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Space-time Matrix Method for Mixed Near-Field and Far-Field Sources Localization

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Abstract—Mixed near-field and far-field sources localization problem has received significant attention recently in some practical applications, such as speaker localization using microphone arrays and guidance systems, etc. This paper presents a novel space-time matrix method to localize mixed near-field and far-field sources. Using the proposed method, both the direction-of-arrival (DOA) and range of a source can be estimated by the same eigen-pair of a defined space-time matrix. Therefore, the pairing of the estimated angles and ranges is automatically determined. Compared with the previous work, the presented method offers a number of advantages over other recently proposed algorithms. For example, it can avoid not only parameters matching problem but also aperture loss problem. It has lower computational complexity since the proposed method does not require the high-order statistics or any parameter search. Simulation results show the performance of the proposed algorithm.

1. INTRODUCTION

Source localization from noisy observations is a fundamental problem in array signal processing. Various algorithms have been developed in the past decades for locating far-field sources (FFSs) [1–3] or near-field sources (NFSs) [4–7]. For the FFS scenario, only DOA parameter needs to be estimated; however, for the NFS scenario, both DOA and range parameters of NFSs should be estimated since the NFSs are located closely to the array. The signals received at an array are often the mixture of FFSs and NFSs, in some practical applications such as speaker localization using microphone arrays, etc. In such scenario, each speaker may be in the near-field (NF) or far-field (FF) of the received array.

A lot of work has been done on the mixed NF and FF sources localization recently. For example, a two-stage MUSIC algorithm based on cumulant is proposed to solve the mixed sources localization issue [8]. Despite its effectiveness, it has a high computational burden since this method needs multiple particular cumulant matrices (which are based on high-order statistics) and three times eigenvalue decomposition of some high-dimensional matrices. To alleviate the computational overhead, a MUSIC-based one-dimensional (1-D) spectral peak search algorithm is presented in [9]. However, this method has great loss of array aperture. Via ESPRIT-like and polynomial rooting methods, an effective mixed sources localization algorithm is given in [10]. It can obtain better estimation performance with reduced computational complexity and find the DOAs of $N-2$ sources when the receiving system is equipped with a uniform linear array (ULA) of N sensors. An improved method is presented in [11]. The proposed approach can estimate DOAs and powers of FFSs by the MUSIC spectral function. And then, an oblique projection technique is adopted to eliminate the FFSs so that DOAs and ranges of NFSs can be estimated by exploiting the symmetry property of ULA. A maximum likelihood localization method based on data supported optimization (DSO) is proposed in [12]. In this method, a two-stage estimation technique is exploited to obtain the data supported grid points. A mixed sources localization based on sparse signal reconstruction is presented in [13]. The DOAs of mixed sources are estimated

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by constructing the cumulant domain data. After that, the ranges are estimated by constructing the mixed over complete basis to get the sparse representation of the receiving array output. A two-stage matrix differencing algorithm [14] is proposed. Firstly, the proposed method exploits the property of the Toeplitz structure associated with the covariance matrix of the FFSs to eliminate the FF components. Secondly, DOAs and ranges of NFSs are estimated by an ESPRIT-like solution. Finally, the DOAs of FFSs are estimated via MUSIC algorithm. The method [15] exploits the recursive relationship of spherical harmonics and spherical array to estimate the DOAs of multiple mixed sources. It can avoid high-order statistics computation and parameter search. However, it can not discriminate far-field and near-field signals.

In this paper, a new space-time matrix method is developed to resolve the mixed sources localization problem when the FFSs and NFSs coexist. The DOAs and ranges of all incoming signal sources can be estimated by the eigen-pairs of a defined space-time matrix based on second order statistics. The outline of the paper is organized as follows. The data model is described in Section 2. Section 3 introduces the space-time matrix method. Section 4 shows some simulation results. Finally, the conclusion is given in Section 5.

2. DATA MODEL

Consider K narrow band signal sources $s_k(t)$ $(1 \leq k \leq K)$, generated by the NFSs and FFSs, impinging on a ULA with M omni-directional sensors. Assume that the first K_1 sources are the FFSs and that other $K - K_1$ sources are the NFSs.

The ULA geometry is depicted in Fig. 1. Employing the first antenna of the ULA as the phase reference, the signal received at the mth antenna can be written as

$$y_m(t) = \sum_{k=1}^{K} a_{mk} s_k(t) + n_m(t) \qquad (m = 1, 2, \ldots, M) \tag{1}$$

where a_{mk} denotes the mth antenna responding to signal $s_k(t)$ from direction θ_k, and range r_k. $n_m(t)$ stands for the noise output of the mth antenna. In matrix form, the array output can be given by

$$\mathbf{Y}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{Y}(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T$ is the array output vector. $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$ represents the signal waveform vector. $\mathbf{n}(t) = [n_1(t), \ldots, n_M(t)]^T$ stands for the array noise vector. $\mathbf{A} = [\mathbf{a}(\theta_1, r_1), \mathbf{a}(\theta_2, r_2), \ldots, \mathbf{a}(\theta_K, r_K)]$ is the $N \times K$ array steering matrix of the mixed NFSs and FFSs. In the array steering matrix, the steering vector $\mathbf{a}(\theta_k, r_k) = [a_{1k}, a_{2k}, \ldots, a_{Mk}]$ with $a_{mk} = \exp\{j\big((m-1)\alpha_k + (m-1)^2\beta_k\big)\}$, in which $\alpha_k = -\frac{2\pi d}{\lambda}\sin(\theta_k)\cdot\beta_k = \frac{\pi d^2}{\lambda r_k}\cos^2(\theta_k)\cdot r_k \in [0.62(D^2/\lambda)^{1/2}, +\infty)$. d denotes the distance between two adjacent sensors. λ stands for the wavelength of sources. $\theta_k \in [-\pi/2, \pi/2]$ and r_k are the DOA and range of the kth source relative to the origin, respectively. D represents the array aperture.

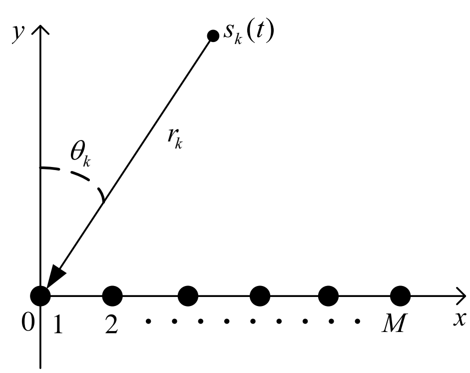

Figure 1. Uniform linear array configuration.

The common assumptions are listed first.

(A1): $K < M$, all sources are not fully correlated.

(A2): $n_i(t)$ $(i = 1, \ldots, M)$ is a complex Gaussian random process with zero-mean and equal variance σ_n^2. The noise $n_i(t)$ is uncorrelated with $s_k(t)$ $(k = 1, \ldots, K)$.

Under the above assumptions, it can be easily seen that

$$R_{n_m n_l}(\tau) = \mathrm{E}\{n_m(t) n_l^*(t+\tau)\} = \sigma_n^2 \delta(\tau)\delta(m-l) \tag{3}$$

$$R_{n_m s_k}(\tau) = \mathrm{E}\{n_m(t) s_k^*(t+\tau)\} = 0 \tag{4}$$

$$R_{s_m s_k}(\tau) = \mathrm{E}\{s_m(t) s_k^*(t+\tau)\} = s_{k,k}(\tau)\delta(m-k) \tag{5}$$

where $\delta(\cdot)$ is the Dirac function. The superscript $(\cdot)^*$ represents the conjugate operation. $\mathrm{E}\{\cdot\}$ denotes the statistical average operation. $s_{k,k}(\tau) = \mathrm{E}\{s_k(t) s_k^*(t+\tau)\}$.

Given the observed signal $\mathbf{y}(t)$, the task is to estimate the two-dimensional parameters $(\theta_k, r_k)(k = 1, \ldots, K)$.

3. ALGORITHM FORMULATION

To develop an effective joint DOA and range estimation algorithm, we define the correlation functions $r_{i,M}(\tau)$ and $r_{i,M-1}(\tau)$ as follows

$$r_{i,M}(\tau) = \mathrm{E}\{y_i(t) y_M^*(t+\tau)\} \ (i = 1, \ldots, M) \tag{6}$$

$$r_{i,M-1}(\tau) = \mathrm{E}\{y_i(t) y_{M-1}^*(t+\tau)\} \ (i = 1, \ldots, M) \tag{7}$$

where $y_i(t)$ is given by (1).

From (3)$\sim$(7), we have the following equations

$$\begin{aligned} r_{i,M}(\tau) &= \mathrm{E}\left\{\left(\sum_{k=1}^{K} s_k(t) a_{ik} + n_i(t)\right)\left(\sum_{p=1}^{K} s_p^*(t+\tau) a_{Mp}^* + n_M^*(t+\tau)\right)\right\} \\ &= \sum_{k=1}^{K} \mathrm{E}\left\{s_k(t) s_k^*(t+\tau)\right\} a_{Mk}^* a_{ik} + \sigma_n^2 \delta(\tau)\delta(i-M) \\ &= \sum_{k=1}^{K} \left(s_{k,k}(\tau) a_{Mk}^*\right) a_{ik} \quad (\tau \neq 0,\ i = 1, 2, \ldots, M) \end{aligned} \tag{8}$$

Similarly, $r_{i,M-1}(\tau)$ has the following expression

$$\begin{aligned} r_{i,M-1}(\tau) &= \mathrm{E}\{y_i(t) y_{M-1}^*(t+\tau)\} \\ &= \sum_{k=1}^{K} \left(s_{k,k}(\tau) a_{(M-1)k}^*\right) a_{ik} \quad (\tau \neq 0,\ i = 1, 2, \ldots, M) \\ &= \sum_{k=1}^{K} \left(s_{k,k}(\tau) a_{Mk}^*\right) a_{ik} \frac{a_{(M-1)k}^*}{a_{Mk}^*} \end{aligned} \tag{9}$$

Define that three vectors $\mathbf{r}_1(\tau)$, $\mathbf{r}_2(\tau)$, and $\mathbf{r}_s(\tau)$ as follows

$$\mathbf{r}_1(\tau) = [r_{1M}(\tau), r_{2M}(\tau), \ldots, r_{MM}(\tau)]^T \tag{10}$$

$$\mathbf{r}_2(\tau) = [r_{1(M-1)}(\tau), r_{2M}(\tau), \ldots, r_{MM}(\tau)]^T \tag{11}$$

$$\mathbf{r}_s(\tau) = [s_{1,1}(\tau) a_{M1}^*, s_{2,2}(\tau) a_{M2}^*, \ldots, s_{K,K}(\tau) a_{MK}^*]^T \tag{12}$$

Thus,

$$\mathbf{r}_1(\tau) = \mathbf{A}\mathbf{r}_s(\tau) \tag{13}$$

$$\mathbf{r}_2(\tau) = \mathbf{A}\boldsymbol{\Phi}\mathbf{r}_s(\tau) \tag{14}$$

where the matrix $\mathbf{\Phi}$ has the following form

$$\mathbf{\Phi} = \mathrm{diag}\left\{\frac{a^*_{(M-1)1}}{a^*_{M1}}, \frac{a^*_{(M-1)2}}{a^*_{M2}}, \ldots, \frac{a^*_{(M-1)K}}{a^*_{MK}}\right\} = \mathrm{diag}\left\{e^{j(\alpha_1+(2M-3)\beta_1)}, \ldots, e^{j(\alpha_K+(2M-3)\beta_K)}\right\} \tag{15}$$

Collect N times "pseudo-snapshot" for $\mathbf{r}_1(\tau)$, $\mathbf{r}_2(\tau)$, respectively, i.e., $\tau = Ts, 2Ts, \ldots, NTs$, we can obtain the following "pseudo-snapshot matrices"

$$\mathbf{X} = [\mathbf{r}_1(T_s), \mathbf{r}_1(2T_s), \ldots, \mathbf{r}_1(NT_s)] \tag{16}$$

$$\mathbf{Y} = [\mathbf{r}_2(T_s), \mathbf{r}_2(2T_s), \ldots, \mathbf{r}_2(NT_s)] \tag{17}$$

Invoking the expressions of $\mathbf{r}_1(\tau)$ and $\mathbf{r}_2(\tau)$ in (13)$\sim$ (14), we have the following relationship

$$\mathbf{X} = \mathbf{A}\mathbf{R}_s \tag{18}$$

$$\mathbf{Y} = \mathbf{A}\mathbf{\Phi}\mathbf{R}_s \tag{19}$$

where $\mathbf{R}_s = [\mathbf{r}_s(T_s), \mathbf{r}_s(2T_s), \ldots, \mathbf{r}_s(NT_s)]$.

Making use of (18) and (19), we define a space-time matrix $\mathbf{R}$ as [16, 17]

$$\mathbf{R} = \mathbf{Y}\mathbf{X}^\dagger \tag{20}$$

where the superscript $(\cdot)^\dagger$ stands for matrix pseudo inverse (Moore-Penrose inverse), then we have the following Theorem 1.

Theorem 1. Assume that there are K (near-field or far-field) narrow-band sources, with complex baseband representations $s_k(t)$ $(1 \leq k \leq K)$ such that the kth source arrives a ULA from direction θ_k and range r_k. If there are no same elements on the diagonal of matrix $\mathbf{\Phi}$, and $\mathbf{R}_s$ is the a full rank matrix, then, the K nonzero eigenvalues of $\mathbf{R}$ are equal to the K elements on the diagonal of matrix $\mathbf{\Phi}$, and the corresponding eigenvectors are equal to the corresponding column vectors of $\mathbf{R}$, namely, $\mathbf{RA}$=$\mathbf{A\Phi}$.

Proof: Under the above assumptions, it is easy to know that $\mathbf{A}$ is a full rank matrix. Furthermore, we can draw a conclusion that rank($\mathbf{X}$) = rank($\mathbf{A}$) = rank($\mathbf{R}_s$) = K. Thus, we have the following equations

$$\mathbf{X}^\dagger = \mathbf{R}_s^H\left(\mathbf{R}_s\mathbf{R}_s^H\right)^{-1}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H \tag{21}$$

From (19) $\sim$ (21), the following equation can be obtained

$$\begin{aligned}\mathbf{RA} &= \mathbf{Y}\mathbf{X}^\dagger\mathbf{A} = \Big(\mathbf{A}\mathbf{\Phi}\mathbf{R}_s\Big)\Big(\mathbf{R}_s^H(\mathbf{R}_s\mathbf{R}_s^H)^{-1}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\Big)\mathbf{A} \\ &= \Big(\mathbf{A}\mathbf{\Phi}\Big)\Big((\mathbf{R}_s\mathbf{R}_s^H)(\mathbf{R}_s\mathbf{R}_s^H)^{-1}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{A}\Big) = \mathbf{A}\mathbf{\Phi}\end{aligned} \tag{22}$$

This concludes the proof.

Remarks:

(1) From Theorem 1, it can be easily seen that the array steering matrix $\mathbf{A}$ and the diagonal matrix $\mathbf{\Phi}$ can be obtained by computing the eigendecomposition of the space-time matrix $\mathbf{R}$. Then the incoming angle θ_k and range r_k can be estimated by making use of the kth eigen-pair of the matrix $\mathbf{R}$, that is, the paring of the estimated two-dimensional parameters is automatically determined.

(2) If there are several sources are close in the angle of incidence θ or range r, but there are no same elements on the diagonal of matrix $\mathbf{\Phi}$, then Theorem 1 is still true, namely, it can resolve the incoming rays with very close θ angles or very close r ranges under the aforementioned conditional restriction.

The procedure of the proposed method is concluded as follows.

(1) Collect the data matrices $\mathbf{X}$ and $\mathbf{Y}$, according to (16) and (17), respectively.

(2) Calculate $\mathbf{X}^\dagger$ and the space-time matrix $\mathbf{R}$.

(3) Compute the eigen-pairs $(\lambda_k, \boldsymbol{\xi}_k)$ of $\mathbf{R}$ $(k = 1, \ldots, K)$, where λ_k is the kth eigenvalue of $\mathbf{R}$ and $\boldsymbol{\xi}_k$ is the corresponding eigenvector.

(4) Estimate α_k and β_k as follows

$$\hat{\beta}_k = \frac{\mu_k - \nu_k}{M-2}, \quad \hat{\alpha_k} = \nu_k - (M-1) \times \hat{\beta}_k \tag{23}$$

where $\nu_k = \text{angle}(\lambda_k)$, $\mu_k = \frac{1}{M-1}\text{angle}(\sum_{m=2}^{M} \frac{\xi^*_{k,m-1}\xi_{k,m}}{\xi^2_{k,1}})$ with $\xi_{k,m}$ denoting the mth element of the kth eigenvector $\boldsymbol{\xi}_k$.

(5) Estimate DOA θ_k and range r_k as follows

$$\begin{cases} \hat{\theta}_k = \sin^{-1}\left(-\hat{\lambda}_k \times \dfrac{\lambda}{2\pi d}\right) \\ \hat{r}_k = \dfrac{\pi d^2 \cos^2(\hat{\theta}_k)}{\lambda \hat{\beta}_k} \end{cases} \tag{24}$$

(6) According to the size of range estimates $\hat{r}_k$, $k = 1, \ldots, K$, determine the type (NFS or FFS) of sources.

In fact, according to (24), we can easily determine that the kth source $s_k(t)$ is near-field or far-field one. When $\hat{r}_k \in [0.62(D^2/\lambda)^{1/2}, 2D^2/\lambda]$ (Fresnel region), we can determine that the source $s_k(t)$ corresponding to $\hat{r}_k$ is a near-field source. On the contrary, when $\hat{r}_k \in (2D^2/\lambda, +\infty)$, we can determine that the source $s_k(t)$ corresponding to $\hat{r}_k$ is a far-field source.

4. SIMULATION RESULTS

In this section, several simulation results are provided to illustrate the performance of the proposed space-time matrix method (STMM).

Consider a ULA composed of 7 sensors with quarter-wavelength inter-sensor spacing. The input signal-to-noise ratio (SNR) is defined as $10\log10(\sigma_s^2/\sigma_n^2)$, where σ_s^2 denotes the power of signal source $s(t)$, and σ_n^2 stands for the noise power. Assume that there are one far-field and two near-field sources are incoming on the ULA, and they are located at $(20°, 45\lambda)$, $(-30°, 5\lambda)$ and $(20°, 2\lambda)$, respectively. The number of snapshots at each sensor is $N = 50$. All results provided are based on 200 independent runs. We use the root-mean-square-error (RMSE) RMSE_θ and RMSE_r as the performance measure. They are defined as

$$\begin{cases} \text{RMSE}_\theta = \sqrt{\text{E}\left\{\sum_{k=1}^{K}(\theta_k - \hat{\theta}_k)^2\right\}} \\ \text{RMSE}_r = \sqrt{\text{E}\left\{\sum_{k=1}^{K}(r_k - \hat{r}_k)^2\right\}} \end{cases} \tag{25}$$

where $\hat{\theta}_k$ and $\hat{r}_k$ are the estimate of θ_k and r_k, for $k = 1, 2, \ldots, K$.

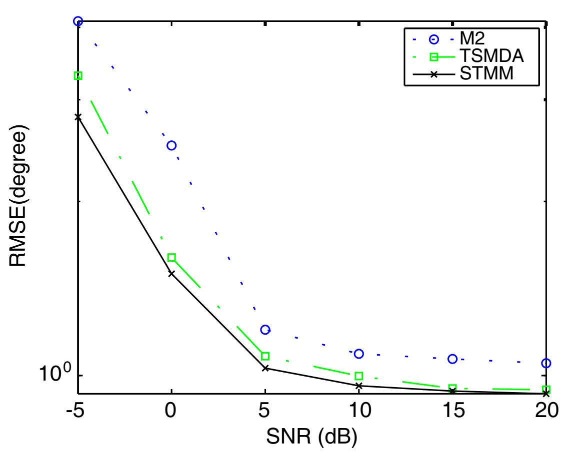

Figure 2. RMSE_θ curves versus SNR.

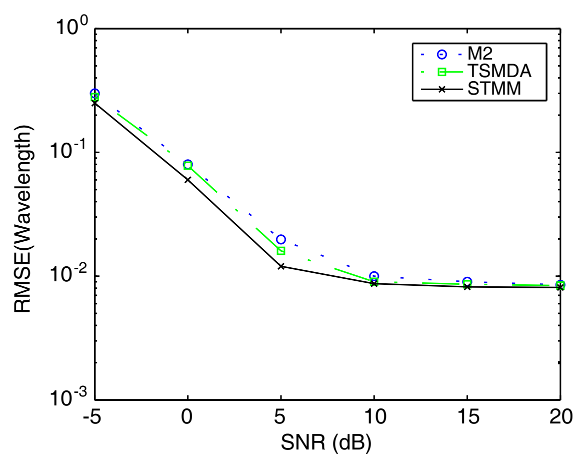

Figure 3. RMSE_r curves versus SNR.

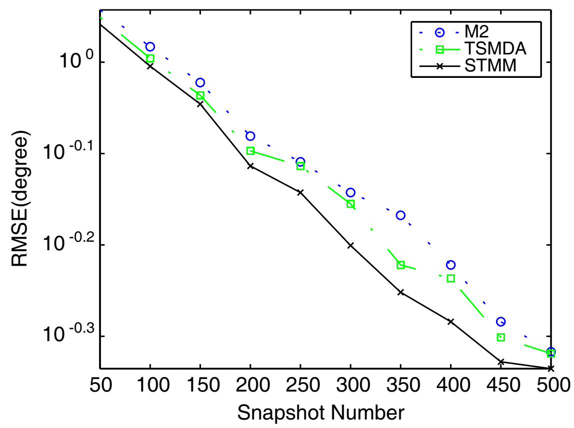

Figure 4. RMSE$_\theta$ curves versus snapshot number.

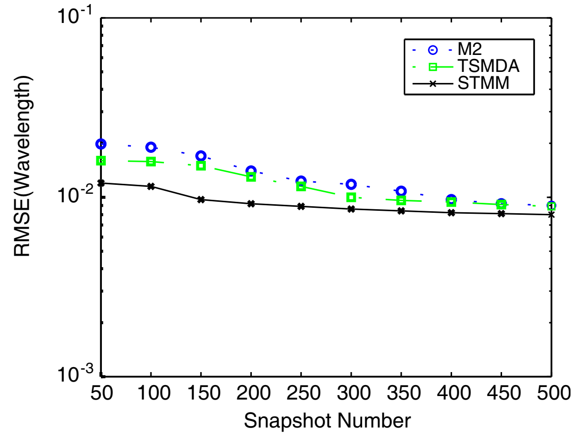

Figure 5. RMSE$_r$ curves versus snapshot number.

Figures 2 and 3 give RMSE$_\theta$ and RMSE$_r$ curves with SNRs ranging from -5 to 20 dB, respectively. The solid line stands for the RMSE curve of the proposed STMM. The dotted line represents the RMSE curve of the method presented in [10] (For the sake of convenience, this method is referred to as M2). The dashdotted line denotes the RMSE curve of the proposed method in [14] (this method is named as TSMDA for short), respectively. From Figs. 2 and 3, we can note that the STMM outperforms the M2 and TSMDA in DOA and range estimates.

When SNR is set to 10 dB and the snapshot number varies from 50 to 500. The RMSE$_\theta$ and RMSE$_r$ curves of the aforementioned three algorithms are shown in Fig. 4 and Fig. 5, respectively. From Figs. 4 and 5, we can see that the proposed method has higher estimation accuracy than that of M2 and TSMDA.

5. CONCLUSIONS

In this paper, we present a space-time matrix method for mixed sources localization. The eigen-pairs of the defined space-time matrix are utilized to estimate the DOAs and ranges. So, the paring of the estimated parameters is automatically determined. The presented approach has a lower computational complexity, but it exhibits superior performance, such as a smaller estimation error and better robustness to SNR change.

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