\pi^2}{2m^*}\left(\frac{n'^2}{L_x^2}+\frac{\ell'^2}{L_y^2}\right)-R_{s,m}}{k_BT}} N_{\omega_0}\right]$$

$$M_{s,m} = a_1^4\cos^4\beta_1 e^{-\frac{R_{s,m}}{2k_BT}}\left(\frac{4m^{*2}R_{s,m}^2}{\hbar^4}\right) K_2\left(\frac{|R_{s,m}|}{2k_BT}\right)$$
$$\left[e^{-\frac{\hbar^2\pi^2}{2m^*k_BT}\left(\frac{n^2}{L_x^2}+\frac{\ell^2}{L_y^2}\right)}\left(N_{\omega_0}+1\right) - e^{-\frac{\frac{\hbar^2\pi^2}{2m^*}\left(\frac{n'^2}{L_x^2}+\frac{\ell'^2}{L_y^2}\right)-R_{s,m}}{k_BT}} N_{\omega_0}\right]$$

In formulas for $Q_{s,m}$, $F_{s,m}$, $M_{s,m}$, we obtain contribution of the Bose-Einstein distribution function for optical phonons $N_{\omega_0} = \frac{1}{e^{\frac{\hbar\omega_0}{k_BT}}-1}$.

$R_{s,m} = \frac{\hbar^2\pi^2}{2m^*}\left(\frac{n'^2-n^2}{L_x^2} + \frac{\ell'^2-\ell^2}{L_y^2}\right) + \hbar\omega_0 - s\hbar\frac{\Omega_1+\Omega_2}{2} - m\hbar\omega$, with $s = -2, -1, 0, 1, 2$; $m = -1, 1$.

Quantity $R_{s,m}$ includes contributions of the quantized energy in the restricted directions before and after scattering, phonon energy, photon energy of three EMWs. $a_1 = \frac{4e\sqrt{F_1^2+F_2^2+2F_1F_2\cos(\Delta\Omega\tau+\Delta\alpha)}}{m^*(\Omega_1+\Omega_2)^2}$; $A_{n,\ell,n',\ell'} = [C_1(1-\delta_{n,n'}) + L_1\delta_{n,n'}][C_2(1-\delta_{\ell,\ell'}) + L_2\delta_{\ell,\ell'}]$, where $C_1 = \frac{1}{L_x}[\frac{\pi}{3} + \frac{(n^2+n'^2)}{2\pi(n^2-n'^2)^2} + \frac{5(n^2+n'^2)}{2\pi n^2n'^2}]$, $C_2 = \frac{1}{L_y}[\frac{\pi}{3} + \frac{(\ell^2+\ell'^2)}{2\pi(\ell^2-\ell'^2)^2} + \frac{5(\ell^2+\ell'^2)}{2\pi\ell^2\ell'^2}]$, $L_1 = \frac{1}{L_x}(\frac{3\pi}{2} + \frac{105}{16\pi n^2})$, $L_2 = \frac{1}{L_y}(\frac{3\pi}{2} + \frac{105}{16\pi\ell^2})$, $Z_1 = \sum_{n=1}^{+\infty} e^{-\frac{\hbar^2\pi^2n^2}{2m^*k_BTL_x^2}}$; $Z_2 = \sum_{\ell=1}^{+\infty} e^{-\frac{\hbar^2\pi^2\ell^2}{2m^*k_BTL_y^2}}$.

Quantities Z_1, Z_2 appear while we standardize balance distribution function of the electron. k_B is the Boltzmann constant, n_0 is the electron density in RQW. β_1 is the angle between the vector $\vec{E}_{01}$ and the positive direction of the Oz axis, β_2 is the angle between the vector $\vec{E}_{02}$ and the positive direction of the Oz axis. $\vec{F}_1$ and $\vec{F}_2$ are the intensities of two laser radiations that create laser radiation modulated by amplitude (with the intensity $\vec{E}_{01}$ and the frequency Ω).

Equation (14) is the expression of ACF of a weak EMW in the presence of external laser radiation modulated by amplitude in a RQW. From this expression, we see that ACF of a weak EMW is independent of E_{02}; only dependent on F_1, F_2, Ω_1, Ω_2, ω, $\Delta\Omega$, T, L_x and L_y. This expression is different from expressions in the published works for the normal bulk semiconductors, quantum wells and cylindrical quantum wires [8–10]. When $\Delta\Omega = 0$, the above results will come back the case of absorption of a weak EMW in the presence of non-modulated laser radiation. From expression (14), when we set $F_1 = 0$ and $F_2 = 0$, we will receive expression of ACF of a weak EMW in the absence of laser radiation in RQW. In Section 3, we will show clearly that under the influence of laser radiation (non-modulated or modulated by amplitude), under proper conditions, ACF of a weak EMW can gets negative values, i.e., the weak EMW can be increased; and in some conditions, when laser radiation is modulated by amplitude, ability to increase a weak EMW can be enhanced in comparison with the use of non-modulated laser radiation.

3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, in order to show clearly that ACF of a weak EMW can gets negative values and in some conditions, when laser radiation is modulated by amplitude, ability to increase a weak EMW can be enhanced in comparison with the use of non-modulated laser radiation, we numerically calculated the ACF for the specific case of a $GaAs/GaAsAl$ RQW. The parameters used in the calculations are as follows [13]: $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m^* = 0.066m_0$, m_0 being the mass of free electron, $n_0 = 10^{23}\,\mathrm{m}^{-3}$, $\hbar\omega_0 = 36.25\,\mathrm{meV}$, $\beta_1 = \frac{\pi}{3}$, $\beta_2 = \frac{\pi}{6}$, $\alpha_1 = \frac{\pi}{3}$, $\alpha_2 = \frac{\pi}{6}$.

Figure 1 describes the dependence of α on the temperature T, with $L_x = 24\,\mathrm{nm}$, $L_y = 26\,\mathrm{nm}$, $\omega = 10^{13}\,\mathrm{Hz}$, $F_1 = 10.10^6\,\mathrm{V/m}$, $F_2 = 15.10^6\,\mathrm{V/m}$, $\Omega_1 = 3.10^{13}\,\mathrm{Hz}$, Ω_2 get five different values: $2.6 \times 10^{13}\,\mathrm{Hz}$, $3 \times 10^{13}\,\mathrm{Hz}$, $3.4 \times 10^{13}\,\mathrm{Hz}$, $3.8 \times 10^{13}\,\mathrm{Hz}$, $4 \times 10^{13}\,\mathrm{Hz}$. The five different values of Ω_2 corresponding to the five different values of $\Delta\Omega$: $0.4 \times 10^{13}\,\mathrm{Hz}$, $0\,\mathrm{Hz}$, $-0.4 \times 10^{13}\,\mathrm{Hz}$, $-0.8 \times 10^{13}\,\mathrm{Hz}$, $-10^{13}\,\mathrm{Hz}$. From the Figure 1, we see that there are the temperature regions at which the ACF of a weak EMW in the presence of laser radiation modulated by amplitude is smaller than one in the presence of non-modulated laser radiation and that there are the temperature regions at which the ACF of a weak EMW in the presence of laser radiation modulated by amplitude gets values greater than one for the case of non-modulated laser radiation. In addition, ACF can gets negative values, i.e., ACF of a weak EMW becomes increased coefficient of a weak EMW. For example, when $T > 108\,\mathrm{K}$, the ACF of a weak EMW gets negative values, and the ACF of a weak EMW in the presence of laser radiation modulated by amplitude (with four cases of $\Delta\Omega \neq 0\,\mathrm{Hz}$) gets values greater than one for the case of non-modulated laser radiation ($\Delta\Omega = 0\,\mathrm{Hz}$). It means that ability to increase a weak EMW in the presence of laser radiation modulated by amplitude is decreased in comparison with one for the case of non-modulated laser radiation. When $94\,\mathrm{K} < T < 108\,\mathrm{K}$, the ACF of a weak EMW in the presence of laser radiation modulated by amplitude (with $\Delta\Omega = -0.4 \times 10^{13}\,\mathrm{Hz}$) is more negative than one for the case of non-modulated laser radiation ($\Delta\Omega = 0\,\mathrm{Hz}$), i.e., ability to increase a weak EMW is enhanced when strong EMW is modulated by amplitude with $\Delta\Omega = -0.4 \times 10^{13}\,\mathrm{Hz}$.

Figure 2 describes the dependence of α on $\Delta\Omega$ ($|\Delta\Omega|$ is the modulated frequency), also with the above conditions and seven different values of T. From the Figure 2, we see that the curves can have a maximum or a minimum in the investigative interval.

Both Figures 1 and 2 show that at high temperature region, ACF is almost independent of $\Delta\Omega$, i.e., the amplitude modulation of laser radiation hardly affects ability to increase a weak EMW in the presence of laser radiation.

Figure 3 describes the dependence of α on the frequency Ω_1 of either laser radiation with $L_x = 24\,\mathrm{nm}$, $L_y = 26\,\mathrm{nm}$, $T = 95\,\mathrm{K}$, $F_1 = 10.10^6\,\mathrm{V/m}$, $F_2 = 15.10^6\,\mathrm{V/m}$, $\omega = 10^{13}\,\mathrm{Hz}$ and two different values of $\Delta\Omega$. The curves in this figure have a maximum and a minimum in the investigative interval. This figure show that the ACF of a weak EMW can have negative values. When $\Omega_1 \in \left[3 \times 10^{13}, 3.4 \times 10^{13}\right]\,\mathrm{Hz}$, ACF of a weak EMW in the presence of strong EMW modulated by amplitude (with $\Delta\Omega = -0.4 \times 10^{13}\,\mathrm{Hz}$) is more negative than one in the presence of non-modulated strong EMW ($\Delta\Omega = 0\,\mathrm{Hz}$), i.e., ability to increase a weak EMW is enhanced when strong EMW is modulated by amplitude with $\Delta\Omega = -0.4 \times 10^{13}\,\mathrm{Hz}$.

Figure 4 describes the dependence of α on the frequency ω of the weak EMW, with $L_x = 24\,\mathrm{nm}$, $L_y = 26\,\mathrm{nm}$, $T = 30\,\mathrm{K}$, $F_1 = 5.10^6\,\mathrm{V/m}$, $F_2 = 11.10^6\,\mathrm{V/m}$, $\Omega_1 = 3.10^{13}\,\mathrm{Hz}$, and five different values of Ω_2 corresponding to the five different values of $\Delta\Omega$. From Figure 4, we see that the curves have a maximum (peak) at $\omega = \omega_0$ and smaller maxima (peaks) at $\omega \neq \omega_0$. In this figure, we also see that the ACF of a weak EMW can have negative values and that there are the frequency ω regions at which the ACF of a weak EMW in the presence of laser radiation modulated by amplitude is smaller or greater than one in the presence of non-modulated laser radiation.

Figure 5 describes the dependence of α on the intensity F_1 of either laser radiation with $L_x = 24\,\text{nm}$, $L_y = 26\,\text{nm}$, $T = 95\,\text{K}$, $F_2 = 3.10^6\,\text{V/m}$, $\omega = 10^{13}\,\text{Hz}$, $\Omega_1 = 3.10^{13}\,\text{Hz}$, and five different values of Ω_2 corresponding to the five different values of $\Delta\Omega$. From Figure 5, we also see that when strong EMW is modulated by amplitude, there are the F_1 regions at which ability to increase a weak EMW is enhanced

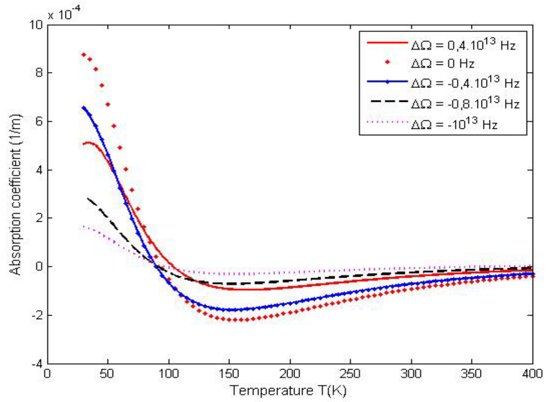

Figure 1. The dependence of α on T.

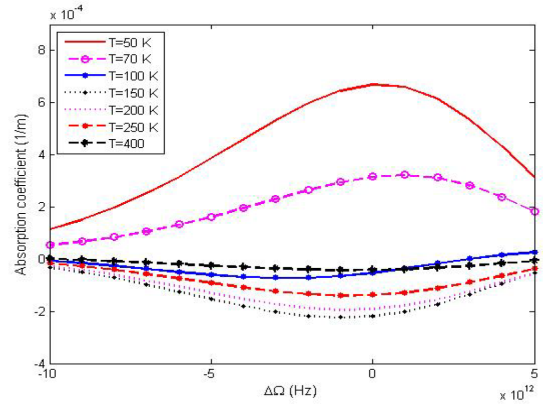

Figure 2. The dependence of α on $\Delta\Omega$.

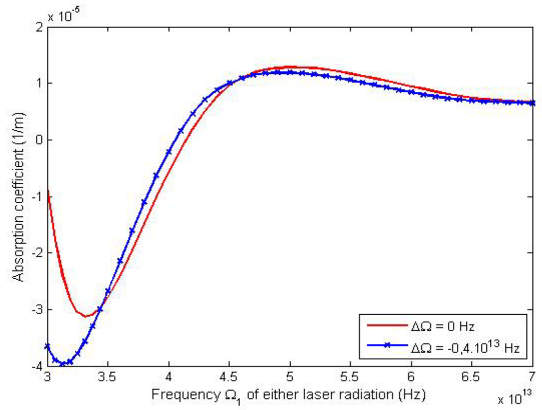

Figure 3. The dependence of α on Ω_1.

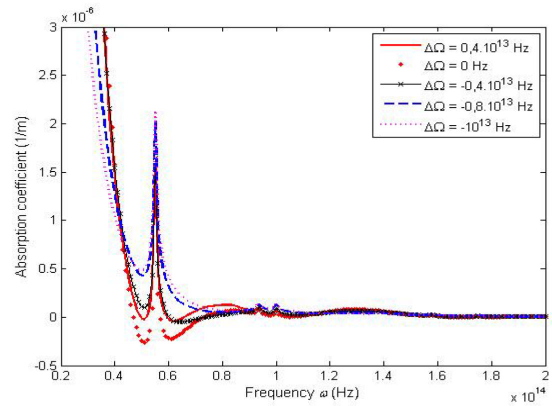

Figure 4. The dependence of α on ω.

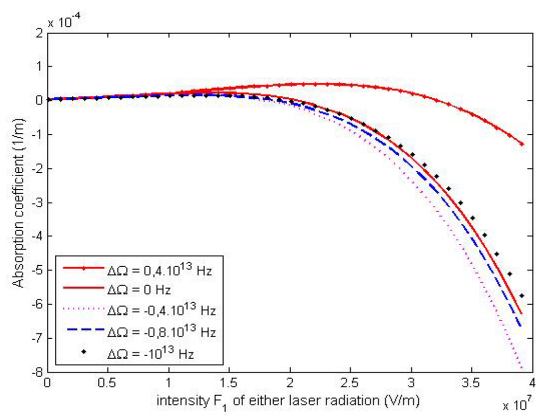

Figure 5. The dependence of α on F_1.

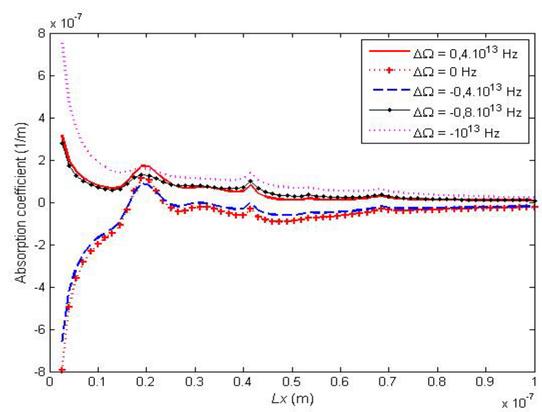

Figure 6. The dependence of α on L_x.

and that there are the F_1 regions at which ability to increase a weak EMW is decreased in comparison with the case of non-modulated laser radiation.

Figure 6 describes the dependence of α on the L_x with $L_y = 26\,\text{nm}$, $T = 90\,\text{K}$, $F_1 = 7.10^6\,\text{V/m}$, $F_2 = 11.10^6\,\text{V/m}$, $\omega = 7.10^{13}\,\text{Hz}$, $\Omega_1 = 3.10^{13}\,\text{Hz}$, and five different values of Ω_2 corresponding to the five different values of $\Delta\Omega$. From this figure, we also see that the curves have many maxima (peaks). Figure 6 also shows that the ACF of a weak EMW can have negative values and that in some conditions, ability to increase a weak EMW can be decreased when strong EMW is modulated by amplitude. When $L_x \in [1, 16]\,\text{nm}$ or $L_x \in [24, 100]\,\text{nm}$, the ACF of a weak EMW in the presence of non-modulated strong EMW gets negative values, i.e., the weak EMW is increased; but when strong EMW is modulated by amplitude (with $\Delta\Omega = -10^{13}\,\text{Hz}$, $-0.8 \times 10^{13}\,\text{Hz}$, $0.4 \times 10^{13}\,\text{Hz}$), the ACF of a weak EMW gets positive values, i.e., the weak EMW is absorbed.

Summary, under influence of laser radiation (non-modulated or modulated by amplitude), ACF of a weak EMW in a RQW can have negative values. Negative ACF of a weak EMW speaks ability to increase a weak EMW. This is different from the similar problem in bulk semiconductors and from the case of the absence of laser radiation. This effect has also appeared in quantum wells and cylindrical quantum wires [9, 10]. In addition, if laser radiation is modulated by amplitude, in some conditions, ability to increase a weak EMW can be enhanced.

4. CONCLUSIONS

In this research, we investigate negative absorption coefficient of a weak EMW caused by electrons confined in RQWs in the presence of laser radiation modulated by amplitude. We obtain an analytical expression of the ACF of a weak EMW in the presence of laser radiation modulated by amplitude in a RQW for the case of electron-optical phonon scattering. The expression shows that the ACF of a weak EMW is independent of E_{02} and is only dependent on F_1, F_2, Ω_1, Ω_2, ω, $\Delta\Omega$, T, L_x and L_y. From this expression, we can receive the expression of the ACF of a weak EMW in the absence of laser radiation in RQW by setting $F_1 = 0$ and $F_2 = 0$. The ACF is numerically calculated for the specific case of $GaAs/GaAsAl$ RQW. These results show that under the influence of laser radiation (non-modulated or modulated by amplitude), the ACF of a weak EMW in a RQW can have negative values. Negative ACF of a weak EMW speaks ability to increase a weak EMW in the presence of laser radiation. This is different from a similar problem in bulk semiconductors and from the case without laser radiation. If laser radiation is modulated by amplitude, in some conditions, the ACF can get more negative values than one for the case of non-modulated laser radiation, i.e., ability to increase a weak EMW can be enhanced in comparison with the case of non-modulated laser radiation. So, when we want to enhance ability to increase a weak EMW, we need only to modulate amplitude of laser radiation and chose proper conditions of system.

ACKNOWLEDGMENT

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APPENDIX A.

In this appendix, we write out some steps to obtain Equation (12) as follows.

Replacing Hamiltonian H into Equation (10), we will obtain the following equation:

$$\frac{\partial n_{n,\ell,\vec{p}_z}(t)}{\partial t} = -\frac{1}{\hbar^2} \sum_{n',\ell',\vec{q}} C_{\vec{q}} I_{n,\ell,n',\ell'}(\vec{q}_\perp) \int_{-\infty}^{t} (H_1 + H_2 + H_3 + H_4)\, dt_2 \tag{A1}$$

where:

$$H_1 = \left[\sum_{n_4,\ell_4,\vec{q}_1} C_{\vec{q}_1} I_{n',\ell',n_4,\ell_4}(\vec{q}_{1\perp}) \left\langle a^+_{n_4,\ell_4,\vec{p}_z+\vec{q}_z+\vec{q}_{1z}} a_{n,\ell,\vec{p}_z} \left(b_{\vec{q}_1} + b^+_{-\vec{q}_1} \right) b_{\vec{q}} \right\rangle_{t_2} \right.$$

$$- \sum_{n_3,\ell_3,\vec{q}_1} C_{\vec{q}_1} I_{n,\ell,n_3,\ell_3}(\vec{q}_{1\perp}) \Big\langle a^+_{n',\ell',\vec{p}_z+\vec{q}_z} a_{n_3,\ell_3,\vec{p}_z-\vec{q}_{1z}} b_{\vec{q}} \left(b_{\vec{q}_1} + b^+_{-\vec{q}_1}\right)\Big\rangle_{t_2} \Bigg]$$

$$\times e^{\frac{i}{\hbar}\left[\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z)-\varepsilon_{n,\ell}(\vec{p}_z)-\hbar\omega_{\vec{q}}\right](t-t_2)-\frac{ie}{m^*c}\int\limits_{t_2}^{t}\vec{q}_z\vec{A}_z(t_1)dt_1}$$

$$H_2 = -\Bigg[\sum_{n_4,\ell_4,\vec{q}_1} C^*_{\vec{q}_1} I^*_{n,\ell,n_4,\ell_4}(\vec{q}_{1\perp}) \Big\langle a^+_{n',\ell',\vec{p}_z+\vec{q}_z} a_{n_4,\ell_4,\vec{p}_z+\vec{q}_{1z}} b^+_{-\vec{q}} \left(b^+_{\vec{q}_1} + b_{-\vec{q}_1}\right)\Big\rangle_{t_2}$$

$$- \sum_{n_3,\ell_3,\vec{q}_1} C^*_{\vec{q}_1} I^*_{n',\ell',n_3,\ell_3}(\vec{q}_{1\perp}) \Big\langle a^+_{n_3,\ell_3,\vec{p}_z+\vec{q}_z-\vec{q}_{1z}} a_{n,\ell,\vec{p}_z} \left(b^+_{\vec{q}_1} + b_{-\vec{q}_1}\right) b^+_{-\vec{q}}\Big\rangle_{t_2} \Bigg]$$

$$\times e^{-\frac{i}{\hbar}\left[\varepsilon_{n,\ell}(\vec{p}_z)-\varepsilon_{n',\ell'}(\vec{p}_z+\vec{q}_z)-\hbar\omega_{\vec{q}}\right](t-t_2)-\frac{ie}{m^*c}\int\limits_{t_2}^{t}\vec{q}_z\vec{A}_z(t_1)dt_1}$$

$$H_3 = -\Bigg[\sum_{n_4,\ell_4,\vec{q}_1} C_{\vec{q}_1} I_{n,\ell,n_4,\ell_4}(\vec{q}_{1\perp}) \Big\langle a^+_{n_4,\ell_4,\vec{p}_z+\vec{q}_{1z}} a_{n',\ell',\vec{p}_z-\vec{q}_z} \left(b_{\vec{q}_1} + b^+_{-\vec{q}_1}\right) b_{\vec{q}}\Big\rangle_{t_2}$$

$$- \sum_{n_3,\ell_3,\vec{q}_1} C_{\vec{q}_1} I_{n',\ell',n_3,\ell_3}(\vec{q}_{1\perp}) \Big\langle a^+_{n,\ell,\vec{p}_z} a_{n_3,\ell_3,\vec{p}_z-\vec{q}_z-\vec{q}_{1z}} b_{\vec{q}} \left(b_{\vec{q}_1} + b^+_{-\vec{q}_1}\right)\Big\rangle_{t_2} \Bigg]$$

$$\times e^{\frac{i}{\hbar}\left[\varepsilon_{n,\ell}(\vec{p}_z)-\varepsilon_{n',\ell'}(\vec{p}_z-\vec{q}_z)-\hbar\omega_{\vec{q}}\right](t-t_2)-\frac{ie}{m^*c}\int\limits_{t_2}^{t}\vec{q}_z\vec{A}_z(t_1)dt_1}$$

$$H_4 = \Bigg[\sum_{n_4,\ell_4,\vec{q}_1} C^*_{\vec{q}_1} I^*_{n',\ell',n_4,\ell_4}(\vec{q}_{1\perp}) \Big\langle a^+_{n,\ell,\vec{p}_z} a_{n_4,\ell_4,\vec{p}_z-\vec{q}_z+\vec{q}_{1z}} b^+_{-\vec{q}} \left(b^+_{\vec{q}_1} + b_{-\vec{q}_1}\right)\Big\rangle_{t_2}$$

$$- \sum_{n_3,\ell_3,\vec{q}_1} C^*_{\vec{q}_1} I^*_{n,\ell,n_3,\ell_3}(\vec{q}_{1\perp}) \Big\langle a^+_{n_3,\ell_3,\vec{p}_z-\vec{q}_{1z}} a_{n',\ell',\vec{p}_z-\vec{q}_z} \left(b^+_{\vec{q}_1} + b_{-\vec{q}_1}\right) b^+_{-\vec{q}}\Big\rangle_{t_2} \Bigg]$$

$$\times e^{-\frac{i}{\hbar}\left[\varepsilon_{n',\ell'}(\vec{p}_z-\vec{q}_z)-\varepsilon_{n,\ell}(\vec{p}_z)-\hbar\omega_{\vec{q}}\right](t-t_2)-\frac{ie}{m^*c}\int\limits_{t_2}^{t}\vec{q}_z\vec{A}_z(t_1)dt_1}$$

Equation (A1) is the quantum kinetic equation for electrons in the RQW. We will approximate this equation to the second order of $C_{\vec{q}}$. So, in the expression H_1 we keep only term with $\vec{q}_1 = -\vec{q}$, $n_4 = n$, $\ell_4 = \ell$, $n_3 = n'$, $\ell_3 = \ell'$; in the expression H_2 we keep only term with $\vec{q}_1 = \vec{q}$, $n_4 = n'$, $\ell_4 = \ell'$, $n_3 = n$, $\ell_3 = \ell$; in the expression H_3 we keep only term with $\vec{q}_1 = -\vec{q}$, $n_4 = n'$, $\ell_4 = \ell'$, $n_3 = n$, $\ell_3 = \ell$; in the expression H_4 we keep only term with $\vec{q}_1 = \vec{q}$, $n_4 = n$, $\ell_4 = \ell$, $n_3 = n'$, $\ell_3 = \ell'$; and we make the following approximation: $\langle b^+_{\vec{q}} b_{\vec{q}}\rangle_{t_2} \approx N_{\vec{q}}$, $\langle b^+_{-\vec{q}} b_{-\vec{q}}\rangle_{t_2} \approx N_{-\vec{q}} = N_{\vec{q}}$, $\langle b_{\vec{q}} b^+_{\vec{q}}\rangle_{t_2} = \langle 1 + b^+_{\vec{q}} b_{\vec{q}}\rangle_{t_2} \approx 1 + N_{\vec{q}}$; and ignore $\langle b_{-\vec{q}} b_{\vec{q}}\rangle_{t_2}$, $\langle b^+_{-\vec{q}} b^+_{\vec{q}}\rangle_{t_2}$. Then, we will get Equation (A1) in the approximation to second order of $C_{\vec{q}}$.

The vector potential $\vec{A}(t)$ of field of the three EMWs is found from the following equation: $-\frac{1}{c}\frac{d\vec{A}(t)}{dt} = \vec{E}(t)$.

We replace $\vec{A}_z(t) = (0, 0, A_z)$ into the approximate equation of $n_{n,\ell,\vec{p}_z}(t)$ that has been just found above, where A_z is the z-component of $\vec{A}(t)$. In addition, we use expansion: $e^{\pm \mathrm{i}z\sin\varphi} = \sum\limits_{m=-\infty}^{+\infty} J_m(z)e^{\pm im\varphi}$. Then, we will obtain Equation (12), and it is the quantum kinetic equation for electrons in the RQW in the approximation to second order of $C_{\vec{q}}$.

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