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Constraint-Based Synthesis of Linear Antenna Array Using Modified Invasive Weed Optimization

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Abstract—This paper presents a novel technique for the synthesis of unequally spaced linear antenna array. The modified Invasive Weed Optimization (IWO) algorithm is applied to optimize the antenna element positions for suppressing peak side lobe level (PSLL) and for achieving nulls in specified directions. The novelty of the proposed approach is in the application of a constraint-based static penalty function during optimization of the array. The static penalty function is able to put selective pressure on the PSLL, the first null beam width (FNBW) or the accurate null positioning as desired by the application at hand lending a high degree of flexibility to the synthesis process. Various design examples are considered and the obtained results are validated by comparing with the results obtained using Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO) and Cat Swarm Optimization (CSO). Results demonstrate that the proposed method outperforms the previously published methods in terms of a significant reduction in peak side lobe level while maintaining strong nulls in desired directions. The flexibility and ease of implementation of the modified IWO algorithm in handling the constraints using static penalty function is evident from this analysis, showing the usefulness of the constraint based method in electromagnetic optimization problems.

1. INTRODUCTION

Antenna arrays [1, 2] are being widely used in wireless, satellite, mobile and radar communications systems. They help in improving the system performance by enhancing directivity, improving signal quality, extending system coverage and increasing spectrum efficiency. The performance of the communication system greatly depends on the efficient design of the antenna arrays.

Systems with narrow first null beam width (FNBW) are desired for obtaining high directivity. On the other hand, systems need to maintain low peak side lobe level (PSLL) to avoid interference with other systems operating in same frequency band. The above mentioned requirements of PSLL and FNBW are in contrast to each other as arrays with narrow beam width generally do not produce lower side lobe levels and vice versa, i.e., the performance cannot be improved significantly for one aspect without degrading the other. In many applications it becomes necessary to sacrifice gain and beam width in order to achieve lower side lobe level. Also the increasing EM pollution has prompted the placing of nulls in undesired directions. So it is necessary to design the antenna array with low side lobe levels while maintaining fixed beam width and placing of nulls in undesired directions.

The radiation pattern of the antenna array depends on the structure of the array, distance between the elements and amplitude and phase excitation of individual elements. For the linear array geometry, suppressing peak side lobe level and placing of nulls in specified directions can be achieved in two ways either by optimizing the spacings between the element positions while maintaining uniform excitations, or by employing non uniform excitations of the elements while using periodic placement of antenna elements. Linear antenna array synthesis has been extensively studied from the past 5 decades.

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In order to optimize this type of electromagnetic design problems, evolutionary algorithms such as genetic algorithm (GA) [3–6], simulated annealing (SA) [7], differential evaluation (DE) [8,9], Particle swarm optimization (PSO) [10–17], Ant colony optimization (ACO) [18], Invasive weed optimization (IWO) [20,22] and Cat swarm optimization (CSO) [23] have been successfully applied. All the above mentioned evolutionary algorithms have shown the capability of searching for global solution in electromagnetic optimization problems.

The present work deals with an improved variant of the Invasive weed optimization (IWO) [21] algorithm. The classical IWO can be modified to incorporate a sinusoidal variation in the standard deviation of the randomly distributed solutions during the optimization. This leads to quicker detection of optimize solutions. The present work also introduces constraint handling as a new concept in antenna array synthesis through the application of the static penalty function [24]. The use of the static penalty function allows the designer to set desired values of PSLL, FNBW and null depth. This renders a great degree of flexibility to the synthesis as the different design parameters may be assigned different penalty coefficients depending on the application requirements. To the best of the author's knowledge, constraint handling using static penalty function has not yet been applied to optimize the element positions in linear antenna array synthesis. In this communication, several design examples are considered to show how the modified IWO using constraint handling is useful in controlling the shape of the radiation pattern while determining the optimal performance.

Brief description of the linear antenna array is given in Section 2. Section 3 presents the modified IWO algorithm. The problem formulation without static penalty function is discussed in Section 4. The problem formulation with static penalty function and the effectiveness of this approach is discussed detail in Section 5. Lastly, Sections 6 and 7 highlight the achievements of this research work.

2. LINEAR ANTENNA ARRAY

The geometry of uniform linear antenna array with $2N$ elements placed symmetrically along x axis is considered as shown in Fig. 1.

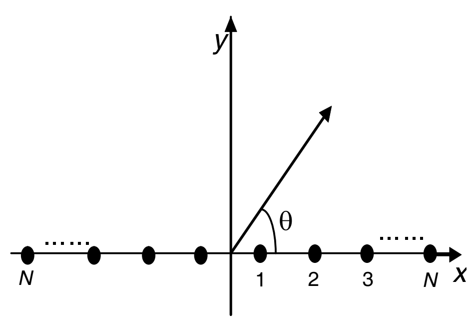

Figure 1. Geometry of the symmetrically placed linear array.

The array factor (AF) for the linear antenna array in the azimuth plane [13,17] is

$$AF(\theta) = 2\sum_{n=1}^{N} I_n \cos\left[kx_n \cos(\theta) + \varphi_n\right] \tag{1}$$

where $k = 2\pi/\lambda$ is the wave number, θ is the azimuth angle and I_n, φ_n and x_n are the excitation amplitude, phase and position of element n respectively.

Let us assume uniform amplitude and phase excitations for all elements, such that, $I_n = 1$ and $\varphi_n = 0$. Thus the array factor can be simplified as follows:

$$AF(\theta) = 2\sum_{n=1}^{N} \cos\left[kx_n \cos(\theta)\right] \tag{2}$$

To achieve the desired radiation pattern with minimum side lobe levels and nulls at desired directions, the optimized positions of $x_1, x_2, \ldots, x_n$ of the corresponding elements need to be determined. In antenna arrays, placement of antenna elements is critical, because if the adjacent elements are placed too close, mutual coupling effects dominate and if they are placed too far, then grating lobes appear. While solving for optimized positions of the elements, the following conditions [14] must be satisfied in order to overcome the disadvantages mentioned above.

$$|x_i - x_j| > 0.25\lambda \tag{3}$$

$$\min\{x_i\} > 0.125\lambda \qquad i = 1, 2, \ldots, N. \quad i \neq j \tag{4}$$

3. MODIFIED INVASIVE WEED OPTIMIZATION

IWO algorithm was first introduced by Mehrabain and Lucas in 2006 [19]. IWO is inspired by weed colonization in nature and is based on weed biology and ecology. Weeds invade the unused resources in the cropping field by means of dispersal. The invade weeds grow to a flowering weeds and produces new seeds based on their strength in the colony. The produced seeds are dispersed randomly over the field and grow to flowering weeds. The total weeds in the colony struggle for existence with competitors and those weeds which have better strength can survive to produce more number of seeds. This concept of seeding, growth and competition of weed colonization in a cropping field is mathematically modelled for solving complex optimization problems. It is simple and robust with accurate global search ability. Distinctive properties of the IWO algorithm are that it allows all of the population (weeds) to participate in the reproduction process and reproduction of weeds happens without mating. This leads to a global search for the optimized solutions. The steps involved in the modified IWO are presented below.

3.1. Initialization

A finite number of seeds are initialized randomly in the N dimensional solution space with random positions. Each seed's position represents one possible solution of the optimization problem.

3.2. Reproduction

In reproduction stage, the fitness value of each seed is determined. This process resembles growing of seed to flowering weed. The magnitude of the fitness value determines the reproductive capability of each seed. The number of reproduced seeds from each seed is calculated based upon the seed's own fitness value and the colony's lowest & highest fitness values. Thus, the number of seeds produced increases linearly from weed with worst fitness to weed with better fitness. That is, those weeds with worst fitness values produce less number of seeds and vice versa. The procedure is illustrates in Fig. 2(a).

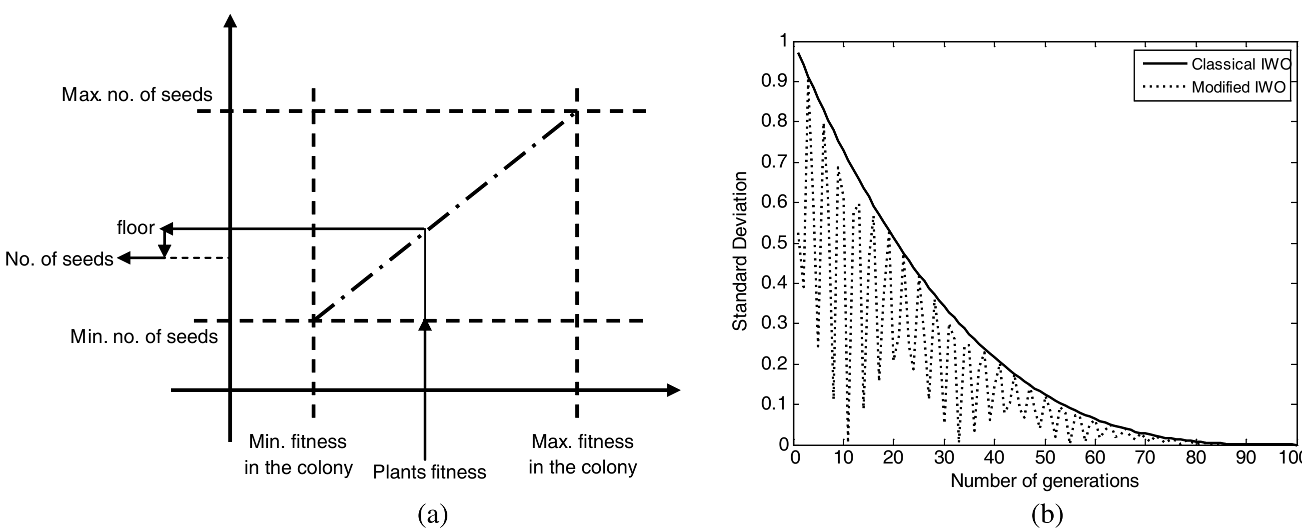

Figure 2. (a) Seeds reproduction procedure [19]. (b) Comparison of the standard deviations for the classical and modified IWO.

A significant advantage of the algorithm is that it allows all weeds to participate in the reproduction process. This is beneficial because under certain conditions, weed with worst fitness value may also have some useful information to contribute during the evolutionary process.

3.3. Spatial Dispersal

The produced seeds are dispersed randomly over the search space by normal distribution with zero mean and varying standard deviation. That is, the produced seeds are scattered around the mother weed, leading to local search. The number of seeds (S) produced by each weed is given by

$$S = Floor\left[S_{\min} + \left(\frac{f - f_{\min}}{f_{\max} - f_{\min}}\right) S_{\max}\right] \tag{5}$$

where $f_{\max}$ and $f_{\min}$ are maximum and minimum numbers of seeds that may be produced from each weed, respectively. $f_{\max}$ and $f_{\min}$ are maximum and minimum fitness values in the colony. It was suggested that the maximum number of seeds between 3 and 5 is a good choice to improve the performance of the IWO optimizer. In addition, the best choice for minimum number of seeds is zero for all the design examples.

The standard deviation (σ_g) of the distribution at generation number g reduces nonlinearly over the generations ranging from initial standard deviation $\sigma_{initial}$ to final standard deviation σ_{final} and is given by:

$$\sigma_g = \frac{(gen_{\max} - g)^{nl}}{(gen_{\max})^{nl}} (\sigma_{initial} - \sigma_{final}) + \sigma_{final} \tag{6}$$

To enhance the performance of IWO, $|\cos(gen)|$ term [21] is added for periodic variation in standard deviation, which helps in exploring the better solutions quickly. The variation of standard deviation in classical and modified IWO is shown in Fig. 2(b). The modified standard deviation is given by:

$$\sigma_g = \frac{(gen_{\max} - g)^{nl}}{(gen_{\max})^{nl}} |\cos(gen)| (\sigma_{initial} - \sigma_{final}) + \sigma_{final} \tag{7}$$

where $gen_{\max}$ is the maximum number of generations and nl the non linear modulation index. It is noticed from the literature [20] that, the value of nl has shown significant effect on the performance of the IWO. It was suggested that the good choice for nl is 3.

3.4. Competitive Exclusion

The new seeds produced grow to flowering weeds and are placed together with parent weeds in the colony. So there is a need of limiting the number of weeds and elimination is done based on the fitness values of the weeds in the colony. Weeds with worst fitness are eliminated until the maximum number of weeds ($P_{\max}$) in the colony is reached. Thus weeds with better fitness survive. Previous work [20–22] has shown that IWO algorithm gives better performance when the $P_{\max}$ is chosen between 10 and 20. The selected weeds go to the next generation. The steps involved in the IWO algorithm are shown in Fig. 3.

4. FORMULATION OF FITNESS FUNCTION WITHOUT STATIC PENALTY

The optimized positions of the elements of a linear antenna array are determined in order to achieve the minimum side lobe levels while placing nulls in specified directions. The fitness function [13, 17] for the optimization is composed of two terms, one for minimizing the side lobe level in between the spatial regions of θ_{li} & θ_{ui}, where i is the desired sidelobe region and the other for achieving deep nulls at θ_k for k nulls. If $\Delta\theta_i = \theta_{ui} - \theta_{li}$, then

$$Fitness = \sum\nolimits_i \frac{1}{\Delta\theta_i} \int_{\theta_{li}}^{\theta_{ui}} |AF(\theta)|^2 \, d\theta + \sum\nolimits_k |AF(\theta_k)|^2 \tag{8}$$

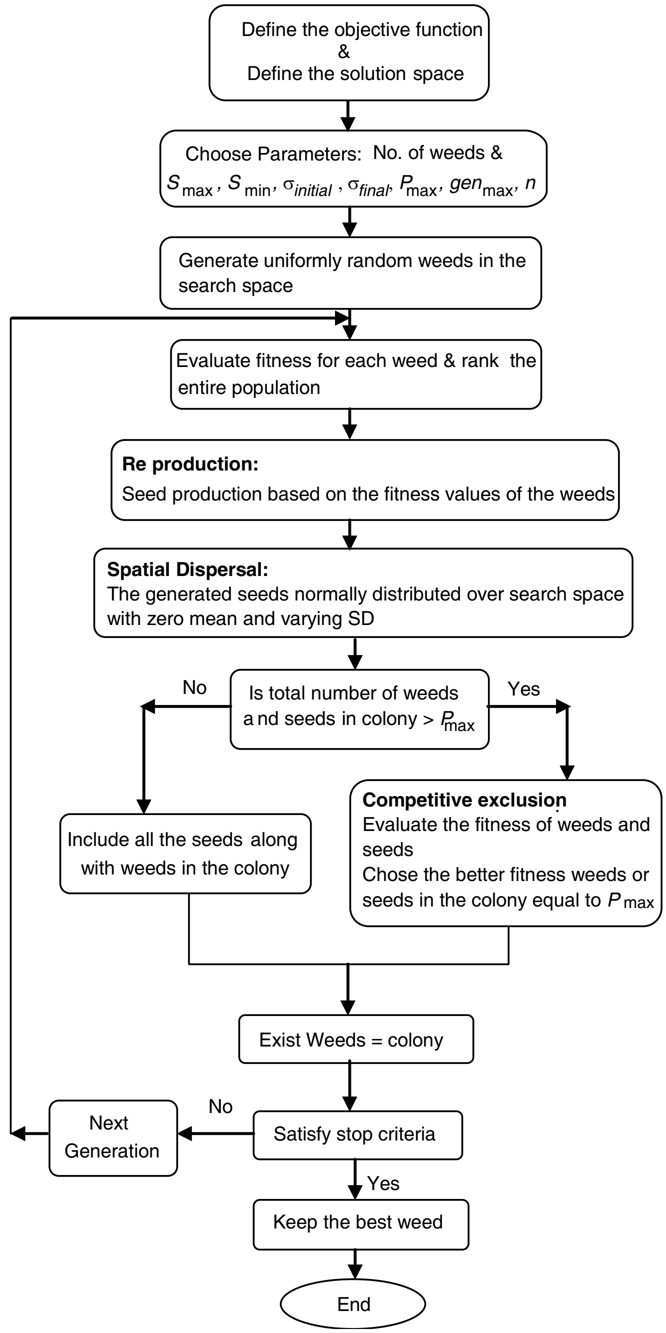

Figure 3. Flow chart of IWO.

4.1. Design Examples

In order to achieve minimum PSLL and deep nulls in specified directions, modified IWO algorithm as discussed in Section 3 of this paper is applied to synthesize the linear antenna array with the topology as discussed in Section 2. For the purpose of comparison with previously published works, the optimum antenna positions of the linear array are computed through the use of PSO & IWO algorithms also. The initial parameters of the PSO, IWO and modified IWO algorithm are given in Table 1. These parameters were set after experimental verification while complying with the guidelines provided in literature [19, 20, 23].

The algorithms are executed 10 times with 2000 generations at each run. Out of these 10 runs,

Table 1. Parameter setup for IWO, modified IWO and PSO.

IWO/Modified IWO		PSO	
Parameter	Value	Parameter	Value
$S_{\max}$	4	Swarm size	20
$S_{\min}$	0	c_1	2
$\sigma_{initail}$	0.1	c_2	2
σ_{finial}	0.00015	ω	Linearly varies from 0.9–0.4
$P_{\max}$	20	$v_{\max}$	4
nl	3	$v_{\min}$	-4
Initial population size	10		

the median run is chosen for illustration. The radiation pattern of the array is computed at 1801 angles in the azimuth region of 0° to 180°. All the computations are performed using MATLAB on a PC operating at 3 GHz with 2 GB of RAM. Different linear array designs are considered and the corresponding element positions are optimized to accomplish the design requirements while maintaining uniform amplitude and phase excitations. The obtained results are compared to the non optimized uniformly spaced array, referred to herein after as conventional array.

4.1.1. Design Example: 28 Element Linear Array with Three Null Conditions

This example illustrates the synthesis of a 28 element array for achieving minimum side lobes with desired nulls at $\theta = 120°$, 122.5° and 125°. The element positions for the PSO, IWO and modified IWO synthesized arrays are given in Table 2. The element positions are normalized with respect to λ. Because of the symmetry of the $2N$ element linear antenna array, positions of the N elements are given in Table 2. Table 3 shows a comparison of the PSLL, FNBW and null depths obtained using PSO, IWO and modified IWO algorithms. The array pattern obtained using the modified IWO algorithm is shown in Fig. 4, along with the conventional and PSO synthesized array pattern.

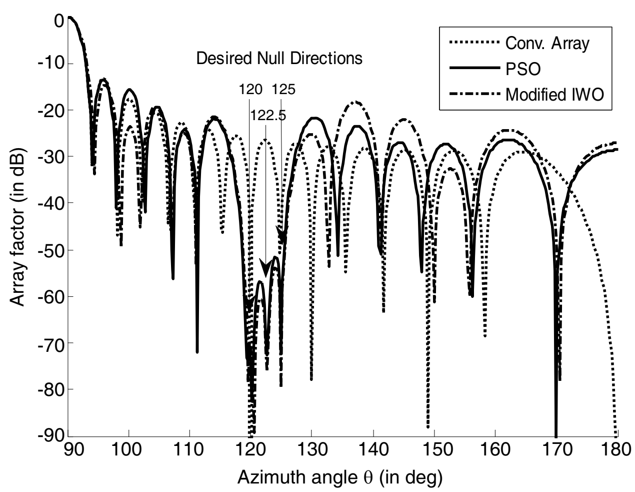

Figure 4. The normalized array pattern of 28-element linear array optimized using modified IWO with desired nulls at 120°, 122.5° and 125°.

The results (Table 3) show that nulls as deep as −70 dB are achieved at the desired directions while maintaining the FNBW of the non-optimized array. The PSLL of the array optimized using modified IWO shows a slight improvement (1.52 dB) over the PSLL of the non-optimized array.

Table 2. Element positions of the 28-element linear array (normalised with respect to λ).

Conventional array	±0.2500	± 0.7500	± 1.2500	± 1.7500	± 2.2500	± 2.7500	± 3.2500
	±3.7500	± 4.2500	± 4.7500	± 5.2500	± 5.7500	± 6.2500	± 6.7500
PSO	±0.2837	± 0.7651	± 1.1648	± 1.6370	± 2.1785	± 2.7001	± 3.2691
	±3.7970	± 4.2400	± 4.8434	± 5.3404	± 5.6945	± 6.1171	± 6.7500
IWO	±0.1914	± 0.8132	± 1.0754	± 1.6731	± 2.0079	± 2.6404	± 2.9455
	±3.5011	± 3.8122	± 4.4296	± 4.9016	± 5.4019	± 5.7494	± 6.3778
Modified IWO	±0.2969	± 0.8159	± 1.2423	± 1.6215	± 2.0730	± 2.5659	± 2.9412
	±3.4122	± 3.9043	± 4.3164	± 5.0199	± 5.5541	± 5.9640	± 6.6972

Table 3. Comparative results for 28-element unequally spaced linear array with three nulls.

S. No.	**Optimization Technique**	**PSLL in dB**	**FNBW in degree**	**Null Depth in dB**		
				120°	**122.5°**	**125°**
1	None (Conv. Array)	-13.23	8.20	NA	NA	NA
2	PSO	−13.50	8.20	−74.02	−72.31	−72.74
3	IWO	−13.57	8.80	−76.19	−75.04	−83.18
4	Modified IWO	−14.76	8.80	−70.60	−76.19	−79.48

Table 4. Element positions of the 32-element linear array (normalised with respect to λ).

Conventional array	±0.2500	± 0.7500	± 1.2500	± 1.7500	± 2.2500	± 2.7500	± 3.2500	± 3.7500
	±4.2500	± 4.7500	± 5.2500	± 5.7500	± 6.2500	± 6.7500	± 7.2500	± 7.7500
PSO	±0.2729	± 0.6479	± 1.1656	± 1.5086	± 1.9528	± 2.3521	± 2.6916	± 3.1717
	±3.4704	± 3.9802	± 4.3605	± 4.9016	± 5.4339	± 6.1638	± 7.0111	± 7.7500
IWO	±0.2895	± 0.6645	± 1.2378	± 1.5284	± 2.0828	± 2.3497	± 2.8400	± 3.1816
	±3.5983	± 4.0294	± 4.4540	± 4.9666	± 5.4994	± 6.2006	± 7.0249	± 7.7500
Modified IWO	±0.2569	± 0.6319	± 1.1058	± 1.4925	± 1.8590	± 2.3389	± 2.5889	± 3.1366
	±3.3866	± 3.9159	± 4.2900	± 4.8222	± 5.3928	± 6.1579	± 7.0067	± 7.7500

4.1.2. Design Example: 32 Element Linear Array with Single Null Condition

Next a 32 element array is synthesized for achieving minimum side lobes with desired nulls at $\theta = 99°$. The optimized element positions are given in Table 4. Table 5 shows a comparison of the PSLL, FNBW and null depths obtained using PSO, IWO and modified IWO algorithms. The array pattern obtained using the modified IWO algorithm is shown in Fig. 5, along with the conventional and PSO synthesized array pattern. Convergence characteristics in terms of the fitness value versus the number of generations using PSO, IWO and modified IWO algorithms are shown in Fig. 6.

As seen from Table 5, all the three optimization algorithms render a deep null of −85 dB or less at the desired direction. The PSLL is lowered by 6 dB from the PSLL of a non-optimized array when modified IWO is used.

4.2. Observations

The design examples illustrate that the optimization technique as proposed in this section is effective in the proper positioning of deep nulls while maintaining a near-constant FNBW. But at the same

Table 5. Comparative results for 32-element unequally spaced linear array synthesis with single null.

S. No.	Optimization Technique	PSLL in dB	FNBW in degree	Null Depth in dB at 99°
1	None (Conv. Array)	−13.23	NA	NA
2	PSO	−18.08	8.50	−87.02
3	IWO	−17.14	8.00	−86.96
4	Modified IWO	−19.22	8.50	−85.72

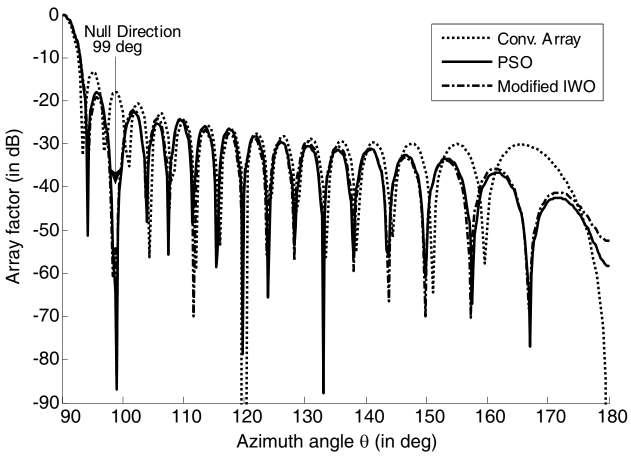

Figure 5. The normalized array pattern of 32-element linear array obtained by modified IWO with desired null at 99°.

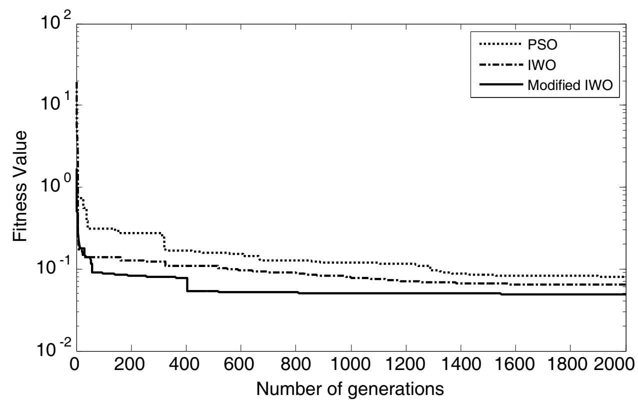

Figure 6. The convergence characteristics of the PSO, IWO and modified IWO algorithms for optimization of a 32 element linear array.

time there is not significant lowering of the sidelobe levels through the use of the fitness function of Equation (8) when nulls are present. In the case that there is no desired null direction, the same techniques will lower the PSLL by an additional 4–5 dB. But, in realistic environment, it is essential to maintain nulls at specified directions to avoid interference and jamming. Thus, it is necessary to modify the fitness function of Section 4 in order to incorporate an additional constraint and put more selective pressure on the algorithm. This is achieved through the use of the static penalty coefficients as shown in Section 5.

5. FORMULATION OF THE FITNESS FUNCTION USING STATIC PENALTY

In Section 4, the fitness function without static penalty function is modelled as single objective function consisting of multiple objectives. The optimization with such a fitness function will find a feasible solution even in the directions where one of the objectives dominates the other. As a result, one of the objectives may not be minimized but the fitness value may be reached as a whole. This is obvious from example of Section 4.1.1 where deep nulls in specified directions are achieved without significant reduction of PSLL.

To overcome the above mentioned drawbacks, it is required to develop an objective function which will force the optimization algorithm to find the feasible solutions by satisfying all constraints. The general optimization problem with inequality constraints is defined as follows

$$\text{Minimize } f(\vec{x}) \quad \text{subject to } g_i(\vec{x}) \leq 0, \quad i = 1, 2, \ldots, m \tag{9}$$

where $\vec{x}$ is the vector of variables, $g_i(\vec{x})$ the ith constraint, and m the total number of inequality constraints.

The main idea behind using penalty functions is to constrain the fitness function so that all the objectives in a multi objective formulation are minimized simultaneously. Penalty function methods transform a constrained problem to an unconstrained one by penalizing those solutions which are infeasible. The penalty function method adds a penalty whenever the solution is far distance from feasibility. One such penalty function is the static penalty function proposed by Homaifar, Lai and Qi [24] in 1994. In the static penalty method the penalty coefficient for each constraint is chosen in such a way that it increases in proportion to higher levels of violation. The static penalty function is incorporated in the fitness function as

$$F(\vec{x}) = f(\vec{x}) + \sum\nolimits_{i=1}^{m} \left(R_{k,i} \times \langle g_i(\vec{x}) \rangle^2 \right) \tag{10}$$

where the function $\langle g_i(\vec{x}) \rangle$ is defined as [26].

$$\langle g_i(\vec{x}) \rangle = \max(0, g_i(\vec{x})) = \begin{cases} 0 & \text{if } g_i(\vec{x}) \le 0 \quad \text{(Constraint is satisfied)} \\ g_i(\vec{x}) & \text{if } g_i(\vec{x}) > 0 \quad \text{(Constraint is violated)} \end{cases}$$

where $R_{k,i}$ is the ith penalty coefficient, $f(\vec{x})$ the unpenalized objective function, $k = 1, 2, \ldots, q$, where q is the number of levels of violation defined by the user.

As seen from Equation (10), if array constraint is violated, then there will be a higher penalty for the violating constraint which is further from feasibility. It puts more pressure on the optimization algorithm to move towards a feasible solution by satisfying all constraints. If no violation occurs, the function $\langle g_i(\vec{x}) \rangle$ is zero, i.e., there is no penalty if all constraints are satisfied. Choosing the degree of penalization is very critical, especially for multi constrained nonlinear optimization problems. High degree of penalty [25] puts more pressure on the optimization algorithm to obtain feasible solutions but also increases the computational burden of the algorithm.

In this communication, the main objective of the optimization is to find the optimum element positions to maintain FNBW of the periodic array, while achieving the desired PSLL and null depth in specific directions. Two penalty coefficients R_1 and R_2 are selected, one for optimizing the PSLL and other for positioning of nulls. To limit the complexity of the optimization process, each penalty carries only one level of violation. The objective function for this design problem is formulated by modifying Equation (10) as

$$F(\vec{x}) = |FNBW_O - FNBW_D| + \left(R_1 \times \max[0, |PSLL_O - PSLL_D|]^2 + R_2 \times \sum\nolimits_{i=1}^{n} \max[0, |Null_{Oi} - Null_{Di}|]^2 \right) \tag{11}$$

where $\vec{x}$ is the element position vector; θ_0 is the angular region excluding the main lobe; $FNBW_O$ & $FNBW_D$ are the optimized FNBW and desired FNBW respectively; $PSLL_O = \max(AF_{\text{dB}}^{\vec{x}}(\theta_i))$ & $PSLL_D$ are the optimized PSLL and desired PSLL respectively; $Null_{Oi} = AF_{\text{dB}}^{\vec{x}}(\theta_i)$ & $Null_{Di}$ are the optimized null depth and desired null depth respectively of the ith null at θ_i for n number of nulls.

It is observed that if R_1 and R_2 are close to each other, the problem resembles the case of no penalty, which as shown in Section 4 does not achieve significant reduction in PSLL. So R_2 is set much lower than R_1, in order to achieve significant reduction in PSLL by putting more selective pressure on the algorithm. Next for determining the values of the penalty coefficients, a detailed study was conducted by varying R_1 between 10^6 (as suggested in the literature [26]) and 10^9 and keeping R_2 significantly (around 4 degrees) lower than R_1. It is seen that as R_1 goes higher than 10^7, there is no significant change in the optimized fitness value. Thus the values of R_1 and R_2 are set at 10^7 and 10^3 respectively for all the design cases.

5.1. Design Example: 28 Element Linear Array with Three Null Conditions

This example illustrates the synthesis of a 28-element array for achieving minimum PSLL with three desired nulls at 120°, 122.5° and 125° as in the example of 4.1. In this case the desired FNBW, PSLL and null depth level are set at 8.2°, −23 dB and 60 dB respectively. To give an idea of the required computational time, this optimization requires 279.22 sec for each run. The array patterns obtained using the PSO and modified IWO with static penalty is shown in Fig. 7, along with the conventional array pattern. The optimized element positions are given in Table 6. Table 7 shows a comparison

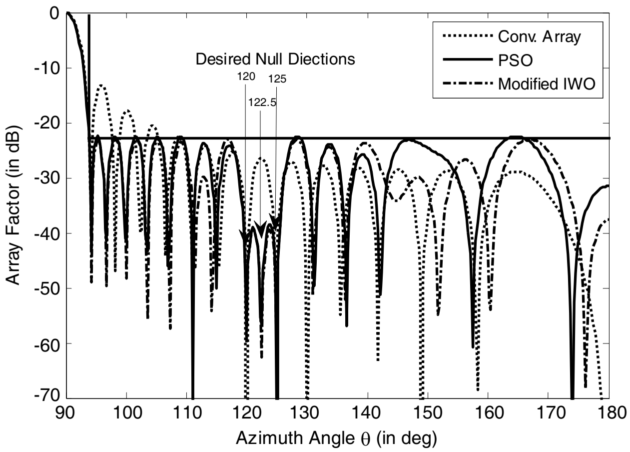

Figure 7. The normalized array pattern of 28-element linear array obtained by modified IWO with constraint handling, with desired nulls at 120°, 122.5° and 125°.

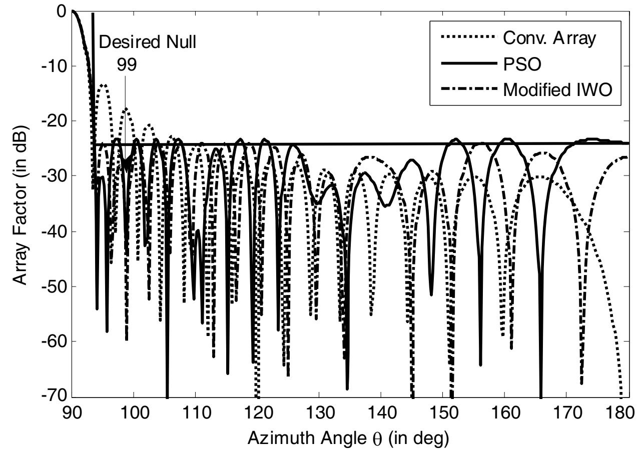

Figure 8. The normalized array pattern of 32-element linear array obtained by modified IWO with constraint handling, with desired null at 99°.

Table 6. Element positions of the 28-element linear array optimized with constraint handling methods (normalised with respect to λ).

PSO	±0.2446	± 0.6709	± 1.1415	± 1.6026	± 2.0332	± 2.5921	± 3.0812
	±3.5812	± 4.1926	± 4.8449	± 5.4488	± 6.3993	± 7.2593	± 7.9994
IWO	±0.2298	± 0.6926	± 1.1628	± 1.5628	± 2.0490	± 2.5959	± 3.0966
	±3.5919	± 4.1856	± 4.8638	± 5.4903	± 6.3904	± 7.2268	± 7.9918
Modified IWO	±0.2702	± 0.6460	± 1.2024	± 1.4786	± 2.0421	± 2.5188	± 3.0842
	±3.5348	± 4.0941	± 4.7710	± 5.3948	± 6.2987	± 7.1605	± 7.9717

Table 7. Comparative results for 28-element unequally spaced linear array optimized with constraint handling methods for three null conditions.

S. No.	**Optimization technique**	**PSLL in dB**	**FNBW in degree**	**Null Depth in dB**		
				120°	**122.5°**	**125°**
1	None (Conv. Array)	−13.23	8.20	NA	NA	NA
2	PSO	−22.50	8.50	−57.40	−56.54	−70.00
3	IWO	−22.16	8.50	−70.00	−65.95	−64.03
4	Modified IWO	−23.00	8.40	−60.10	−62.74	−61.17

of the PSLL, FNBW and null depth obtained using PSO, IWO and modified IWO algorithms with constraint-handling for unequally spaced 28-element linear array synthesis.

As shown in Table 7, all the algorithms perform much better when a static penalty is imposed. In particular, the modified IWO algorithm is able to achieve the desired PSLL of −23 dB while maintaining a FNBW of 8.4° and nulls as deep as −60 dB or better in specified directions. This shows that modified IWO outperforms the PSO and IWO in term of the PSLL achieved. Thus a suppression of 9.77 dB in PSLL level is achieved by using static penalty function with the modified IWO algorithm.

Table 8. Element positions of the 32-element linear array optimized with constraint handling methods (normalised with respect to λ).

PSO	±0.1625	± 0.6615	± 0.9744	± 1.4781	± 1.7485	± 2.2222	± 2.6790	± 3.2085
	±3.6423	± 4.2666	± 4.6911	± 5.3432	± 6.2218	± 7.0281	± 8.0411	± 8.8425
IWO	±0.2246	± 0.6001	± 1.1234	± 1.4387	± 1.9795	± 2.4323	± 2.8471	± 3.4238
	±3.8796	± 4.4507	± 5.0576	± 5.6144	± 6.3873	± 7.1811	± 8.2418	± 8.9923
Modified IWO	±0.1589	± 0.6358	± 1.0477	± 1.4326	± 1.8859	± 2.2855	± 2.7602	± 3.1403
	±3.6810	± 4.1017	± 4.6677	± 5.2277	± 5.9047	± 6.6902	± 7.7680	± 8.5000

Table 9. Comparative results for 32-element unequally spaced linear array optimized with constraint handling methods for single null condition.

S. No.	Optimization Technique	PSLL in dB	FNBW in degree	Null Depth in dB at 99°
1	None (Conv. Array)	−13.23	NA	NA
2	PSO	−23.41	8.50	−43.71
3	IWO	−24.00	8.00	−43.99
3	Modified IWO	−24.00	8.20	−60.11

5.2. Design Example: 32 Element Linear Array with Single Null Condition

This example illustrates the synthesis of a 32-element array for achieving minimum PSLL with the desired null at 99° as in the example of Section 4.2. In this case the desired FNBW, PSLL and null depth level are set at 7.4°, −23 dB and −60 dB respectively. R_1 and R_2 are set at 10^7 and 10^3 respectively. The computational time for finding the optimal solution is 240.22 sec. The optimized element positions are given in Table 8. The obtained normalized array pattern using the optimized element positions from the PSO and modified IWO algorithms are shown in Fig. 8, along with the conventional array pattern. Table 9 shows a comparison of the PSLL, FNBW and null depth obtained using PSO, IWO and modified IWO algorithms with constraint handling. Convergence characteristics in terms of the fitness value versus the number of generations using PSO, IWO and modified IWO algorithms each with static penalty imposed during the optimization are shown in Fig. 9. It is seen from Table 9 that the solution achieved has a PSLL of −24 dB while not significantly enhancing the FNBW of the conventional array geometry. At the same time, the obtained null level at 99° is −60.11 dB.

6. SALIENT OBSERVATIONS

6.1. Convergence Characteristics

The convergence characteristics (Figs. 6 & 9) show that the modified IWO outperforms the PSO and IWO algorithms in terms of both the convergence rate as well as the fitness value at convergence. The final fitness value transforms to a lower PSLL level. It is also evident from Fig. 8 that with the incorporation of the static penalty function, the modified IWO has a much faster convergence as compared to the other algorithms.

6.2. Comparison with Published Work

The synthesis of 28 and 32-element linear array has been explored previously by many researchers [13, 18, 23]. The optimized array obtained by the novel method proposed in this paper may be compared with previously published work [Tables 10 and 11]. The results demonstrate that the

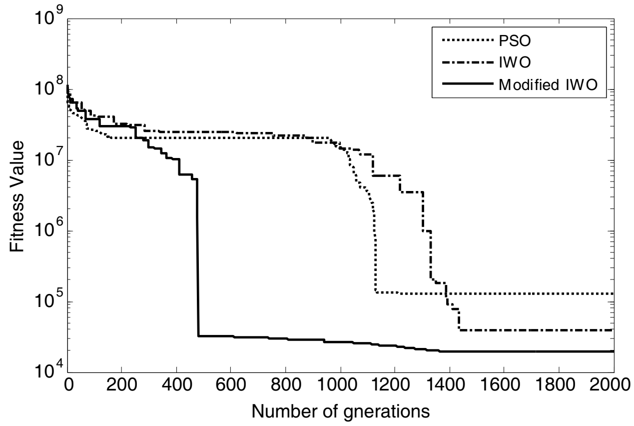

Figure 9. The convergence characteristics of the PSO, IWO and modified IWO with constraint handling for a 32 element linear array.

Table 10. Comparison with published results for 28-element unequally spaced linear array synthesis with three null conditions.

S. No.	28 element Array	PSLL in dB	FNBW in degree	Null Depth in dB		
				120°	122.5°	125°
1	PSO [13]	−13.23	8.20	−52.74	−51.66	−61.46
2	ACO [18]	−14.88	8.40	−57.42	−59.20	−60.46
3	CSO [23]	−13.23	8.20	−75.00	−67.05	−65.32
4	Modified IWO	−14.76	8.80	−70.60	−76.19	−79.48
5	Modified IWO with static penalty method	**−23.00**	**8.40**	**−60.10**	**−62.74**	**−61.17**

Table 11. Comparison with published results for 32-element unequally spaced linear array synthesis for single null condition.

S. No.	32 element Array	PSLL in dB	FNBW in degree	Null Depth in dB at 99°
1	PSO [13]	−18.73	8.20	−62.12
2	ACO [18]	−17.52	7.70	−60.00
3	CSO [23]	−18.80	8.20	−80.00
4	Modified IWO	−19.22	8.50	−85.72
5	Modified IWO with static penalty method	**−24.00**	**8.20**	**−60.11**

modified IWO with static penalty method provides an efficient way to control the shape of the radiation pattern with low PSLL and strong nulls in desired directions.

It is seen from Table 10 that for a 28-element array with three null conditions, the modified IWO algorithm with static penalty function offers suppression of PSLL by around 9.77 dB, 8.12 dB and 9.77 dB as compared to arrays optimized with PSO [13], ACO [18] and CSO [23] respectively. At the same time, the FNBW is maintained at a value close to that of the unoptimized array and strong nulls (as deep as −60 dB) are achieved at the specified directions.

For a 32-element array (Table 11) with single null condition, the modified IWO algorithm with static penalty method offers suppression of PSLL by around 5.27 dB, 6.48 dB and 5.20 dB as compared to arrays optimized with PSO [13], ACO [18] and CSO [23] respectively. The proposed method also has the least FNBW with a strong null (-60 dB) in the specified direction.

The above results demonstrate that although the previously published methods are susceptible to the number of null conditions imposed on the antenna array, the method using static penalty functions as proposed in this paper performs well irrespective of the null conditions desired for the application.

7. CONCLUSION

In this communication, the static penalty function is incorporated with a modified IWO algorithm for the synthesis of unequally spaced linear antenna array. This proposed approach provides an efficient way to control the shape of the radiation pattern in terms of PSLL, FNBW and null positioning. Results show that there is a significant reduction in PSLL for a specific desired FNBW while maintaining strong deep nulls in specified directions. This novel approach succeeds in achieving better results as compared to previously published works. It also exhibits good performance in terms of the convergence rate as compared to the classical algorithms. By this method, it is possible to have more flexibility in the design of the arrays as we can put selective pressure on the PSLL, the FNBW or the null depth of the optimized array. Thus the constraint handling based modified IWO shows good potential as an algorithm for solving complex multimodal problems in electromagnetic engineering.

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