Progress In Electromagnetics Research M, Vol. 36, 161–168, 2014

Effect of Dust Grain Parameters on Ion Beam Driven Ion Cyclotron Waves in a Magnetized Plasma

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Abstract—Excitation of electrostatic ion cyclotron waves (EICW) in a magnetized dusty plasma by an ion beam is studied taking into account the effect of dust particle size, dust particle charge and dust particle number density variations. The presence of dust grain charge fluctuations modifies the dispersion relation for ion cyclotron waves in dusty plasma. It is shown that in the absence of ion beam, the ion cyclotron mode damps due to dust charge fluctuations and an additional damping dust charge fluctuation mode is induced in plasma. The ion beam propagating parallel to the magnetic field drives ion cyclotron waves to instability via Cerenkov interaction. Using the analytical and numerical results the influence of the relative density of negatively charged dust particles on growth rate of ion cyclotron waves is studied. The dust grain size distribution has also significant contributions on the growth rate of ion cyclotron waves.

1. INTRODUCTION

Dusty plasmas are fully or partially ionized low-temperature gases comprising of electrons, ions, neutral molecules and electrically charged extremely heavy dust grains. It is well established that the presence of dust grains can considerably change collective properties of a plasma. The dust charging process introduces new physics by modifying plasma dielectric properties [1–3]. The most affected are plasma modes with frequencies of the order of dust charging frequency. The growth rate of the instabilities is found to be dust-dependent [4–9]. Moreover, in real situations, dust grains exhibit a size distribution. Dust particles usually attach plasma free electrons inducing a new plasma equilibrium and the occurrence of new phenomena.

There has been a great deal of interest in studying the collective properties of a dusty plasma [10–15], particularly for low frequency wave propagation. The dust charge may fluctuate due to a variety of reasons. Several authors have tried to incorporate the effect of these charge fluctuations on wave propagation and found interesting results [16–19]. Cui and Goree [18] have developed a numerical simulation code to characterize the charge fluctuation. They have used a discrete charging model where the grain charge fluctuates due to the random absorption of ions and electrons at the grain surface. D'Angelo [20] has examined the Rayleigh-Taylor instability in a dusty plasma and found that the presence of negatively charged dust decreases the range of unstable wavenumbers. Tsytovich and Havnes [1] has considered a novel kinetic effect arising from inhomogeneities in dust charges and demonstrated collisionless drift wave instability. Mahanta et al. [21] have showed that dust charge fluctuation lead to damping of lower-hybrid like waves in cold dusty plasma. Islam et al. [22] have found that the damping of lower-hybrid mode in a magnetized dusty plasma is the sum of the Landau damping and the dust charge fluctuation effect. Chow and Rosenberg [23] have investigated the effect of

Received 10 April 2014, Accepted 21 May 2014, Scheduled 27 May 2014
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charged dust on the collisionless electrostatic ion cyclotron instability and have showed that the critical electron drift decreases as the charge density carried by negatively charged dust increases, but in the absence of dust charge fluctuations.

In this paper, we consider electrostatic ion cyclotron waves (EICW) in dusty, homogeneous plasma immersed in a static and uniform magnetic field. Effects of dust charge fluctuations, temperature of plasma electrons, dust grain size, number density of dust grains, and ion beam velocity on EICW are studied using fluid theory. Section 2 presents the instability analysis and derive the dispersion relation of EICW in the presence of dust charge fluctuations. The growth rate of the instability has been obtained using first order perturbation technique. Section 3 gives a brief discussion of our results and comments on the excitation of electrostatic ion cyclotron instability in the presence of dust charge fluctuations by an ion beam.

2. INSTABILITY ANALYSIS

We consider a homogeneous and uniformly magnetized dusty plasma consisting of electrons, ions and negatively charged dust grains. An ion beam is propagating along z-axis parallel to the magnetic field with velocity $v_{bo}\hat{z}$. At equilibrium, the plasma system is quasineutral, i.e., $n_{io}+n_{bo} = n_{eo}+Z_d n_{do}$, where n_{io}, n_{bo}, n_{eo}, n_{do} are the number densities of ions, beam ions, electrons and dust grains, respectively, Z_d $(= Q_{do}/e)$ is the number of electrons residing on the surface of negatively charged dust grains; Q_{do} being the equilibrium dust grain charge and e is the magnitude of the electronic charge. Let us consider an electrostatic wave, such as ion-cyclotron mode, propagating nearly perpendicular to the external static magnetic field ($\mathbf{B}_s//\hat{z}$) with propagation vector $\vec{k}$ lying in the x-z plane. In the presence of perturbation $\phi = \phi_0 e^{-i(\omega t - k_x x - k_z z)}$ the equilibrium is disturbed, and the dust acquires a perturbed charge Q_{d1}. Then, the quasi-neutrality condition is given by

$$en_{i1} + en_{b1} = en_{e1} + Q_{do}n_{d1} + Q_{d1}n_{d0} \tag{1}$$

where the subscript 1 refers to a perturbed quantity due to electrostatic perturbation.

The response of plasma electrons, plasma ions, beam ions and negatively charged dust particles is governed by the equations of motion and continuity, which on linearization yield density perturbations

$$n_{e1} = \frac{\omega_{pe}^2\phi}{4\pi e v_{te}^2}, \tag{2}$$

$$n_{i1} = \frac{\omega_{pi}^2\phi}{4\pi e}\left[\frac{k_x^2}{\omega^2-\omega_{ci}^2}+\frac{k_z^2}{\omega^2}\right], \tag{3}$$

$$n_{d1} = -\frac{\omega_{pd}^2\phi}{4\pi Q_{do}}\frac{k^2}{\omega^2}, \tag{4}$$

and

$$n_{b1} = \frac{\omega_{pb}^2\phi}{4\pi e}\left[\frac{k_x^2}{\bar{\omega}^2-\omega_{ci}^2}+\frac{k_z^2}{\bar{\omega}^2}\right], \tag{5}$$

where we have assumed short parallel wavelength limit, i.e., $\omega \ll k_z v_{te}$, and where $\bar{\omega} = \omega - k_z v_{bo}$, $v_{te}^2 = \frac{T_e}{m_e}$, $\omega_{pe}^2 = \frac{4\pi n_{eo}e^2}{m_e}$, $\omega_{pi}^2 = \frac{4\pi n_{io}e^2}{m_i}$, $\omega_{ci} = \frac{eB_s}{m_i c}$, $\omega_{pd}^2 = \frac{4\pi n_{do}Q_{do}^2}{m_d}$, $\omega_{cd} = \frac{Q_{do}B_s}{m_d c}$ and $k^2 = k_x^2 + k_z^2$.

The dust response is taken as unmagnetized as $\omega \gg \omega_{cd}$. Applying probe theory to a dust grain, the charge on a dust grain is known to be balanced with plasma currents on the given surface, and following Jana et al. [16], we obtain the dust charge fluctuation,

$$Q_{d1} = \frac{i\,|I_{eo}|}{(\omega+i\eta)}\left(\frac{n_{i1}}{n_{io}}-\frac{n_{e1}}{n_{eo}}\right). \tag{6}$$

Substituting the value of n_{e1} and n_{i1} from Eqs. (2) and (3) in Eq. (6), we obtain

$$Q_{d1} = \frac{-i\,|I_{eo}|\,\phi}{(\omega+i\eta)}\left[\frac{e}{m_i}\left(\frac{k_x^2}{\omega^2-\omega_{ci}^2}+\frac{k_z^2}{\omega^2}\right)-\frac{e}{m_e v_{te}^2}\right]. \tag{7}$$

Using Eqs. (2)–(5) and Eq. (7) in the quasi-neutrality condition given by Eq. (1), we obtain

$$\begin{aligned}&\left[\omega^4-\left(\omega_{ci}^2+\frac{\omega_{pi}^2 v_{te}^2}{\omega_{pe}^2}k^2+\frac{\omega_{pd}^2 v_{te}^2}{\omega_{pe}^2}k^2\right)\omega^2+\left(\frac{\omega_{pi}^2 v_{te}^2}{\omega_{pe}^2}k_z^2\omega_{ci}^2+\frac{\omega_{pd}^2 v_{te}^2}{\omega_{pe}^2}k^2\omega_{ci}^2\right)\right](\omega+i\eta)\\&+i\beta\left[\omega^4-\left(\omega_{ci}^2+k^2C_s^2\right)\omega^2+k_z^2C_s^2\omega_{ci}^2\right]\\=\;&\frac{\omega_{pb}^2 v_{te}^2}{\omega_{pe}^2}\omega^2\left(\omega^2-\omega_{ci}^2\right)(\omega+i\eta)\left[\frac{k_x^2}{\left[(\omega-k_z v_{b0})^2-\omega_{ci}^2\right]}+\frac{k_z^2}{(\omega-k_z v_{b0})^2}\right],\end{aligned}\tag{8}$$

where $\beta=\frac{|I_{eo}|n_{do}}{en_{eo}}$ is the coupling parameter given as $\beta=0.1\pi a^2 n_{do}v_{te}$, $\eta=0.01\omega_{pe}\frac{n_{eo}}{n_{io}}\frac{a}{\lambda_D}$ is the time scale of delay and $C_s=\sqrt{\frac{T_e}{m_i}}$ is the ion acoustic speed.

Considering only Cerenkov interaction term, Eq. (8) can be rewritten in the limit $k_z \ll k$ for ICW as:

$$\left(\omega^2-\omega_1^2\right)(\omega+i\eta)+i\beta\left(\omega^2-\omega_2^2\right)=\frac{\omega_{pb}^2 v_{te}^2}{\omega_{pe}^2}\left(\omega^2-\omega_{ci}^2\right)(\omega+i\eta)\frac{k_z^2}{(\omega-k_z v_{b0})^2}$$

or

$$\left(\omega^2-\omega_1^2\right)\left(\omega+i\eta+i\beta'\right)=-i\omega_1^2\beta'+\frac{\omega_{pb}^2 v_{te}^2}{\omega_{pe}^2}\left(\omega^2-\omega_{ci}^2\right)(\omega+i\eta)\frac{k_z^2}{(\omega-k_z v_{b0})^2}\tag{9}$$

where $\beta'=\beta\left(\omega^2-\omega_2^2\right)/\omega^2$.

In the limit of vanishing beam density, Eq. (9) yield

$$\omega_1^2=\omega_{ci}^2+k^2C_s^2\frac{n_{io}}{n_{eo}}\left(1+\frac{\omega_{pd}^2}{\omega_{pi}^2}\right),\quad\text{and}\tag{10}$$

$$\omega_2^2=\omega_{ci}^2+k^2C_s^2\tag{11}$$

In the dispersion relation given by Eq. (9), the first bracket on the left hand side represents the ion cyclotron mode with dust particles having frequency $\omega=\omega_1$ and the second bracket indicates the damped low-frequency dust charge fluctuation mode. The first term on the right hand side is a coupling term proportional to the dust charge fluctuation and the second term describes the beam mode with Cerenkov term.

A perturbative solution of Eq. (9) in the absence of beam yields two roots

$$\omega=\omega_1-i\beta'/2,\tag{12}$$

and

$$\omega=-i\eta\left(1+\eta\beta'/\omega_1^2\right),\tag{13}$$

where Eq. (12) represents the ion cyclotron mode damping due to dust charge fluctuations and Eq. (13) represents the damping dust charge fluctuation mode.

For Cerenkov interaction of ICW with beam mode, $(\omega-k_z v_{bo})\approx 0$. From a perturbative solution of Eq. (9), i.e., $\omega=\omega_1+\Delta$ and $\omega=k_z v_{bo}+\Delta$, where Δ is the small frequency mismatch, we obtain

$$\Delta^2(\Delta+A+iB)=C+iD,\tag{14}$$

where $A=\beta'\frac{\omega_1}{2}\frac{(\eta+\beta')}{\left[\omega_1^2+(\eta+\beta')^2\right]}$, $B=\beta'\frac{\omega_1}{2}\frac{\omega_1}{\left[\omega_1^2+(\eta+\beta')^2\right]}$, $C=\frac{k_z^2\omega_{pb}^2 v_{te}^2\left(\omega_1^2-\omega_{ci}^2\right)}{2\omega_{pe}^2\omega_1}\frac{\omega_1^2+\eta(\eta+\beta')}{\left[\omega_1^2+(\eta+\beta')^2\right]}$, and $D=\frac{-\beta' k_z^2\omega_{pb}^2 v_{te}^2\left(\omega_1^2-\omega_{ci}^2\right)}{2\omega_{pe}^2\left[\omega_1^2+(\eta+\beta')^2\right]}$.

If β' is small, i.e., damping due to dust charge fluctuations is small or A, $B \ll \Delta$, then Eq. (14) reduces to

$$\Delta^3=C+iD$$

which yields a growth rate $\gamma=\mathrm{Im}(\Delta)$ or

$$\gamma=\left(C^2+D^2\right)^{1/6}\cos\left[\left(2\cos^{-1}(C/r)+\pi\right)/6\right]\tag{15}$$

If β' can not be neglected, i.e., dust charge fluctuations are appreciable, then A, $B \gg \Delta$ and Eq. (14) becomes

$$\Delta^2 = (C + iD)/(A + iB)$$

which yields a growth rate

$$\gamma = r^{1/2} \sin\left[\frac{1}{2}\cos^{-1}\frac{1}{r}\frac{(AC + BD)}{(A^2 + B^2)}\right] \tag{16}$$

where $r = \left[\frac{(AC+BD)^2+(BC-AD)^2}{(A^2+B^2)}\right]^{1/2}$.

3. DISCUSSION AND CONCLUSIONS

The plasma parameters used in the present calculations are: ion plasma density $n_{io} = 3 \times 10^{10}\,\mathrm{cm}^{-3}$, beam density $n_{bo} = 3 \times 10^8\,\mathrm{cm}^{-3}$, static magnetic field $B_S = 5000\,\mathrm{G}$, mass of ion $m_i = 39 \times 1836 m_e$ (K-plasma), mass of dust grain $m_d \approx 10^{12} m_i$. The relative density of negatively charged dust grains δ $(= n_{io}/n_{eo})$ has been varied from 1 to 5. The above mentioned parameters are typical dusty plasma parameters corresponding to experimental results on the electrostatic ion cyclotron instability in dusty plasma reported by Barkan et al. [25]. The dusty plasma produced in a Q-machine or in dc discharges or r_f discharges also have similar plasma parameters. The sizes of dust particles, which are observed in tokamaks TEXTOR, TFTR etc. range from 100 nm to 100 µm, therefore we have observed the variation of growth rate with the dust grain size for a maximum dust grain size of 100 µm. As an example of a space environment in which negatively charged dust can cause the ion cyclotron to become unstable is the convective current system that connects Jupiter with its satellite I_o.

We have plotted the dispersion curves of EIC waves for different values of δ using Eq. (10) along with the beam modes of different velocities, and found that they intersect each other, indicating excitation of ion cyclotron instability. Fig. 1 shows the frequency versus normalized parallel wave number k_z/k curves of EIC waves with a beam mode of velocity 2.5×10^6 cm/s propagating in z-direction. The frequencies and the corresponding normalized wave numbers are obtained from the points of intersection between the plasma mode and beam modes of different velocities, and are given as Table 1. From Table 1, we can say that the unstable frequency of the excited waves gradually increases with an increase in the value of δ.

Table 1. Unstable wave frequencies ω (rad/s) and normalized parallel wave numbers k_z/k for different values of δ from Fig. 1.

δ	k_z/k	ω (rad/s) $\times 10^6$
1	0.0491	1.279
2	0.0510	1.333
3	0.0539	1.396
4	0.0561	1.463
5	0.0593	1.547

In Fig. 2, we have plotted the growth rate γ versus normalized perpendicular wave number k_x/k for different values of δ, using Eq. (15), where we have considered the dust charge fluctuations effect as small. It can be seen that the growth rate of the unstable mode first increases with k_x/k, reaches a maximum and then decreases, for all values of δ. The value of k_x/k at maximum growth decreases as δ increases, similar to the results obtained theoretically in the absence of beam [24]. The effect of dust charge fluctuations is directly related to the parameters β and η. β is the coupling parameter which is like an effective collision frequency of electrons with the dust grains. The β terms arise due to coupling of the ion cyclotron mode with the dust charge fluctuations which result as a response to collective plasma perturbations. The charge fluctuations on dust particles have a natural decay rate of order η. Physically, the charge fluctuations decay because any deviation of the grain potential from the equilibrium floating potential is opposed by electron and ion currents into the grains.

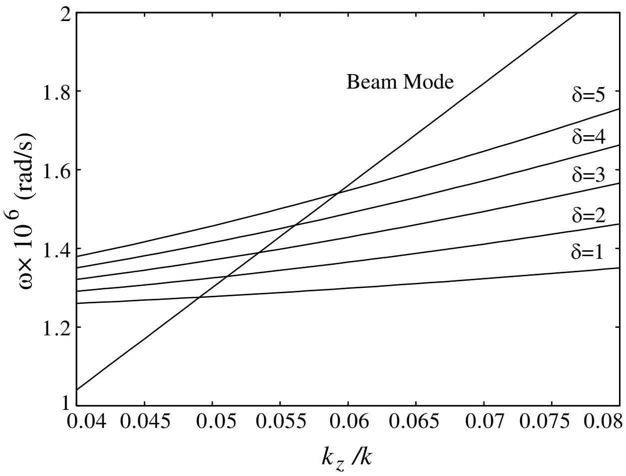

Figure 1. Dispersion curves of EIC waves over a magnetized dusty plasma for different values of δ $(= n_{io}/n_{eo})$ and beam mode of velocity 2.5×10^6 cm/s.

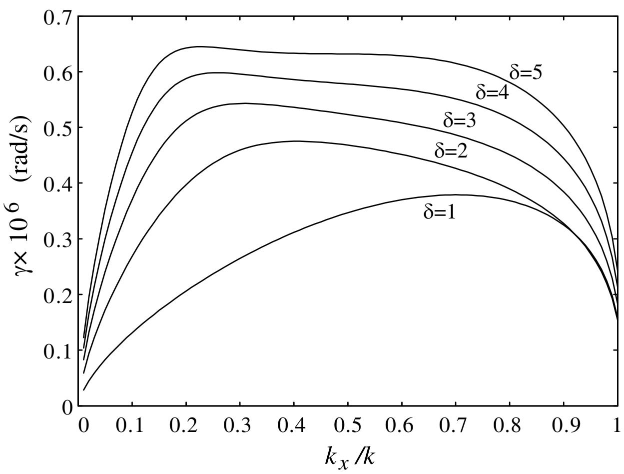

Figure 2. Growth rate of the unstable mode as a function of normalized perpendicular wave number for small dust charge fluctuations.

Considering the dust charge fluctuation effect appreciable, we have also plotted the growth rate as a function of k_x/k, using Eq. (16) in Fig. 3 for $\delta = 2$–5 ($\delta = 1$ indicates the absence of dust grains in plasma). For all the values of δ, the growth rate decreases with an increase in the value of k_x/k. The mechanism of damping is that the electrons from the ambient plasma of a dust grain will gain energy from the IC mode to flow into the dust surface to stop its charge fluctuation. The IC wave continuously gives its energy to the electrons and the waves will be damped. Also in the case of appreciable dust charge fluctuations, the beam effect and the dust charge fluctuation effect both need to be considered. When the ion beam enters the dusty plasma, it interacts with the dust grains and releases secondary electrons due to impact ionization. This reduces the average dust grain charge Q_{do} and thus leads to a decrease in growth rate. However, it can be observed from Figs. 2 and 3 that the growth rate increases with an increase in the relative density of negatively charged dust grains δ. It may be attributed to the fact that as the value of δ increases, the electron plasma density n_{eo} decreases with respect to ion plasma density n_{io}. For increased values of δ, the ions have an effective mass $m_{\text{ieff}} = \frac{m_i}{\delta}$, that is less than m_i and their greater mobility leads to increased wave generation.

Figure 4 displays the variation of growth rate with respect to dust grain size 'a' for four values of δ. The growth rate first increases with an increase in the radius of dust grains, attains a maximum value at a critical dust grain size and then decreases for a further increase in the dust grain size. By observing curves for different values of δ, we can say that the critical values of dust grain size 'a' decreases with δ. It is because, when we gradually increase 'a', the number of electrons striking the dust grain surface increases. As a result, the dust grain acquires much more electrons and its surface potential increases, increasing the average dust grain charge Q_{d0}, which increases the growth rate of IC instability. However, when dust grain radius exceeds a critical value (i.e., the intergrain spacing becomes smaller than a critical value), the dust grain surface potential starts to decrease, which decreases the average dust grain charge, and the growth rate also starts decreasing. The critical value of dust grain size decreases with increase in the value of δ i.e., the relative density of negatively charged dust grains, because the intergrain spacing reaches its critical value at lower dust grain size for large δ. From Fig. 4, it is observed that the dust grain size has a critical value of 61 μm and 37 μm for $\delta = 2$ and $\delta = 5$, respectively [10].

In Fig. 5, we have plotted the variation of growth rate with respect to the ratio of electron temperature to ion temperature T_e/T_i. The growth rate increases with an increase in the value of electron temperature, for all values of δ. For isolated dust grains, the dust grain charge Q_{d0} depends only on the dust particle size while for non-isolated dust grains, Q_{d0} does not only depend on the dust grain size, but also on the dust particle number density n_{do}. As n_{do} increases, Q_{d0} decreases more

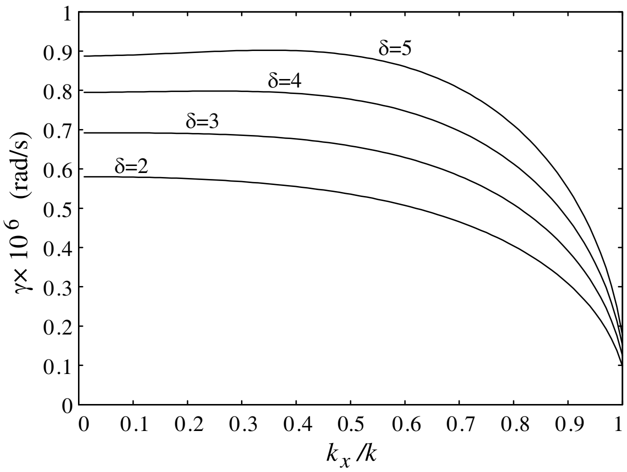

Figure 3. Growth rate of the unstable mode as a function of normalized perpendicular wave number for appreciable dust charge fluctuations.

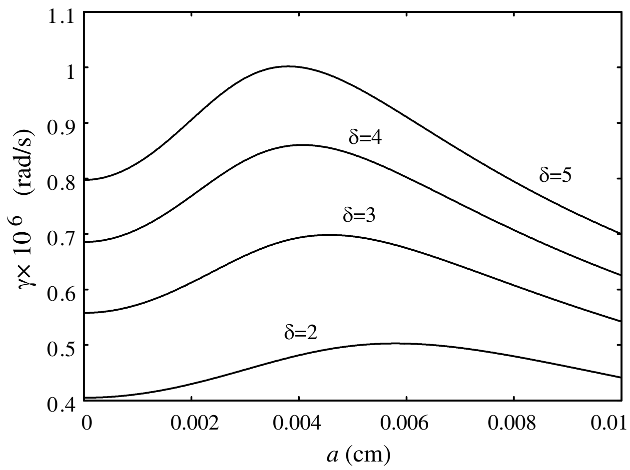

Figure 4. Comparison of growth rate of the unstable mode as a function of dust grain size for different values of δ.

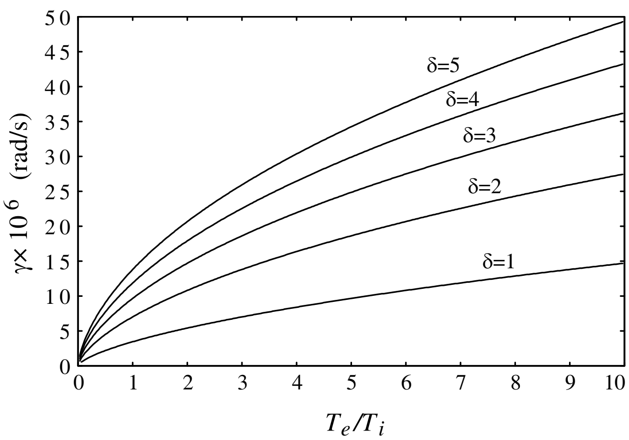

Figure 5. Growth rate versus ratio of electron to ion temperature for different values of δ.

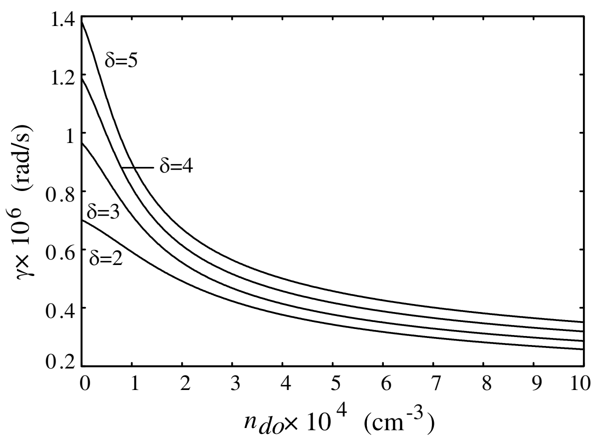

Figure 6. Comparison of growth rate of the unstable mode as a function of dust grain number density for different values of δ.

rapidly in the non-isolated case (cf. Eq. (1)). The variation of the growth rate as a function of the dust particle number density is shown in Fig. 6 for dust grain size of 100 μm.

The growth rate decreases with an increase in the dust grain number density. It is because an increase in the dust grain number density means that the dust grains together have a large apetite for the electrons, but the number of available electrons per dust grain decreases. This reduces the average dust grain charge Q_{d0}, and thus leads to a decrease in growth rate. The growth rate does not change significantly with the dust grain number density, for dust grain sizes less than 100 μm. Some of our theoretical results are in line with the experimental observations of Barkan et al. [25] and Merlino et al. [26]. It has been observed experimentally that for a given value of the electron drift speed along the magnetic field, the wave amplitude is higher when the dust particles are present.

In conclusion, the electrostatic IC waves are driven to instability by an ion beam via Cerenkov interaction. The frequency of the IC mode increases with an increase in the relative density of negatively charged dust grains (cf. Fig. 2). An increase in the dust population enhances the growth rate of the ICWs through the effect of capturing electrons, modifying the dispersion properties of ICWs in plasmas. A unique dissipative mechanism in the presence of dust grains is the dust charging process. The dust charging induces the existence of an additional short wavelength damping mode [cf. Eq. (13)]. The charging model for dust particles used here is valid only for conducting dust particles such as graphite or magnetite, found in interstellar clouds, terrestrial aerosols and laboratory plasmas or tokamaks. Presence of ion beam excites the electrostatic IC mode, whose growth rate depends significantly on the dust grain size, dust grain charge fluctuations, electron temperature and dust grain number density.

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