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A Design Approach for Dual-Band Wilkinson Power Divider with Two Pairs of Coupled-Line Sections

Xin Xu* **and Xiaohong Tang**

Abstract—This paper presents a design approach for dual-band Wilkinson power dividers. Two pairs of parallel coupled-lines are used to replace the quarter-wavelength transformers in the conventional Wilkinson power divider to obtain dual-band operation. Using even- and odd-mode analysis, the closed-form design equations are derived for design parameters and design procedures of the proposed dual-band power divider are given. For verification purpose, a practical power divider, which operates at 1 GHz and 2.1 GHz with 3 dB power dividing ratio, is designed, fabricated and tested. The simulated and measured results are in good agreement.

1. INTRODUCTION

Power dividers are widely used in microwave and millimeter-wave circuits and systems, such as antenna feeder networks, power amplifiers, balanced mixers, phase shifters, and data modulators. The original and useful power divider is the Wilkinson divider with excellent isolation between two output ports at the centre frequency [1]. To achieve dual-band operation, several modified power dividers have been developed [2–10]. Compared with conventional transmission line, the coupled-line has advantages of compact structure and flexible design parameters due to introducing even- and odd-mode impedances. The use of cascaded coupled-line structure has been proposed in [11, 12]. In addition, in [13–15], a new method has been developed by replacing them with a pair of coupled-lines. The power divider given in [13] uses coupled-lines to replace some part of the quarter-wavelength transformer in Wilkinson divider. However, it can only operate over a range of small frequency ratio ($r < 1.64$). The dual-band power divider in [14] can operate at a larger frequency ratio between 1.5 and 3. But the electrical length of the coupled-line section is fixed at 90°, which reduces the design flexibility. In addition, although the design flexibility is no longer a problem in [15], the length of the couple-line section is increased due to the additional short-circuited stub.

In this paper, a new circuit configuration is proposed for dual-band divider. It consists of two pairs of parallel coupled-lines and two isolation resistors. The coupled-lines play a very important role in obtaining the dual-band operation. The dual-band operation is fully controlled by the coupled line parameters only, without any additional components. Based on even- and odd-mode analysis, the final closed-form design equations with flexible parameter selection are given. In addition, the simulated results and fabricated power divider certify this proposed structure and corresponding design approach.

2. CIRCUIT STRUCTURE AND DESIGN THEORY

Figure 1 shows the structure of the proposed dual-band power divider. It consists of two pairs of parallel coupled-lines and two lumped resistors. The coupled-line sections are the key element in obtaining dual-band operation. The dual-band characteristics of the divider are determined by the coupled-line parameters only. The resistors are required for the output ports matching and isolation.

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* Corresponding author: Xin Xu (xindereck0512@163.com).
The authors are with the School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, P. R. China.

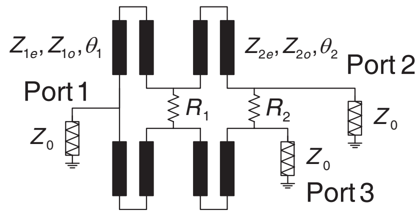

Figure 1. Proposed dual-band Wilkinson power divider.

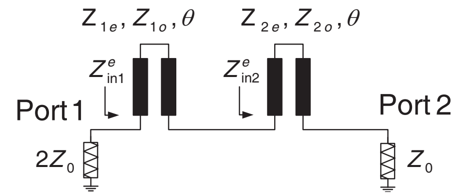

Figure 2. Circuit of the proposed power divider for even-mode analysis.

2.1. Even-Mode Analysis

Figure 2 shows the equivalent circuit model of the power divider when port 1 is excited. Its power can be equally divided to port 2 and port 3, and the center vertical plane becomes a perfect magnetic wall. Thus, no current flows through the isolation resistors. To match the input port 1, the value of each input equivalent impedance should satisfy the following relationship:

$$\begin{cases} Z^e_{in2} = aZ_0 \\ Z^e_{in1} = aZ^e_{in2} \end{cases} \tag{1}$$

where a is the impedance transform ratio of each coupled-line section. The $ABCD$ matrixes of each coupled-line section are given as [16]

$$\begin{cases} A_i = D_i = \dfrac{k_i - \tan^2\theta}{k_i + \tan^2\theta} \\ B_i = \dfrac{2jZ_{ie}\tan\theta}{k_i + \tan^2\theta} \\ C_i = \dfrac{2j\tan\theta}{Z_{io}(k_i + \tan^2\theta)} \end{cases} \tag{2}$$

where $k_i = \frac{Z_{ie}}{Z_{io}}$ $(i = 1, 2)$ is the impedance ratio that determines the degree of coupling of the coupled section. The coupling C_i in dB is defined as

$$C_i = 20\lg\frac{k_i - 1}{k_i + 1} \tag{3}$$

The input impedance can be expressed in another form:

$$Z_{in} = \frac{AZ_0 + jB}{jCZ_0 + D} \tag{4}$$

Based on Equations (1), (2), and (4), the characteristic impedance of each coupled-line section is

$$\begin{cases} Z_1^2 = Z_{1e}Z_{1o} = a^3Z_0^2 \\ Z_2^2 = Z_{2e}Z_{2o} = aZ_0^2 \end{cases} \tag{5}$$

Then the even-mode $ABCD$ parameters of the power divider can be derived from Figure 2:

$$\begin{cases} A = \dfrac{1}{g_1g_2}\left[\left(k_1 - \tan^2\theta\right)\left(k_2 - \tan^2\theta\right) - 4\dfrac{Z_{1e}}{Z_{2o}}\tan^2\theta\right] \\ B = \dfrac{2j}{g_1g_2}\left[Z_{2e}\tan\theta\left(k_1 - \tan^2\theta\right) + Z_{1e}\tan\theta\left(k_2 - \tan^2\theta\right)\right] \\ C = \dfrac{2j}{g_1g_2}\left[\dfrac{\tan\theta}{Z_{2o}}\left(k_1 - \tan^2\theta\right) + \dfrac{\tan\theta}{Z_{1o}}\left(k_2 - \tan^2\theta\right)\right] \\ D = \dfrac{1}{g_1g_2}\left[\left(k_1 - \tan^2\theta\right)\left(k_2 - \tan^2\theta\right) - 4\dfrac{Z_{2e}}{Z_{1o}}\tan^2\theta\right] \end{cases} \tag{6}$$

where $g_i = k_i + \tan^2\theta$. The input scattering parameter S_{11} can be obtained as

$$S_{11} = \frac{\left(\begin{array}{l} Z_0\left[-\left(k_1-\tan^2\theta\right)\left(k_2-\tan^2\theta\right)-4\dfrac{Z_{1e}}{Z_{2o}}\tan^2\theta+8\dfrac{Z_{2e}}{Z_{1o}}\tan^2\theta\right] \\ +2j\tan\theta\left[\left(Z_{2e}-\dfrac{2Z_0^2}{Z_{2o}}\right)\left(k_1-\tan^2\theta\right)+\left(Z_{1e}-\dfrac{2Z_0^2}{Z_{1o}}\right)\left(k_2-\tan^2\theta\right)\right] \end{array}\right)}{\left(\begin{array}{l} Z_0\left[3\left(k_1-\tan^2\theta\right)\left(k_2-\tan^2\theta\right)-4\dfrac{Z_{1e}}{Z_{2o}}\tan\theta^2-8\dfrac{Z_{2e}}{Z_{1o}}\tan\theta^2\right] \\ +2j\tan\theta\left[\left(Z_{2e}+\dfrac{2Z_0^2}{Z_{2o}}\right)\left(k_1-\tan^2\theta\right)+\left(Z_{1e}+\dfrac{2Z_0^2}{Z_{1o}}\right)\left(k_2-\tan^2\theta\right)\right] \end{array}\right)} \tag{7}$$

For the ideal input matching, where $S_{11} = 0$, the real and imaginary parts of Equation (7) can be written separately as

$$\tan^4\theta-\left(k_1+k_2-4a\sqrt{k_1k_2}+\frac{8\sqrt{k_1k_2}}{a}\right)\tan^2\theta+k_1k_2=0 \tag{8a}$$

$$\tan^2\theta=\sqrt{k_1k_2}\frac{\sqrt{k_1}\left(\sqrt{a}-\dfrac{2}{\sqrt{a}}\right)+\sqrt{k_2}\left(\sqrt{a^3}-\dfrac{2}{\sqrt{a^3}}\right)}{\sqrt{k_2}\left(\sqrt{a}-\dfrac{2}{\sqrt{a}}\right)+\sqrt{k_1}\left(\sqrt{a^3}-\dfrac{2}{\sqrt{a^3}}\right)} \tag{8b}$$

In order to solve (8a) and (8b), the following cases should be discussed.

2.1.1. Case 1

If $a^2 = 2$, (8b) is simplified as

$$\tan^2\theta=\sqrt{k_1k_2} \tag{9}$$

By substituting (9) into (8a), we can obtain

$$k_1=k_2=k \tag{10}$$

Then (9) is found as

$$\tan\theta=\pm\sqrt{k} \tag{11}$$

It is shown that there is only one transmission zero in each passband at θ_1 and θ_2

$$\begin{cases}\theta_1=\arctan\sqrt{k}\\ \theta_2=\pi-\theta_1\end{cases} \tag{12}$$

the frequency ratio is then given by :

$$r=\frac{\theta_2}{\theta_1}=\frac{\pi-\arctan\sqrt{k}}{\arctan\sqrt{k}} \tag{13}$$

From (12) and (13), we have

$$k=\tan^2\frac{\pi}{1+r} \tag{14}$$

Thus, as long as the frequency ratio r is determined, the electrical length θ_1 of the coupled-lines and the impedance ratio k can be calculated from (12) and (14). Then the even- and odd-mode characteristic impedances of each coupled-line sections can be specified from (5).

2.1.2. Case 2

If $k_1 = k_2 = k$ and $a^2 \neq 2$. At the center frequencies θ_1 and θ_2, (7) is simplified as

$$S_{11}=\frac{a^2-2}{a^2+2} \tag{15}$$

Assume that $|S_{11}| \leq -20\,\mathrm{dB}$, from (15) we can obtain

$$1.636\leq a^2\leq 2.444 \tag{16}$$

Figure 3 shows the VSWR responses for different value of a^2 when $k_1 = k_2 = 3$. As a^2 is decreased from 2 to 1.636, VSWR increases to 1.2 at the desired frequency and the bandwidth is slightly extended. On the contrary, the bandwidth is decreased as a^2 enlarged to 2.444.

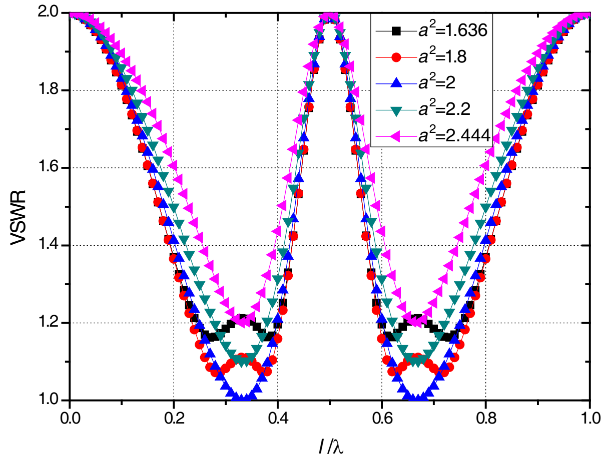

Figure 3. VSWR response curves with $k_1 = k_2 = 3$.

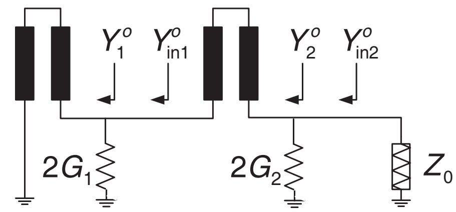

Figure 4. Circuit of the proposed power divider for odd-mode analysis.

2.2. Odd-Mode Analysis

Under odd-mode excitation, the equivalent circuit of the proposed divider is redrawn in Figure 4. Assuming that the two output ports are perfectly matched and isolated, the output admittance should be equal to the system admittance Y_0.

$$Y_{in2}^o = Y_2^o + 2G_2 = Y_0 \tag{17}$$

where $Y_2^o = \frac{1}{Z_{20}^2 Y_{in1}^o}$, $Y_{in1}^o = 2G_1$. Solving (17), we can obtain

$$Z_0 = \frac{G_1 a \pm \sqrt{G_1^2 a^2 - 4G_1 G_2 a}}{4G_1 G_2 a} \tag{18}$$

For a given Z_0 of single value, $G_1^2 a^2 - 4G_1 G_2 a = 0$ must be satisfied. It can be simplified as follow:

$$\frac{G_1}{G_2} = \frac{4}{a} = 2\sqrt{2} \tag{19}$$

substituting (19) into (18), we can obtain the closed-form design equation

$$G_2 = \frac{1}{4Z_0} \tag{20}$$

So according to (19) and (20), the values of isolated resistors R_1 and R_2 can be calculated directly.

3. DESIGN EXAMPLE AND MEASUREMENT

Base on the above analysis, the design procedure of the proposed power divider can be described as follows:

a) Choose two frequencies f_1 and f_2, and calculate the corresponding frequency ratio r;
b) Calculate the electrical length θ_1 from (12) and impedance ratio k from (14);
c) Calculate the even- and odd-mode impedances of each coupled-line section from (5) by k.

Figure 5(a) plots the calculated design parameters of the proposed dual-band power divider for various frequency ratios r. As the frequency ratio r increases, the even-mode characteristic impedances Z_{1e} and Z_{2e} decrease, while the odd-mode characteristic impedances Z_{1o} and Z_{2o} increase. In Figure 5(b), the required coupling coefficient of the coupled lines is reduced as the frequency ratio increases. It can also be observed from Figures 5(a) and (b) that, as the frequency ratio r increases to

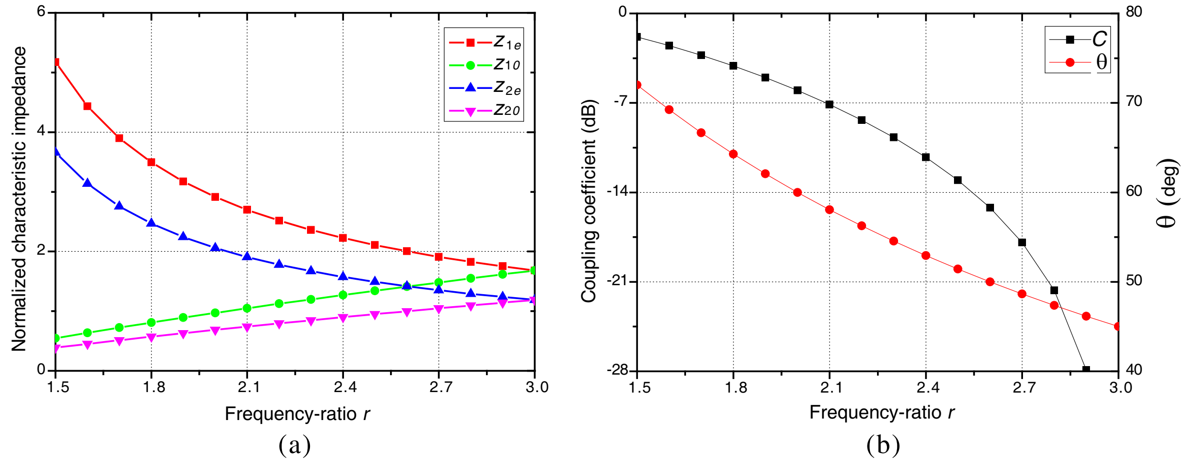

Figure 5. Design parameters of the proposed dual-band power divider. (a) Normalized even- and odd-mode characteristic impedances. (b) Coupling coefficients of and electric length of the coupled lines.

3, the even- and odd-mode characteristic impedances become equal, and the coupled lines become the conventional transmission lines.

Based on Equations (5), (12), (13), and (14), five examples for different r are calculated and listed in Table 1. The corresponding frequency responses are illustrated in Figure 6.

As show in Figure 6, the center frequency of the first band is fixed at $f_1 = 1\,\text{GHz}$ and f_2 is changing from 2.1 to 2.5 GHz. It is observed that the proposed dual-band power divider is effective and its analytical method and design equations are convenient.

Table 1. Design parameters of the dual-band power dividers. $Z_0 = 50\,\Omega$.

r	$\theta@f_1(\,^\circ)$	C (dB)	Z_{1e} (Ω)	Z_{1o} (Ω)	Z_{2e}(Ω)	Z_{2o} (Ω)	R_1 (Ω)	R_2 (Ω)
2.1	58.06	-7.12	134.91	52.41	95.39	37.06		
2.2	56.25	-8.34	125.85	56.18	88.99	39.73		
2.3	54.55	-9.71	118.09	59.88	83.50	42.34	70.7	200
2.4	52.94	-11.25	111.37	63.49	78.75	44.90		
2.5	51.43	-13.06	105.43	67.07	74.55	47.42		

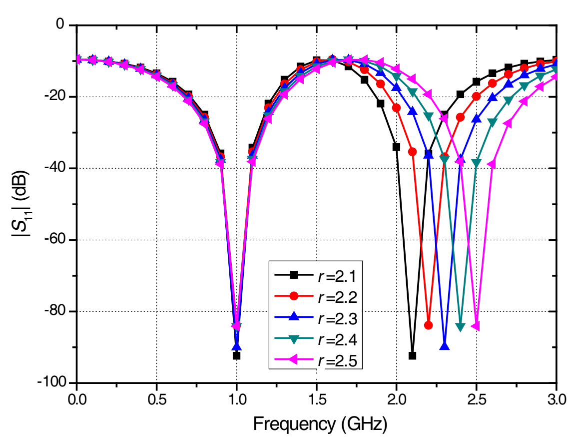

Figure 6. Frequency responses of dual-band power dividers in five cases ($r = 2.1 \sim 2.5$).

To validate the design and analyzed equations of the proposed power divider, a power divider with frequency ratio ($r = 2.1$) working at 1 GHz and 2.1 GHz is designed. It is fabricated on a Rogers RO4350 board with dielectric constant ($\varepsilon_r = 3.66$) and thickness of 0.508 mm. The even- and odd-mode impedances can be obtained from Table 1. The final resistors in our fabricated power divider are $R_1 = 68\,\Omega$ and $R_2 = 200\,\Omega$, which can correspond to commercial available values. A three dimensional (3D) view of the proposed power divider is illustrated in Figure 7. The lumped chip resistors are modeled as vacuum boxes with the same dimension, and the lumped RLC boundaries are assigned to these models. Figure 8 shows the photograph of the fabricated power divider.

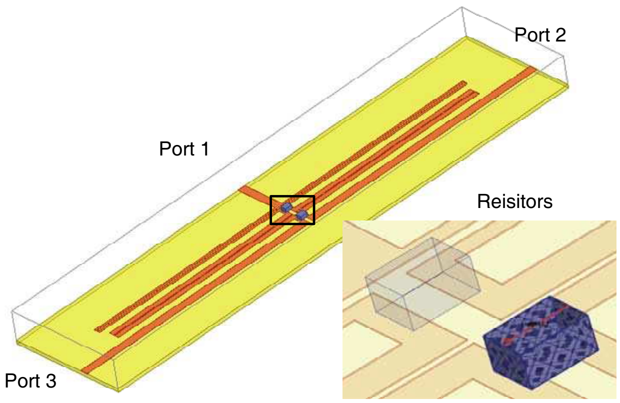

Figure 7. 3D model of the proposed power divider.

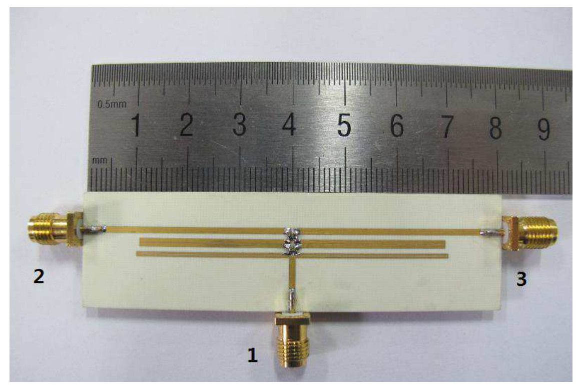

Figure 8. Photograph of the fabricated power divider.

Figure 9 shows the simulated and measured results along with the full wave electromagnetic simulation results by HFSS. The measured results of ports matching are smaller than -20 dB for both 1 and 2.1 GHz as shown in Figure 9(a). The measured transmission value $|S_{21}|$ are -3.45 dB for 1 GHz and -3.77 dB for 2.1 GHz. Figure 9(b) shows that the measured output matching $|S_{22}|$ and isolation $|S_{32}|$ are all better than 20 dB for both 1 and 2.1 GHz. Therefore, the fabricated power divider has a small insertion loss, good matching, and isolation performance at the desired dual bands.

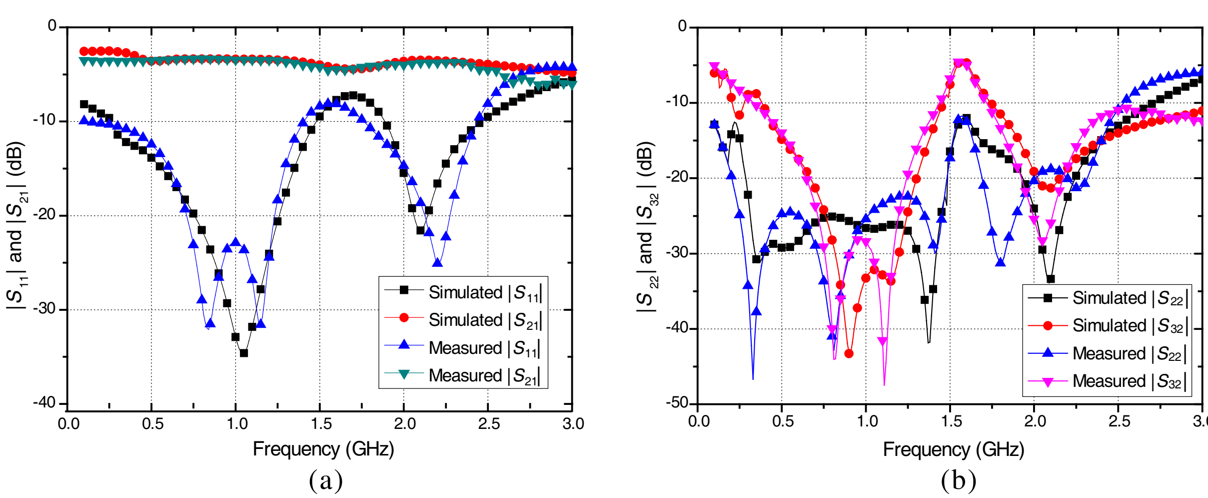

Figure 9. Simulated and measured S-parameters. (a) Input matching and transmission. (b) Output matching and isolation.

4. CONCLUSION

A design method for dual-band Wilkinson power divider with two pairs of parallel coupled-lines sections is presented in this paper. Using even- and odd-mode analysis, the closed-form design equations are derived for design parameters and design procedures of the proposed dual-band power divider are given. Finally, the proposed structure and the corresponding analytical design method are verified through five simulated examples and a fabricated power divider.

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