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Interaction of Electromagnetic Waves with a Moving Slab: Fundamental Dyadic Method

Atieh Kashaninejad-Rad, Ali Abdolali*, and Mohammad M. Salary

Abstract—This paper concerns with the interaction of electromagnetic waves with a moving slab. Consider a homogeneous isotropic slab moving uniformly in an arbitrary direction surrounded by an isotropic medium (free space). In this paper a new simple and systematic method is proposed for analyzing reflection and transmission of obliquely incident electromagnetic waves by a moving slab based on the concept of propagators. In the previous works complex relations were arrived but using this novel method those complexities will not appear thus the method may be extended to more complex structures. In this method, first, electric and magnetic fields are decomposed into their tangential and normal components then each constitutive dyadic is decomposed into a two-dimensional dyadic in transverse plane and two two-dimensional vectors in this plane. Substituting these dyadics into Maxwell's equations gives a first order differential equation which contains fundamental dyadic of the medium. From the solution of this equation, fields inside the slab may be expressed in terms of fields at the front surface of the slab and the propagator matrix which is an exponential function of fundamental dyadic. Using this method the up-going and down-going tangential electromagnetic fields may be obtained at the same time. As a limiting case a slab with vanishing velocity is discussed using this method, and reflection and transmission coefficients of this slab are derived, which ends in Fresnel's equations. At last, several typical examples are provided to exemplify the applicability of the proposed method. Moreover, the results are compared with the method of Lorentz transformation. A good agreement is observed between the results which verifies the validity of the proposed method.

1. INTRODUCTION

The problem of interaction of electromagnetic waves with moving media has long been a subject of interest due to its wide application in various areas such as optics, radio sciences, and astrophysics and numerous investigations have been carried out in this area [1–20]. Two methods are mainly used to analyze scattering from such structure: Lorentz transformation [2] and transformed constitutive relations [1]. Using these methods, we will arrive at complicated relations. In a moving slab it is not possible to decompose the fields inside the slab into TE and TM polarizations. Moreover electric and magnetic fields are coupled in the constitutive relations and thus dealing with such problem is difficult using the common approaches. In this contribution we propose a new method based on the concept of propagators, which is simple and more systematic. Moreover the application scope of the method may be expanded to include more complex structures.

In [21, 22] the concept of propagators is introduced to obtain the fields inside a stationary bianisotropic stratified medium excited by an arbitrary electromagnetic wave incidence. In [21] an exact solution is given for the fields at the front surface of the structure and then an exponential function propagates the field in the correct way from front surface to other positions. In [22] it is shown that all the wave propagation properties of a bianisotropic slab may be expressed by a propagator which is a function of thickness and electromagnetic parameters of slab. The propagator maps the tangential

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* Corresponding author: Ali Abdolali (abdolali@iust.ac.ir).
The authors are with the Department of Electrical Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran.

electric and magnetic fields at one position to the other positions so it is enough to obtain the fields at the front surface of the structure.

In this paper, we use the same approach to analyze scattering of electromagnetic waves by a homogeneous isotropic slab moving uniformly in an arbitrary direction surrounded by an isotropic medium (free space). It is noteworthy that constitutive relations pertaining to an isotropic slab transform into constitutive relations of a bi-isotropic slab if the slab is moving with a constant velocity, so using the concept of propagators will ease the solution. At last, several examples are provided to show the applicability of the proposed method. Moreover, the results are compared with method of Lorentz transformation. A good agreement is observed between the results, which verifies the validity of the proposed method.

2. THEORY

2.1. Dispersion Relation

The geometry of the problem is shown in Fig. 1. Consider a homogeneous isotropic slab moving uniformly in an arbitrary direction surround by free space. A uniform plane electromagnetic wave is incident at an angle θ_i from free space on the front surface of the slab.

By matching the phases of incident, reflected, and transmitted waves at the boundaries between the two media, noting that the boundaries are at $z = v_z t$ and $z = v_z t - d$ when $t > 0$, one can obtain the equalities for the wave vectors k and the frequencies ω of incident (i), reflected (r), and transmitted (t) waves and the waves inside the slab (a, b):

$$\begin{aligned} k_i^x(x+v_xt)+k_i^y(y+v_yt)+k_i^z(v_zt)-\omega_it &= k_r^x(x+v_xt)+k_r^y(y+v_yt)+k_r^z(v_zt)-\omega_rt \\ &= k_{a,b}^x(x+v_xt)+k_{a,b}^y(y+v_yt)+k_{a,b}^z(v_zt)-\omega_{a,b}t \\ &= k_t^x(x+v_xt)+k_t^y(y+v_yt)+k_t^z(v_zt)-\omega_tt \end{aligned} \tag{1}$$

As Equation (1) holds for all x, y, and t, it yields:

$$k_r^x = k_{a,b}^x = k_t^x = k_i^x \tag{2}$$

$$k_r^y = k_{a,b}^y = k_t^y = k_i^y \tag{3}$$

$$k_r^z v_z - \omega_r = k_a^z v_z - \omega_a = k_b^z v_z - \omega_b = k_t^z v_z - \omega_t = k_i^z v_z - \omega_i \tag{4}$$

As it is shown in Fig. 1, the plane of incidence is xz and so $k_i^x = k_0 \sin(\theta_i)$, $k_i^y = 0$ and $k_i^z = -k_0 \cos(\theta_i)$, in which $k_0 = \frac{\omega_i}{c_0}$ and $c_0 = 1/\sqrt{\varepsilon_0\mu_0}$. According to dispersion relation for the reflected wave $k_r^2 = \omega_r^2\varepsilon_0\mu_0$

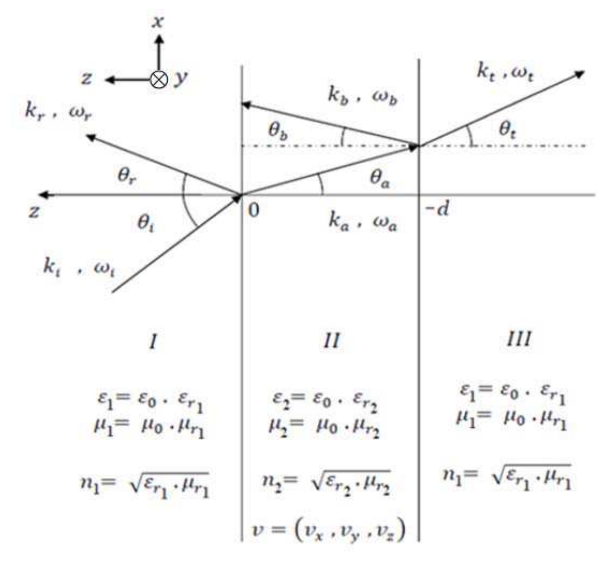

Figure 1. Structure of the problem.

and (4), we have:

$$k_r^z = k_0\left(2\beta_z + \cos(\theta_i) + \beta_z^2\cos(\theta_i)\right) / \left(1-\beta_z^2\right) \tag{5}$$

$$\omega_r = \omega_i\left(1 + 2\beta_z\cos(\theta_i) + \beta_z^2\right) / \left(1-\beta_z^2\right) \tag{6}$$

In which $\beta = \frac{v}{c_0}$.

Constitutive relations of a moving isotropic medium is similar to that of a stationary bianisotropic medium. According to [2] these relations may be expressed as below:

$$\vec{D} = \varepsilon\bar{\bar{A}}\cdot\vec{E} + \vec{\Omega}\times\vec{H} \tag{7}$$

$$\vec{B} = \mu\bar{\bar{A}}\cdot\vec{H} - \vec{\Omega}\times\vec{E} \tag{8}$$

in which:

$$\bar{\bar{A}} = \frac{1-\beta^2}{1-n^2\beta^2}\left[\bar{\bar{I}} - \frac{n^2-1}{1-\beta^2}\vec{\beta}\vec{\beta}\right] \tag{9}$$

$$\vec{\Omega} = \frac{n^2-1}{1-n^2\beta^2}\frac{\vec{\beta}}{c_0} \tag{10}$$

In the above relations $n = \sqrt{\varepsilon_r\mu_r}$ is relative index of refraction of the stationary slab. Due to simplification we can write:

$$\vec{\Omega}\times \equiv \bar{\bar{\Omega}}. \tag{11}$$

$$\bar{\bar{\Omega}} = \left(\frac{n^2-1}{1-n^2\beta^2}\right)\frac{1}{c_0^2}\begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix} \tag{12}$$

Using (7), (8), (12), the dispersion relation of a moving medium may be obtained, as follows:

$$\left(1-n^2\beta^2\right)\left(\vec{k}^{a,b}\cdot\vec{N}\right)^2 + 2\frac{\omega_{a,b}}{c_0}\beta\left(n^2-1\right)\left(\vec{k}^{a,b}\cdot\vec{N}\right) + \left(1-\beta^2\right)\left(\vec{k}^{a,b}\times\vec{N}\right)^2 + \left(\frac{\omega_{a,b}}{c_0}\right)^2\left(\beta^2-n^2\right) = 0 \tag{13}$$

In which $\vec{N} = \vec{v}/v$ is the unit vector along the velocity vector. Using (4) and (13), we have:

$$k_a^z = k_0(\beta_z(1+\beta_z\cos(\theta_i)) - q)/(1-\beta_z^2) \tag{14}$$

$$k_b^z = k_0\left(\beta_z(1+\beta_z\cos(\theta_i)) + q\right) / \left(1-\beta_z^2\right) \tag{15}$$

$$\omega_a = \omega_i\left(1+\beta_z\cos(\theta_i) - \beta_z q\right) / \left(1-\beta_z^2\right) \tag{16}$$

$$\omega_b = \omega_i\left(1+\beta_z\cos(\theta_i) + \beta_z q\right) / \left(1-\beta_z^2\right) \tag{17}$$

in which:

$$q = \left(1-\beta_z^2\right)\left(\gamma^2(n^2-1)(1-\beta_x\sin(\theta_i)+\beta_z\cos(\theta_i))^2 - \sin^2(\theta_i)\right) + \left(1+\beta_z\cos(\theta_i)\right)^2\Big)^{1/2} \tag{18}$$

$$\gamma = 1/\left(1-\beta^2\right)^{1/2} \tag{19}$$

According to the dispersion relation for the transmitted wave $k_t^2 = \omega_t^2\varepsilon_0\mu_0$ and (4) we have:

$$k_t^z = -k_0\cos(\theta_i) \tag{20}$$

$$\omega_t = \omega_i \tag{21}$$

2.2. Extension of the Fundamental Dyadic Method for a Moving Slab

For the purpose of obtaining the fundamental equation, the fundamental dyadic ($\bar{\bar{M}}$) and the wave propagator for a moving slab, we will use Maxwell equations and the constitutive relations in moving media.

For a general bianisotropic medium, the constitutive parameters are given by:

$$\vec{D} = \varepsilon_0\left(\bar{\bar{\varepsilon}}_r\vec{E} + \eta_0\bar{\bar{\xi}}_r\vec{H}\right) \tag{22}$$

$$\vec{B} = \frac{1}{c_0}\left(\bar{\bar{\zeta}}_r\vec{E} + \eta_0\bar{\bar{\mu}}_r\vec{H}\right) \tag{23}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}, \quad c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}, \quad k_0 = \frac{\omega}{c_0}, \quad \bar{\bar{\xi}}_r = \bar{\bar{\chi}} - j\bar{\bar{\kappa}}, \quad \zeta_r = \bar{\bar{\chi}} + j\bar{\bar{\kappa}} \tag{24}$$

By comparing above relations with (7)–(10) for a moving slab it can be deduced:

$$\bar{\bar{\varepsilon}}_r = \varepsilon_r \bar{\bar{A}} \tag{25}$$

$$\bar{\bar{\xi}}_r = \frac{\bar{\bar{\Omega}}}{\sqrt{\varepsilon_0\mu_0}} \tag{26}$$

$$\bar{\bar{\zeta}}_r = -\frac{\bar{\bar{\Omega}}}{\sqrt{\varepsilon_0\mu_0}} \tag{27}$$

$$\bar{\bar{\mu}}_r = \mu_r \bar{\bar{A}} \tag{28}$$

Assuming time dependence convention as $e^{j\omega t}$, Maxwell equations inside the moving slab can be written as:

$$\begin{aligned} \vec{E}_2 &= \vec{E}_{02} e^{-jk_x x - jk_y y - jk_2^z z} \cdot e^{j\omega_2 t} = \vec{E}_{02} e^{-jk_x x - jk_y y} \cdot e^{-j(k_2^z v_z - \omega_2)t} \\ z &= v_z t \end{aligned} \tag{29}$$

$$\nabla \times \vec{E}_2 = -\frac{\partial \vec{B}_2}{\partial t} \tag{30}$$

$$\vec{H}_2 = \vec{H}_{02} e^{-jk_x x - jk_y y - jk_2^z z} \cdot e^{j\omega_2 t} = \vec{H}_{02} e^{-jk_x x - jk_y y} \cdot e^{-j(k_2^z v_z - \omega_2)t} \tag{31}$$

$$-jk_y E_2^z - \frac{dE_2^y}{dz} = j(k_2^z v_z - \omega_2) B_2^x \tag{32}$$

$$jk_x E_2^z + \frac{dE_2^x}{dz} = j(k_2^z v_z - \omega_2) B_2^y \tag{33}$$

$$-jk_x E_2^y + jk_y E_2^x = j(k_2^z v_z - \omega_2) B_2^z \tag{34}$$

$$\nabla \times \vec{H}_2 = \frac{\partial \vec{D}_2}{\partial t} \tag{35}$$

$$-jk_y H_2^z - \frac{dH_2^y}{dz} = -j(k_2^z v_z - \omega_2) D_2^x \tag{36}$$

$$jk_x H_2^z + \frac{dH_2^x}{dz} = -j(k_2^z v_z - \omega_2) D_2^y \tag{37}$$

$$-jk_x H_2^y + jk_y H_2^x = -j(k_2^z v_z - \omega_2) D_2^z \tag{38}$$

Then the fundamental equation for one-dimensional propagation becomes:

$$\frac{d}{dz}\begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} = -j \begin{bmatrix} k_x & 0 \\ k_y & 0 \\ 0 & -k_y \\ 0 & k_x \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} - j(k_2^z v_z - \omega_2) \begin{bmatrix} -B_y \\ B_x \\ \eta_0 D_x \\ \eta_0 D_y \end{bmatrix} \tag{39}$$

Two dimensional rotation dyadic is defined as:

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{40}$$

$$J \cdot \vec{k}_t = J \cdot \begin{bmatrix} k_x \\ k_y \end{bmatrix} = \begin{bmatrix} -k_y \\ k_x \end{bmatrix} \tag{41}$$

$$J \cdot \vec{B}_{xy} = \begin{bmatrix} -B_y \\ B_x \end{bmatrix} \tag{42}$$

It should be noted that k_2^z is identical to k_a^z and is the propagation constant of up-going wave in the slab. Thus by substituting $k_2^z v_z - \omega_2 = k_i^z v_z - \omega_i$ according to Equation (4), we can write:

$$\frac{d}{dz}\begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} = -j \begin{bmatrix} k_x & 0 \\ k_y & 0 \\ 0 & -k_y \\ 0 & k_x \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} - j\left(k_i^z \frac{v_z}{c_0} - \frac{\omega_i}{c_0}\right) \begin{bmatrix} -c_0 B_y \\ c_0 B_x \\ \eta_0 c_0 D_x \\ \eta_0 c_0 D_y \end{bmatrix}$$

$$= -j \begin{bmatrix} \vec{k}_t & 0 \\ 0 & J \cdot \vec{k}_t \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} - j(k_i^z \beta_z - k_0) \begin{bmatrix} c_0 J \cdot \vec{B}_{xy} \\ \eta_0 c_0 \vec{D}_{xy} \end{bmatrix} \tag{43}$$

$$(k_i^z \beta_z - k_0) \begin{bmatrix} c_0 \eta_0 D_z(z) \\ c_0 B_z(z) \end{bmatrix} = - \begin{bmatrix} 0 & \vec{k}_t \\ J \cdot \vec{k}_t & 0 \end{bmatrix} \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} \tag{44}$$

It is appropriate to decompose each constitutive dyadic as:

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\varepsilon}_{\perp\perp} & \boldsymbol{\varepsilon}_{\perp} \\ \boldsymbol{\varepsilon}_{z} & \varepsilon_{zz} \end{bmatrix} \tag{45}$$

The dyadic $\varepsilon_{\perp\perp}$ is a two-dimensional dyadic in the x-y plane, and the vectors ε_z, and $\varepsilon_\perp$ are two two-dimensional vectors in this plane. ε_{zz} is a scalar.

$$\begin{bmatrix} c_0 J \cdot \vec{B}_{xy} \\ c_0 \eta_0 \vec{D}_{xy} \end{bmatrix} = \begin{bmatrix} J \cdot \boldsymbol{\zeta}_{\perp\perp} & -J \cdot \boldsymbol{\mu}_{\perp\perp} \cdot J \\ \boldsymbol{\varepsilon}_{\perp\perp} & -\boldsymbol{\xi}_{\perp\perp} \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} + \begin{bmatrix} J \cdot \boldsymbol{\zeta}_{\perp} & J \cdot \boldsymbol{\mu}_{\perp} \\ \boldsymbol{\varepsilon}_{\perp} & \boldsymbol{\xi}_{\perp} \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} \tag{46}$$

$$\begin{bmatrix} \varepsilon_{zz} & \xi_{zz} \\ \zeta_{zz} & \mu_{zz} \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} = \begin{bmatrix} c_0 \eta_0 D_z(z) \\ c_0 B_z(z) \end{bmatrix} + \begin{bmatrix} -\boldsymbol{\varepsilon}_z & \boldsymbol{\xi}_{zz} \cdot J \\ -\boldsymbol{\zeta}_z & \boldsymbol{\mu}_z \cdot J \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} \tag{47}$$

$$\begin{aligned} \frac{d}{dz} \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} &= -j(k_i^z \beta_z - k_0) \begin{bmatrix} J \cdot \boldsymbol{\zeta}_{\perp\perp} & -J \cdot \boldsymbol{\mu}_{\perp\perp} \cdot J \\ \boldsymbol{\varepsilon}_{\perp\perp} & -\boldsymbol{\xi}_{\perp\perp} \cdot J \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} \\ & \quad -j(k_i^z \beta_z - k_0) \begin{bmatrix} \frac{\vec{k}_t}{k_i^z \beta_z - k_0} + J \cdot \zeta_\perp & J \cdot \mu_\perp \\ \varepsilon_\perp & \frac{J \cdot \vec{k}_t}{k_i^z \beta_z - k_0} + \xi_\perp \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} \end{aligned} \tag{48}$$

$$\begin{bmatrix} \varepsilon_{zz} & \xi_{zz} \\ \zeta_{zz} & \mu_{zz} \end{bmatrix} \cdot \begin{bmatrix} E_z(z) \\ \eta_0 H_z(z) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\varepsilon}_z & \frac{-\vec{k}_t}{k_i^z \beta_z - k_0} + \boldsymbol{\xi}_{zz} \cdot J \\ \frac{-J \cdot \vec{k}_t}{k_i^z \beta_z - k_0} - \boldsymbol{\zeta}_z & \boldsymbol{\mu}_z \cdot J \end{bmatrix} \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} \tag{49}$$

$$\begin{aligned} \frac{d}{dz} \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} &= -j(k_i^z \beta_z - k_0) \left\{ \begin{bmatrix} J \cdot \zeta_{\perp\perp} & -J \cdot \mu_{\perp\perp} \cdot J \\ \varepsilon_{\perp\perp} & -\xi_{\perp\perp} \cdot J \end{bmatrix} \right. \\ & \quad + \begin{bmatrix} \frac{k_t}{k_i^z \beta_z - k_0} + J \cdot \zeta_\perp & J \cdot \mu_\perp \\ \varepsilon_\perp & \frac{J \cdot k_t}{k_i^z \beta_z - k_0} + \xi_\perp \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{zz} & \xi_{zz} \\ \zeta_{zz} & \mu_{zz} \end{bmatrix}^{-1} \\ & \quad \left. \cdot \begin{bmatrix} -\boldsymbol{\varepsilon}_z & \frac{-k_t}{k_i^z \beta_z - k_0} + \boldsymbol{\xi}_{zz} \cdot J \\ \frac{-J \cdot k_t}{k_i^z \beta_z - k_0} + -\boldsymbol{\zeta}_z & \boldsymbol{\mu}_z \cdot J \end{bmatrix} \right\} \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} \end{aligned} \tag{50}$$

$$\frac{d}{dz} \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} = -j(k_i^z \beta_z - k_0) \bar{\bar{M}} \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} \tag{51}$$

$$\begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} = \bar{\bar{P}} \cdot \begin{bmatrix} \vec{E}_{xy}(z = v_z t) \\ \eta_0 J \cdot \vec{H}_{xy}(z = v_z t) \end{bmatrix} \tag{52}$$

$$\bar{\bar{P}} = \exp\left(-j(k_i^z \beta_z - k_0) z \bar{\bar{M}}\right) \tag{53}$$

In Equations (51)–(53), the fundamental equation, the fundamental dyadic, $\bar{\bar{M}}$, and the wave propagator, $\bar{\bar{P}}$, are obtained.

2.3. Obtaining the EM Waves on the Front Surface of the Moving Slab ($z = v_z t$)

To obtain $\begin{bmatrix} \vec{E}_{xy}(z = v_z t) \\ \eta_0 J \cdot \vec{H}_{xy}(z = v_z t) \end{bmatrix}$ (fields on the front surface of the moving slab) we employ boundary conditions of a moving slab. The boundaries are at $z = v_z t$ and $z = v_z t - d$ when $t > 0$ (Fig. 1). A uniform plane electromagnetic wave is incident at an angle θ_i from free space on the front surface of the slab. The plane of incidence is xz. By splitting incident wave into two orthogonal polarizations TE_y, TM_y we have:

$$\vec{E}^{inc}(\bar{r}) = \left[-E_{inc}^{TE}\hat{y} + E_{inc}^{TM}\left(\hat{k}^{inc}\times\hat{y}\right)\right]e^{-jk_0\hat{k}^{inc}\cdot\bar{r}} = \vec{E}_{inc}e^{-jk_0\hat{k}^{inc}\cdot\bar{r}} \tag{54}$$

$$\vec{H}^{inc}(\bar{r}) = \frac{1}{\eta_0}\left[-E_{inc}^{TM}\hat{y} - E_{inc}^{TE}\left(\hat{k}^{inc}\times\hat{y}\right)\right]e^{-jk_0\hat{k}^{inc}\cdot\bar{r}} = \vec{H}_{inc}e^{-jk_0\hat{k}^{inc}\cdot\bar{r}} \tag{55}$$

$$\vec{B}^{inc} = \mu_0\vec{H}^{inc}, \quad \vec{D}^{inc} = \varepsilon_0\vec{E}^{inc} \tag{56}$$

$$\hat{k}^{inc}\cdot\bar{r} = x\sin\theta_i - z\cos\theta_i \tag{57}$$

$$\left|\vec{k}^{inc}\right| = k_0 \tag{58}$$

$$\hat{k}^{inc}\times\hat{y} = \hat{z}\sin\theta_i + \hat{x}\cos\theta_i \tag{59}$$

From the boundary condition of tangential components and phase matching, it can be concluded that $k_x = k_0\sin\theta_i$ and $k_y = 0$. By taking the Fourier transform with respect to x, we have:

$$\vec{E}^{inc}(k_x, z) = \vec{E}_{inc}\cdot e^{jk_0\cos\theta_i}\cdot\delta(k_x - k_0\sin\theta_i) \tag{60}$$

$$\vec{H}^{inc}(k_x, z) = \vec{H}_{inc}\cdot e^{jk_0\cos\theta_i}\cdot\delta\left(k_x - k_0\sin\theta_i\right) \tag{61}$$

The reflected wave may be expressed as:

$$\vec{E}^{r}(\bar{r}) = \left[-E_r^{TE}\hat{y} + E_r^{TM}\left(\hat{k}^r\times\hat{y}\right)\right].e^{-j\vec{k}^r\cdot\bar{r}} = \vec{E}_r\cdot e^{-j\vec{k}^r\cdot\bar{r}} \tag{62}$$

$$\vec{H}^{r}(\bar{r}) = \frac{1}{\eta_0}\left[-E_r^{TE}\left(\hat{k}^r\times\hat{y}\right) - E_r^{TM}\hat{y}\right]\cdot e^{-j\vec{k}^r\cdot\bar{r}} = \vec{H}_r\cdot e^{-j\vec{k}^r\cdot\bar{r}} \tag{63}$$

$$\vec{B}^{r} = \mu_0\vec{H}^{r}, \quad \vec{D}^{r} = \varepsilon_0\vec{E}^{r} \tag{64}$$

in which:

$$\vec{k}^r\cdot\bar{r} = xk_0\sin\theta_i + zk_r^z \tag{65}$$

$$\left|\vec{k}^r\right| = \frac{\omega_r}{c_0} \tag{66}$$

$$\hat{k}^r\times\hat{y} = \hat{z}\frac{k_0}{\omega_r/c_0}\sin\theta_i - \hat{x}\frac{k_r^z}{\omega_r/c_0} \tag{67}$$

Notice that the fields inside the moving isotropic slab may be obtained from (52).

The moving boundary conditions require the continuity of $\hat{z}\times\vec{E}_{total} - (\hat{z}\cdot\vec{v})\vec{B}_{total}$ and $\hat{z}\times\vec{H}_{total} + (\hat{z}\cdot\vec{v})\vec{D}_{total}$ at the both boundaries. Total fields in region 1 are obtained as the sum of incident and reflected waves as $\vec{E}_{total} = \vec{E}^i + \vec{E}^r$ and $\vec{H}_{total} = \vec{H}^i + \vec{H}^r$. By applying cross product, we have:

$$\hat{z}\times\vec{E} - (\hat{z}\cdot\vec{v})\vec{B} = (-E_y - v_zB_x)\hat{x} + (E_x - v_zB_y)\hat{y} - v_zB_z\hat{z} \tag{68}$$

$$\hat{z}\times\vec{H} + (\hat{z}\cdot\vec{v})\vec{D} = (-H_y + v_zD_x)\hat{x} + (H_x + v_zD_y)\hat{y} + v_zD_z\hat{z} \tag{69}$$

Boundary conditions for tangential components require the continuity of $\begin{bmatrix}(-E_y - v_zB_x)\hat{x}\\(E_x - v_zB_y)\hat{y}\end{bmatrix}$ and $\begin{bmatrix}(-H_y + v_zD_x)\hat{x}\\(H_x + v_zD_y)\hat{y}\end{bmatrix}$ on both boundaries. On the other hand, according to the form of decomposition of the fields in the fundamental equation, the fundamental dyadic and the wave propagator it is appropriate to arrange the fields in the form of $\begin{bmatrix}\vec{E}_{xy}(z)\\\eta_0 J\cdot\vec{H}_{xy}(z)\end{bmatrix}$. Thus we express the first boundary condition in the form of $\begin{bmatrix}(E_x - v_zB_y)\hat{y}\\-1\times(-E_y - v_zB_x)\hat{x}\end{bmatrix}$ and the second one in the form of $\begin{bmatrix}\eta_0(-H_y + v_zD_x)\hat{x}\\\eta_0(H_x + v_zD_y)\hat{y}\end{bmatrix}$. Then the moving boundary conditions at $z = v_z t$ may be expressed as:

$$\begin{bmatrix} (E_x - v_z B_y) \\ -1 \times (-E_y - v_z B_x) \\ \eta_0(-H_y + v_z D_x) \\ \eta_0(H_x + v_z D_y) \end{bmatrix}^{(1)}_{z=v_z t} = \begin{bmatrix} (E_x - v_z B_y) \\ -1 \times (-E_y - v_z B_x) \\ \eta_0(-H_y + v_z D_x) \\ \eta_0(H_x + v_z D_y) \end{bmatrix}^{(2)}_{z=v_z t} \tag{70}$$

By substituting relations (54)–(67), the left-hand side of (70) can be written as follows:

$$\begin{aligned}
&\begin{bmatrix} (E_x - v_z B_y) \\ -1 \times (-E_y - v_z B_x) \\ \eta_0(-H_y + v_z D_x) \\ \eta_0(H_x + v_z D_y) \end{bmatrix}^{(1)}_{z=v_z t} \\
&= \begin{bmatrix} \left(\cos\theta_i E_{inc}^{TM} + v_z \frac{\mu_0}{\eta_0} E_{inc}^{TM}\right) e^{jk_0 v_z t\cdot\cos\theta_i} + \left(-\frac{k_r^z}{\omega_r/c_0} E_r^{TM} + v_z \frac{\mu_0}{\eta_0} E_r^{TM}\right) e^{-jv_z t\cdot k_r^z} \\ \left(-E_{inc}^{TE} - v_z \frac{\mu_0}{\eta_0} \cos\theta_i E_{inc}^{TE}\right) e^{jk_0 v_z t\cdot\cos\theta_i} + \left(-E_r^{TE} + v_z \frac{\mu_0}{\eta_0} \frac{k_r^z}{\omega_r/c_0} E_r^{TE}\right) e^{-jv_z t\cdot k_r^z} \\ \left(E_{inc}^{TM} + v_z \eta_0 \varepsilon_0 \cos\theta_i E_{inc}^{TM}\right) e^{jk_0 v_z t\cdot\cos\theta_i} + \left(E_r^{TM} - v_z \eta_0 \varepsilon_0 \frac{k_r^z}{\omega_r/c_0} E_r^{TM}\right) e^{-jv_z t\cdot k_r^z} \\ \left(-\cos\theta_i E_{inc}^{TE} - v_z \eta_0 \varepsilon_0 E_{inc}^{TE}\right) e^{jk_0 v_z t\cdot\cos\theta_i} + \left(\frac{k_r^z}{\omega_r/c_0} E_r^{TE} - v_z \eta_0 \varepsilon_0 E_r^{TE}\right) e^{-jv_z t\cdot k_r^z} \end{bmatrix} \\
&= \begin{bmatrix} \begin{bmatrix} 0 & \left(\cos\theta_i + v_z \frac{\mu_0}{\eta_0}\right) e^{jk_0 v_z t\cdot\cos\theta_i} \\ \left(-1 - v_z \frac{\mu_0}{\eta_0} \cos\theta_i\right) e^{jk_0 v_z t\cdot\cos\theta_i} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & (1 + \eta_0 v_z \varepsilon_0 \cos\theta_i) e^{jk_0 v_z t\cdot\cos\theta_i} \\ (-\cos\theta_i - \eta_0 v_z \varepsilon_0) e^{jk_0 v_z t\cdot\cos\theta_i} & 0 \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} \\
&\quad + \begin{bmatrix} \begin{bmatrix} 0 & \left(-\frac{k_r^z}{\omega_r/c_0} + v_z \frac{\mu_0}{\eta_0}\right) e^{-jv_z t. k_r^z} \\ \left(-1 + v_z \frac{\mu_0}{\eta_0} \frac{k_r^z}{\omega_r/c_0}\right) e^{-jv_z t. k_r^z} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \left(1 - \eta_0 v_z \varepsilon_0 \frac{k_r^z}{\omega_r/c_0}\right) e^{-jv_z t\cdot k_r^z} \\ \left(\frac{k_r^z}{\omega_r/c_0} - \eta_0 v_z \varepsilon_0\right) e^{-jv_z t\cdot k_r^z} & 0 \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \end{bmatrix} \\
&= \begin{bmatrix} Q \\ U \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} + \begin{bmatrix} V \\ W \end{bmatrix} \cdot \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \end{bmatrix}
\end{aligned} \tag{71}$$

And for the right-hand side of (70) we have:

$$\begin{aligned}
\begin{bmatrix} (E_x - v_z B_y) \\ -1 \times (-E_y - v_z B_x) \\ \eta_0(-H_y + v_z D_x) \\ \eta_0(H_x + v_z D_y) \end{bmatrix}^{(2)}_{z=v_z t} &= \begin{bmatrix} E_x \\ E_y \\ -\eta_0 H_y \\ \eta_0 H_x \end{bmatrix} + \frac{v_z}{c_0} \begin{bmatrix} -c_0 B_y \\ c_0 B_x \\ c_0 \eta_0 D_x \\ c_0 \eta_0 D_y \end{bmatrix} = \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix} + \frac{v_z}{c_0} \begin{bmatrix} c_0 J \cdot \vec{B}_{xy} \\ \eta_0 c_0 \vec{D}_{xy} \end{bmatrix} \\
&= \left(\bar{\bar{I}}_4 + \frac{v_z}{c_0} \left\{ \begin{bmatrix} J \cdot \boldsymbol{\zeta}_{\perp\perp} & -J \cdot \boldsymbol{\mu}_{\perp\perp} \cdot J \\ \boldsymbol{\varepsilon}_{\perp\perp} & -\boldsymbol{\xi}_{\perp\perp} \cdot J \end{bmatrix} + \begin{bmatrix} J \cdot \boldsymbol{\zeta}_{\perp} & J \cdot \boldsymbol{\mu}_{\perp} \\ \boldsymbol{\varepsilon}_{\perp} & \boldsymbol{\xi}_{\perp} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{zz} & \xi_{zz} \\ \zeta_{zz} & \mu_{zz} \end{bmatrix}^{-1} \right.\right. \\
&\quad \left.\left. \cdot \begin{bmatrix} -\boldsymbol{\varepsilon}_z & \frac{-\vec{k}_t}{k_i^z \beta_z - k_0} + \boldsymbol{\xi}_{zz} \cdot J \\ \frac{-J\cdot\vec{k}_t}{k_i^z \beta_z - k_0} + -\boldsymbol{\zeta}_z & \boldsymbol{\mu}_z \cdot J \end{bmatrix} \right\} \right) \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix}_{z=v_z t} \\
&= \bar{\bar{A}}_3 \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix}_{z=v_z t}
\end{aligned} \tag{72}$$

$$\begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix}^{(2)}_{z=v_z t} = \bar{\bar{A}}_3^{-1} \cdot \left(\begin{bmatrix} Q \\ U \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} + \begin{bmatrix} V \\ W \end{bmatrix} \cdot \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \end{bmatrix} \right) \tag{73}$$

Now is the time to apply boundary conditions at $z = v_z t - d$:

$$\begin{bmatrix} (E_x - v_z B_y) \\ -1 \times (-E_y - v_z B_x) \\ \eta_0(-H_y + v_z D_x) \\ \eta_0(H_x + v_z D_y) \end{bmatrix}^{(2)}_{z=v_z t-d} = \begin{bmatrix} (E_x - v_z B_y) \\ -1 \times (-E_y - v_z B_x) \\ \eta_0(-H_y + v_z D_x) \\ \eta_0(H_x + v_z D_y) \end{bmatrix}^{(3)}_{z=v_z t-d} \tag{74}$$

The left-hand side of (73) can be written as:

$$\begin{aligned} \bar{\bar{A}}_3 \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix}^{(2)}_{z=v_z t-d} &= \bar{\bar{A}}_3 \cdot \bar{\bar{P}}(z = v_z t - d) \cdot \begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_0 J \cdot \vec{H}_{xy}(z) \end{bmatrix}^{(2)}_{z=v_z t} \\ &= \bar{\bar{A}}_3 \cdot \bar{\bar{P}}(z = v_z t - d) \cdot \bar{\bar{A}}_3^{-1} \cdot \left(\begin{bmatrix} Q \\ U \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} + \begin{bmatrix} V \\ W \end{bmatrix} \cdot \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \end{bmatrix} \right) \end{aligned} \tag{75}$$

And for the right-hand side of (74) we have:

$$\begin{bmatrix} \begin{bmatrix} 0 & \left(-\frac{k_t^z}{\omega_t/c_0} + v_z \frac{\mu_0}{\eta_0}\right) e^{-j(v_z t-d)k_t^z} \\ \left(-1 + v_z \frac{\mu_0}{\eta_0} \frac{k_t^z}{\omega_t/c_0}\right) e^{-j(v_z t-d)k_t^z} & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & \left(1 - \eta_0 v_z \varepsilon_0 \frac{k_t^z}{\omega_t/c_0}\right) e^{-j(v_z t-d)k_t^z} \\ \left(\frac{k_t^z}{\omega_t/c_0} - \eta_0 v_z \varepsilon_0\right) e^{-j(v_z t-d)k_t^z} & 0 \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} E_t^{TE} \\ E_t^{TM} \end{bmatrix} = \begin{bmatrix} L \\ N \end{bmatrix} \cdot \begin{bmatrix} E_t^{TE} \\ E_t^{TM} \end{bmatrix} \tag{76}$$

By combination of (75) and (76), we have:

$$\left(-\bar{\bar{A}}_3 \cdot \bar{\bar{P}}(z = v_z t - d) \cdot \bar{\bar{A}}_3^{-1}\right) \cdot \begin{bmatrix} V \\ W \end{bmatrix} \cdot \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \end{bmatrix} + \begin{bmatrix} L \\ N \end{bmatrix} \cdot \begin{bmatrix} E_t^{TE} \\ E_t^{TM} \end{bmatrix} = \left(\bar{\bar{A}}_3 \cdot \bar{\bar{P}}(z = v_z t - d) \cdot \bar{\bar{A}}_3^{-1}\right) \begin{bmatrix} Q \\ U \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} \tag{77}$$

$$\left(-\bar{\bar{A}}_3 \cdot \bar{\bar{P}}(z = v_z t - d) \cdot \bar{\bar{A}}_3^{-1}\right) \cdot \begin{bmatrix} V \\ W \end{bmatrix} = \begin{bmatrix} V' \\ W' \end{bmatrix} \tag{78}$$

$$\left(-\bar{\bar{A}}_3 \cdot \bar{\bar{P}}(z = v_z t - d) \cdot \bar{\bar{A}}_3^{-1}\right) \cdot \begin{bmatrix} Q \\ U \end{bmatrix} = \begin{bmatrix} Q' \\ U' \end{bmatrix} \tag{79}$$

$$\begin{bmatrix} V' & L \\ W' & N \end{bmatrix} \cdot \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \\ E_t^{TE} \\ E_t^{TM} \end{bmatrix} = \begin{bmatrix} Q' \\ U' \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} \tag{80}$$

In (80) E_r^{TE}, E_r^{TM}, E_t^{TE}, E_t^{TM} are unknowns of the problem from which we can obtain reflection and transmission coefficients. For this purpose, we have:

$$\begin{bmatrix} E_R \\ E_T \end{bmatrix} = \begin{bmatrix} V' & L \\ W' & N \end{bmatrix}^{-1} \cdot \begin{bmatrix} Q' \\ U' \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} = \begin{bmatrix} S \\ Z \end{bmatrix} \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} \tag{81}$$

$$E_R = \begin{bmatrix} E_r^{TE} \\ E_r^{TM} \end{bmatrix} = S \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} \tag{82}$$

$$E_T = \begin{bmatrix} E_t^{TE} \\ E_t^{TM} \end{bmatrix} = Z \cdot \begin{bmatrix} E_{inc}^{TE} \\ E_{inc}^{TM} \end{bmatrix} \tag{83}$$

$$R = \frac{|E^r|}{|E^{inc}|} = \frac{\sqrt{(E_r^x)^2 + (E_r^y)^2 + (E_r^z)^2}}{\sqrt{(E_{inc}^x)^2 + (E_{inc}^y)^2 + (E_{inc}^z)^2}} = \frac{\sqrt{\left(\left(\frac{k_r^z}{\omega_r/c_0}\right)^2 + \left(\frac{k_0}{\omega_r/c_0} \sin\theta_i\right)^2\right) \cdot (E_r^{TM})^2 + (E_r^{TE})^2}}{\sqrt{\left(E_{inc}^{TM}\right)^2 + \left(E_{inc}^{TE}\right)^2}} \tag{84}$$

$$T = \frac{|E^t|}{|E^{inc}|} = \frac{\sqrt{(E_t^x)^2 + (E_t^y)^2 + (E_t^z)^2}}{\sqrt{(E_{inc}^x)^2 + (E_{inc}^y)^2 + (E_{inc}^z)^2}} = \frac{\sqrt{(E_t^{TM})^2 + (E_t^{TE})^2}}{\sqrt{(E_{inc}^{TM})^2 + (E_{inc}^{TE})^2}} \tag{85}$$

Moreover, the fields inside the slab can be obtained from (52) and (73).

The above said was for a moving slab in the free space. For a moving slab in an arbitrary isotropic homogeneous medium with parameters ε_{r1}, μ_{r1}, the following points should be considered:

1. In all the relations c_1, k_1 should be used instead of c_0, k_0. For example: $\beta = \frac{v}{c_1}$, $\vec{\Omega} = \frac{n^2-1}{1-n^2\beta^2}\frac{\vec{\beta}}{c_1}$, $k_i^x = k_1 \sin(\theta_i)$ and $k_i^z = -k_1 \cos(\theta_i)$. In which $k_1 = \frac{\omega_i}{c_1}$ and $c_1 = 1/\sqrt{\varepsilon_1 \mu_1}$. Using dispersion relations for the reflected and transmitted waves $k_r^2 = \omega_r^2 \varepsilon_1 \mu_1$ and $k_t^2 = \omega_t^2 \varepsilon_1 \mu_1$ and according to (4) we have:

$$k_r^z = k_1(2\beta_z + \cos(\theta_i) + \beta_z^2 \cos(\theta_i))/(1-\beta_z^2) \tag{86}$$
$$k_t^z = -k_1 \cos(\theta_i) \tag{87}$$

It is noteworthy that the relations in fundamental dyadic method turn into $\begin{bmatrix} \vec{E}_{xy}(z) \\ \eta_1 J \cdot \vec{H}_{xy}(z) \end{bmatrix}$ and $\begin{bmatrix} c_1 J \cdot \vec{B}_{xy} \\ \eta_1 c_1 \vec{D}_{xy} \end{bmatrix}$.

2. In relations (9)–(10) we have: $n = \sqrt{(\varepsilon_{r2}\mu_{r2})/(\varepsilon_{r1}\mu_{r1})}$. In which ε_{r2}, μ_{r2} are electromagnetic parameters of the slab in the stationary case.

3. Constitutive relations of a stationary bianisotropic slab in this medium are:

$$\vec{D} = \varepsilon_0 \varepsilon_{r1} \left(\frac{\bar{\bar{\varepsilon}}_{r2}}{\varepsilon_{r1}} \vec{E} + \eta_0 \eta_{r1} \frac{\bar{\bar{\xi}}_{r2}}{\varepsilon_{r1}\eta_{r1}} \vec{H} \right) \tag{88}$$

$$\vec{B} = \frac{1}{c_0 c_{r1}} \left(c_{r1} \bar{\bar{\zeta}}_{r2} \vec{E} + \eta_0 \eta_{r1} \frac{c_{r1} \bar{\bar{\mu}}_r}{\eta_{r1}} \vec{H} \right) \tag{89}$$

$$\eta_{r1} = \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}, \quad c_{r1} = \frac{1}{\sqrt{\varepsilon_{r1}\mu_{r1}}}, \quad k_1 = \frac{\omega}{c_1} \tag{90}$$

And thus:

$$\vec{D} = \varepsilon_1 \left(\frac{\bar{\bar{\varepsilon}}_{r2}}{\varepsilon_{r1}} \vec{E} + \eta_1 \frac{\bar{\bar{\xi}}_{r2}}{\varepsilon_{r1}\eta_{r1}} \vec{H} \right) \tag{91}$$

$$\vec{B} = \frac{1}{c_1} \left(c_{r1} \bar{\bar{\zeta}}_{r2} \vec{E} + \eta_1 \frac{c_{r1} \bar{\bar{\mu}}_r}{\eta_{r1}} \vec{H} \right) \tag{92}$$

By comparing these relations with the relation of a moving slab (7)–(10), we have:

$$\bar{\bar{\varepsilon}}_r = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \bar{\bar{A}} \tag{93}$$

$$\bar{\bar{\xi}}_r = \frac{\bar{\bar{\Omega}}}{\sqrt{\varepsilon_0 \mu_0}} \cdot \frac{1}{\sqrt{\varepsilon_{r1}\mu_{r1}}} \tag{94}$$

$$\bar{\bar{\zeta}}_r = -\frac{\bar{\bar{\Omega}}}{\sqrt{\varepsilon_0 \mu_0}} \cdot \frac{1}{\sqrt{\varepsilon_{r1}\mu_{r1}}} \tag{95}$$

$$\bar{\bar{\mu}}_r = \frac{\mu_{r2}}{\mu_{r1}} \bar{\bar{A}} \tag{96}$$

3. DISCUSSION

Here, in order to verify the validity of our method, the limiting case of a slab with vanishing velocity, is considered and a comparison is made between the obtained expressions for reflection and transmission coefficients with Fresnel's equations [23].

In this case we have:

$$v_x = v_y = v_z = 0 \tag{97}$$

$$\bar{\bar{A}} = = \frac{1-\beta^2}{1-n^2\beta^2} \left[\bar{\bar{I}} - \frac{n^2-1}{1-\beta^2} \vec{\beta}\vec{\beta} \right] = \bar{\bar{I}} \tag{98}$$

$$\vec{\Omega} = \frac{n^2-1}{1-n^2\beta^2}\frac{\vec{\beta}}{c_0} = 0 \tag{99}$$

Thus, the constitutive relations of (7) and (8) reduce to:

$$\vec{D} = \varepsilon\vec{E} \tag{100}$$

$$\vec{B} = \mu\vec{H} \tag{101}$$

According to (53) and (72), we have:

$$\bar{\bar{A}}_3 = \bar{\bar{I}}_4 \tag{102}$$

$$\bar{\bar{P}} = \exp\left(jk_0 z\bar{\bar{M}}\right) \tag{103}$$

$\bar{\bar{M}}$ may be obtained as [22] in Section 7.1:

$$\bar{\bar{M}} = \begin{pmatrix} 0 & -\mu I_2 + \frac{1}{\varepsilon k_0^2}k_t k_t \\ -\varepsilon I_2 - \frac{1}{\mu k_0^2}J\cdot k_t k_t\cdot J & 0 \end{pmatrix} \tag{104}$$

For this case, the eigenvalues are given by:

$$\lambda_+^2 = \lambda_-^2 = \lambda^2 = \varepsilon\mu - k_t^2/k_0^2 \tag{105}$$

and $\bar{\bar{P}}$ may be derived as [22] in Section 7.1:

$$\bar{\bar{P}} = \exp(jk_0 d\bar{\bar{M}}) = I_4\cos(k_0 d\lambda) + \frac{j}{\lambda}M\sin(k_0 d\lambda) \tag{106}$$

The propagator may be rewritten in the $\{\hat{e}_\parallel,\, \hat{e}_\perp\}$ system as:

$$\bar{\bar{P}} = \exp\left(jk_0 d\bar{\bar{M}}\right) = \begin{pmatrix} \left(\hat{e}_\parallel\hat{e}_\parallel + \hat{e}_\perp\hat{e}_\perp\right)\cos(k_0 d\lambda) & -j\left(\hat{e}_\parallel\hat{e}_\parallel\frac{\lambda}{\varepsilon_2} + \hat{e}_\perp\hat{e}_\perp\frac{\mu_2}{\lambda}\right)\sin(k_0 d\lambda) \\ -j\left(\hat{e}_\parallel\hat{e}_\parallel\frac{\varepsilon_2}{\lambda} + \hat{e}_\perp\hat{e}_\perp\frac{\lambda}{\mu_2}\right)\sin(k_0 d\lambda) & \left(\hat{e}_\parallel\hat{e}_\parallel + \hat{e}_\perp\hat{e}_\perp\right)\cos(k_0 d\lambda) \end{pmatrix} \tag{107}$$

where $\lambda^2 = \varepsilon_2\mu_2 - k_t^2/k_0^2$. Calculations show that principal of blocks of the scattering dyadic are:

$$\begin{cases} 2T_{11} = \hat{e}_\parallel\hat{e}_\parallel\left(2\cos(k_0 d\lambda) + j\left(\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)\right) \\ \qquad\quad +\hat{e}_\perp\hat{e}_\perp\left(2\cos(k_0 d\lambda) + j\left(\frac{\lambda}{\mu_2\cos\theta_i} + \frac{\mu_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)\right) \\ 2T_{22} = \hat{e}_\parallel\hat{e}_\parallel\left(2\cos(k_0 d\lambda) - j\left(\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)\right) \\ \qquad\quad +\hat{e}_\perp\hat{e}_\perp\left(2\cos(k_0 d\lambda) - j\left(\frac{\lambda}{\mu_2\cos\theta_i} + \frac{\mu_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)\right) \end{cases} \tag{108}$$

and

$$\begin{cases} 2T_{12} = j\left(\hat{e}_\parallel\hat{e}_\parallel\left(-\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right) + \hat{e}_\perp\hat{e}_\perp\left(\frac{\lambda}{\mu_2\cos\theta_i} - \frac{\mu_2\cos\theta_i}{\lambda}\right)\right)\sin(k_0 d\lambda) \\ T_{21} = -T_{12} \end{cases} \tag{109}$$

Consequently, the reflection and transmission dyadics are:

$$\begin{cases} r = \hat{e}_\parallel\hat{e}_\parallel r_{\parallel\parallel} + \hat{e}_\perp\hat{e}_\perp r_{\perp\perp} \\ t = \hat{e}_\parallel\hat{e}_\parallel t_{\parallel\parallel} + \hat{e}_\perp\hat{e}_\perp t_{\perp\perp} \end{cases} \tag{110}$$

where

$$\begin{cases} r_{\parallel\parallel} = \dfrac{j\left(-\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)}{2\cos(k_0 d\lambda) - j\left(\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)} = r_{1\parallel\parallel}\dfrac{1-\exp(2jk_0\lambda d)}{1-r_{1\parallel\parallel}^2\exp(2jk_0\lambda d)} \\ r_{\perp\perp} = \dfrac{j\left(\frac{\lambda}{\mu_2\cos\theta_i} - \frac{\mu_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)}{2\cos(k_0 d\lambda) - j\left(\frac{\lambda}{\mu_2\cos\theta_i} + \frac{\mu_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)} = r_{1\perp\perp}\dfrac{1-\exp(2jk_0\lambda d)}{1-r_{1\perp\perp}^2\exp(2jk_0\lambda d)} \end{cases} \tag{111}$$

$$\begin{cases} t_{\|\|} = \dfrac{2}{2\cos(k_0 d\lambda) - j\left(\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)} = \dfrac{\left(1 - r_{1\|\|}^2\right)\exp(2jk_0\lambda d)}{1 - r_{1\|\|}^2\exp(2jk_0\lambda d)} \\ t_{\perp\perp} = \dfrac{2}{2\cos(k_0 d\lambda) - j\left(\frac{\lambda}{\mu_2\cos\theta_i} + \frac{\mu_2\cos\theta_i}{\lambda}\right)\sin(k_0 d\lambda)} = \dfrac{\left(1 - r_{1\perp\perp}^2\right)\exp(2jk_0\lambda d)}{1 - r_{1\perp\perp}^2\exp(2jk_0\lambda d)} \end{cases} \tag{112}$$

and

$$\begin{cases} r_{1\|\|} = \dfrac{\frac{1}{2}\left(\frac{\lambda}{\varepsilon_2\cos\theta_i} - \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)}{1 + \frac{1}{2}\left(\frac{\lambda}{\varepsilon_2\cos\theta_i} + \frac{\varepsilon_2\cos\theta_i}{\lambda}\right)} = \dfrac{\lambda - \varepsilon_2\cos\theta_i}{\lambda + \varepsilon_2\cos\theta_i} \\ r_{1\perp\perp} = \dfrac{\frac{1}{2}\left(\frac{\mu_2\cos\theta_i}{\lambda} - \frac{\lambda}{\mu_2\cos\theta_i}\right)}{1 + \frac{1}{2}\left(\frac{\mu_2\cos\theta_i}{\lambda} + \frac{\lambda}{\mu_2\cos\theta_i}\right)} = \dfrac{\mu_2\cos\theta_i - \lambda}{\mu_2\cos\theta_i + \lambda} \end{cases} \tag{113}$$

Equations (112) are recognized as Fresnel's equations for reflection and transmission coefficients of a static slab [23].

4. NUMERICAL RESULTS

In this section, we illustrate the analysis presented in the previous sections through some numerical examples. The results of the presented method are compared by the method of Lorentz transformation [2] for TE waves. A great agreement is observed between the results, which confirms the validity of our method. For TM waves similar results are obtained.

The programming task can be easily done in a language that supports matrix manipulations, e.g., MATLAB.

4.1. Example 1

Consider a dielectric slab with electromagnetic parameters $\varepsilon_{r2} = 2$, $\mu_{r2} = 1$ and a thickness of $d = 10\,\text{cm}$ moving with constant velocity of $0.01c_0(\hat{i}+\hat{j}+\hat{k})$ in free space. A TE wave with $\omega_i = 100\,\text{MHz}$ is incident on the front surface of the slab with angle of θ_i. Fig. 2 presents the reflectance versus angle of incidence obtained by Fundamental method and Lorentz transformation. As it can be seen great agreement is observed between the results.

4.2. Example 2

Consider a slab with electromagnetic parameters $\varepsilon_{r2} = 2$, $\mu_{r2} = 3$ and a thickness of $d = 10\,\text{cm}$ moving with constant velocity of $c_0(0.02\hat{i} + 0.03\hat{j} + 0.01\hat{k})$ in free space illuminated by a TE wave with $\omega_i = 100\,\text{MHz}$ and the angle of θ_i. Fig. 3 presents the reflectance versus angle of incidence obtained by Fundamental method and Lorentz transformation. A great agreement is observed between the results.

4.3. Example 3

Consider a slab with electromagnetic parameters $\varepsilon_{r2} = 2$, $\mu_{r2} = 3$ and a thickness of $d = 10\,\text{cm}$ moving with constant velocity of $c_0(0.2\hat{i} + 0.1\hat{j} + 0.5\hat{k})$ in a medium with electromagnetic parameters $\varepsilon_{r2} = 3$, $\mu_{r2} = 5$ excited by a TE wave with $\omega_i = 100\,\text{MHz}$ and the angle of θ_i. Fig. 4 presents the reflectance versus angle of incidence obtained by Fundamental method and Lorentz transformation. A great agreement is observed between the results which verifies that the method holds true for the velocities near to the speed of light.

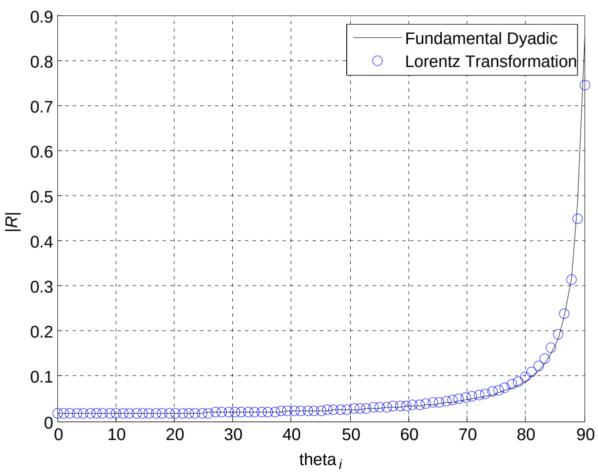

Figure 2. Reflectance of a dielectric slab with electromagnetic parameters $\varepsilon_{r2} = 2$, $\mu_{r2} = 1$ and a thickness of $d = 10\,\mathrm{cm}$ moving with constant velocity of $0.01c_0(\hat{\imath} + \hat{\jmath} + \hat{k})$ in free space illuminated by a TE wave with $\omega_i = 100\,\mathrm{MHz}$ versus the angle of incidence.

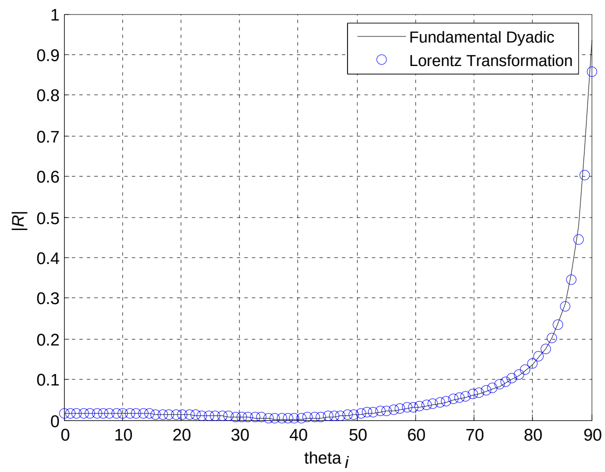

Figure 3. Reflectance of a slab with electromagnetic parameters $\varepsilon_{r2} = 2$, $\mu_{r2} = 3$ and a thickness of $d = 10\,\mathrm{cm}$ moving with constant velocity of $c_0(0.02\hat{\imath} + 0.03\hat{\jmath} + 0.01\hat{k})$ in free space illuminated by a TE wave with $\omega_i = 100\,\mathrm{MHz}$ versus the angle of incidence.

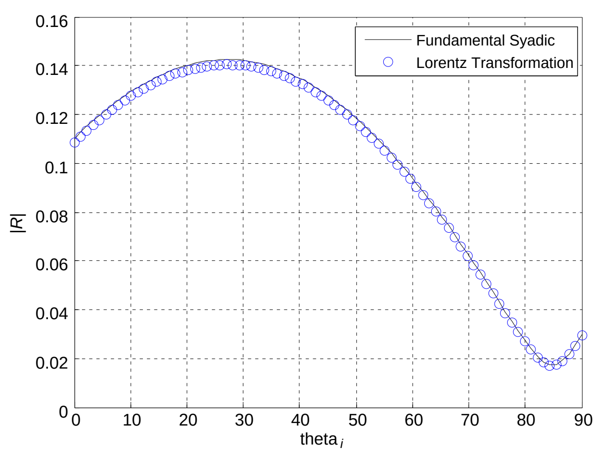

Figure 4. Reflectance of a slab with electromagnetic parameters $\varepsilon_{r2} = 2$, $\mu_{r2} = 3$ and a thickness of $d = 10\,\mathrm{cm}$ moving with constant velocity of $c_0(0.2\hat{\imath} + 0.1\hat{\jmath} + 0.5\hat{k})$ in a medium with electromagnetic parameters $\varepsilon_{r2} = 3$, $\mu_{r2} = 5$ excited by a TE wave with $\omega_i = 100\,\mathrm{MHz}$ versus the angle of incidence.

5. CONCLUSION

In this paper we have presented a new method for analyzing reflection and transmission of obliquely incident electromagnetic waves by a moving slab based on the concept of propagators. The propagators map the total field at any point inside the slab, to the fields on the left-hand side boundary of the slab. This method is simple and systematic and easily gives the reflection and the transmission dyadics of the slab. Moreover the method can be employed to analyze more complicated structures. By considering the limiting case of a slab with vanishing velocity, we have arrived at well-known Fresnel's equations for reflection and transmission coefficients. Furthermore, several numerical examples show the applicability of the analysis and comparison of the results with the method of Lorentz transformation verifies the validity of the method.

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