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# A New High-Resolution and Stable MV-SVD Algorithm for Coherent Signals Detection

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**Abstract**—The performance of smart antenna greatly relies on the efficient use of direction-of-arrival (DOA) estimation techniques for both coherent and non-coherent signals. In practice, DOA estimation problems and difficulties increase when the signals in multipath propagation environments are highly correlated or coherent. Therefore exploring an algorithm which is capable of estimating coherent signals is of great importance. To overcome this problem a new high-resolution modified virtual singular value decomposition (MVSVD) algorithm for DOA estimation of coherent signals is proposed. It is based on the hybrid combination of the virtual array extension singular value decomposition (SVD), and modified MUSIC algorithms. The proposed algorithm provides many features such as: decorrelation of the coherence between the signals without reducing the rank of the covariance matrix or losing the array aperture size; high-resolution and more stability especially at low SNR values; and an increase in the maximal number of detectable signals to $M - 1$, where $M$ is the number of antenna elements.

## 1. INTRODUCTION

DOA estimation is an important research topic in smart antenna systems for both coherent and non-coherent signals. It is widely used in many commercial and military applications such as sonar, radar, mobile communication systems and biomedical engineering.

The most popular subspace algorithms, MUSIC [1, 2] and ESPRIT [3, 4], are well-known algorithms for estimating the DOA and give high-resolution results when the signals are uncorrelated. However, due to the multipath propagation or man-made interference, the signals are usually highly correlated or even coherent. These algorithms can't effectively distinguish coherent signals. Great efforts areexerted to solve this problem such as, the spatial smoothing (SS) technique [5], which is based on the partitioning of the total number of antenna array elements into a number of subarrays, and then averages the subarrays output covariance matrices to form the spatially smoothed covariance matrix, but it detects only $M/2$ sources and gives good results only at high SNR.

Forward/backward spatial smoothing (FBSS) [6] is proposed to improve the SS technique. It increases the number of detectable sources from $M/2$ to $2M/3$. The singular value decomposition (SVD) method [7–9] detects coherent signals. It reconstructs a covariance matrix by using the eigenvector corresponding to the largest eigenvalue, and then performs singular value decomposition on this matrix. However it has poor performance when the SNR is low. It also reduces the rank of data covariance matrix to achieve decorrelation of coherent sources. To overcome the aperture size reduction problem of the SS, FBSS, and SVD methods, Toeplitz [10–12] and VSS [13] algorithms are presented without the need for dimension reduction of the covariance matrix, and there's no loss of the size of the array aperture, but signal detection at low SNR remains an important problem.

In this paper, a modified virtual SVD (MV-SVD) algorithm for coherent signals detection is presented. It is based on the hybrid combination of the virtual array extension, singular value decomposition (SVD), and modified MUSIC algorithms. Firstly, the virtual array extension is used

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to extend the original receiving antenna array from actual $M$ sensors to virtual $(2M-1)$ sensors. This increases the degree of freedom to detect more signals and overcomes the aperture size problem. Secondly, the SVD algorithm is then used to construct an $M \times M$ covariance matrix. Finally, the directions of arrival of the coherent signals are estimated using the modified MUSIC algorithm. The proposed algorithm provides high resolution and more stable results, especially at low SNR values and increases the number of detectable sources to $M-1$. These advantages distinguish the proposed algorithm from other algorithms.

## 2. PROBLEM FORMULATION

This section introduces the SVD algorithm [7] and illustrates its main drawbacks which will be overcome using the proposed algorithm. Assume $N$ narrowband far-field sources impinging on a uniform linear array (ULA) consisting of $M$ antenna elements $(N \leq M)$ with uniform element spacing $d = \lambda/2$, where $\lambda$ is the carrier wavelength of the signal. Consider $K$ snapshots are received from different DOAs $(\theta_1, \theta_2, \theta_3, \ldots, \theta_N)$. The array output vector at a time $t$ is then given by [7]

$$X(t) = AS(t) + n(t) \tag{1}$$

where

$$\begin{aligned}
X(t) &= [x_1(t), x_2(t), \ldots, x_M(t)]^T \\
S(t) &= [s_1(t), s_2(t), \ldots, s_N(t)]^T \\
n(t) &= [n_1(t), n_2(t), \ldots, n_M(t)]^T \\
A(\theta) &= [a(\theta_1), a(\theta_2), \ldots, a(\theta_N)] \\
a(\theta_i) &= \left[1, e^{-j\frac{2\pi}{\lambda}d\sin\theta_i}, \ldots, e^{-j(M-1)\frac{2\pi}{\lambda}d\sin\theta_i}\right]^T
\end{aligned}$$

where $x_m(t)$, $(m = 1, 2, \ldots, M)$ is the input of the $m$th antenna element, $s_n(t)$, $(n = 1, 2, \ldots, N)$ the complex amplitude of the narrow band signals, and $A(\theta)$ the array manifold matrix. The superscript $T$ is a vector or matrix transpose.

Suppose that the signal $S(t)$ and noise $n(t)$ are zero-mean Gaussian processes and uncorrelated. The noise power of each element is $\sigma_n^2$. The array output data covariance matrix are given by [7]

$$R_x = E\left[XX^H\right] = AR_sA^H + \sigma_n^2 I \tag{2}$$

where $R_s = E[SS^H]$ is the source covariance matrix, $E[n(t)n(t)^H] = \sigma_n^2 I$, $I$ the identity matrix, $H$ the conjugation transpose, and $E[\,]$ the expectation.

Singular value decomposition (SVD) algorithm is a kind of effective coherent solution algorithm. It uses the eigenvector corresponding to the largest eigenvalue of the data covariance matrix $R_x$, $e = [e_1, e_2, \ldots, e_M]$ to construct the matrix $Y$ as follows;

$$\mathrm{Y} = \begin{bmatrix} e_1 & e_2 & \ldots & e_p \\ e_2 & e_3 & \ldots & e_{p+1} \\ \ldots & \ldots & \ldots & \ldots \\ e_m & e_{m+1} & \ldots & e_M \end{bmatrix} \tag{3}$$

where $m > N$ and $m + p - 1 = M$.

Then obtain the two matrices $Y_0$ and $Y_1$ from $Y$ as follow [7]:

$$Y_0 = YY^H \tag{4}$$

$$Y_1 = \frac{1}{2}(Y_0 + J_m Y_0^* J_m) \tag{5}$$

where $J_m$ represents the $m \times m$ exchange matrix, and the superscript $(\cdot)^*$ stands for the complex conjugate.

Using the eigenvalue decomposition of $Y_1$ to estimate the DOAs of the signals by MUSIC algorithm according to

$$P_{MUSIC} = \frac{1}{\mathrm{a}(\theta)^H U_n U_n^H a(\theta)} \tag{6}$$

where $U_n$ represents the noise subspace which is formed by small eigenvalues of $Y_1$.

The SVD algorithm reduces the rank of the data covariance matrix which reduces the aperture size to achieve decorrelation of coherent sources. It detects only $2M/3$ coherent sources and gives very poor performance at low SNR. In this paper, the proposed algorithm overcomes these problems. It decorrelates the signals coherence without reducing the array aperture size. It has very high resolution and more stable results especially at low SNR values. Furthermore, it increases the number of detectable coherent sources up to $M-1$.

## 3. PROPOSED MV-SVD ALGORITHM

In this paper, the virtual array extension is used to extend the original receiving antenna array from actual $M$ elements to virtual $2M-1$ elements as shown in Figure 1. Theoretically, it can increase the array aperture size, identify more sources, and achieve higher resolution especially at low SNR. The proposed algorithm performs the following steps:

1. Applying the virtual array extension, the received data array $X(t)$ is used to construct a $(2M-1) \times K$ dimensional matrix $X_1$ according to the literature [13],

$$X_1 = \begin{bmatrix} X(t)' \\ X(t) \end{bmatrix} \tag{7}$$

where $X(t)$ is $(M \times K)$ data receiving array.

$$X(t)' = [x_M(t)^*, x_{M-1}(t)^*, x_M(t)^*, \ldots, x_2(t)^*]^T \tag{8}$$

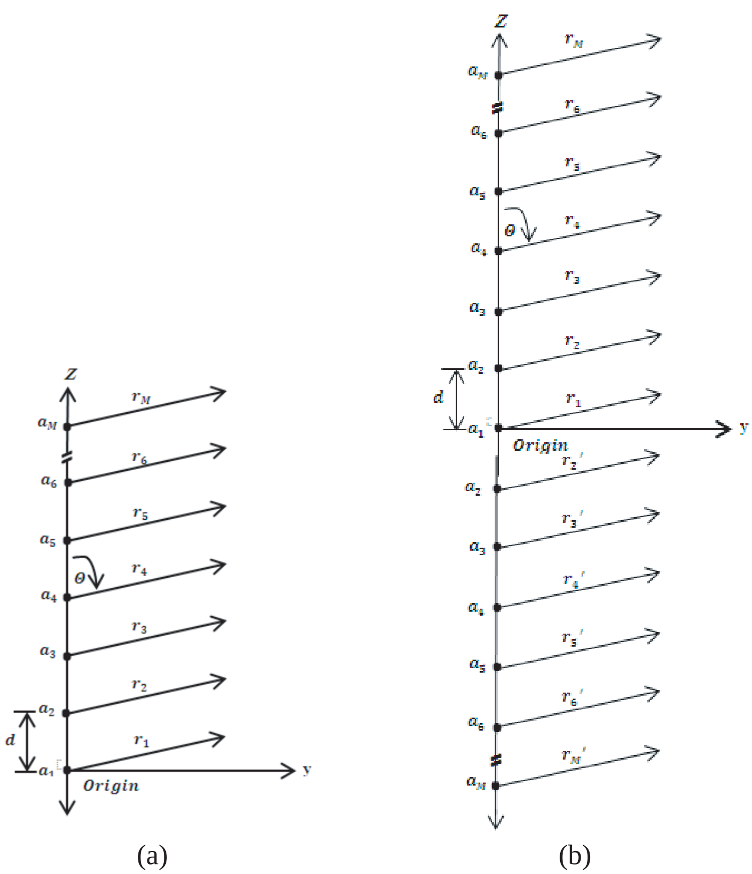



**Figure 1.** (a) $M$ elements ULA array. (b) Equivalent $(2M-1)$ ULA array obtained using the virtual array extension technique.

2. Utilizing $X_1$ to construct a new virtual data covariance matrix $R_{x_1}$ as follows

$$R_{x_1} = E\left[X_1 X_1^H\right] = A_1 R_{s1} A_1^H + \sigma_n^2 I_{2M-1} \quad (9)$$

3. Performing the eigenvalue decomposition of $R_{x_1}$ to obtain the eigenvector corresponding to the largest eigenvalue of the data covariance matrix $e = [e_1, e_2, \ldots, e_{2M-1}]$. This Eigenvector is a linear combination of all signal sources' steering vector. Whether the source is coherent or not, it contains all the signal information. So, we use it to construct a new data covariance matrix $Y$ applying Equation (3) substituting $m = M$, and $m + p - 1 = 2M - 1$.
   The new data covariance matrix $Y$ is $M \times M$ dimensional matrix unlike the SVD algorithm. The MV-SVD algorithm does not reduce the aperture size, but uses the whole aperture size $M$ that increases the number of detectable signals to $M - 1$.
4. Using the new data covariance matrix $Y$ to construct the two matrices $Y_0$ and $Y_1$ applying Equations (4) and (5) to improve the resolution.
5. Obtaining the noise subspaceusing the singular value decomposition of $Y_1$ .
6. The DOAs of the multiple incident signals can be estimated by locating the peaks of the modified MUSIC spectrum as in Equation (10) according to the literature [14]. The modified MUSIC spectrum is used in spite of the original MUSIC spectrum of Equation (5) to enhance the resolution of the DOA estimation process. In Equation (10), as noticed from the denominator, the orthogonality between $a(\theta)$ and $U_n$ will reduce it to a minimum, and hence will increase $P_{MUSIC}(\theta)$ which leads to avery high resolution in detecting the largest peaks of the MUSIC spectrum that correspond to the DOAs of the signals impinging on the array.

$$P_{MUSIC}(\theta) = \frac{\mathrm{a}(\theta)^H RA a(\theta)}{\mathrm{a}(\theta)^H U_n U_n^H a(\theta)} \quad (10)$$

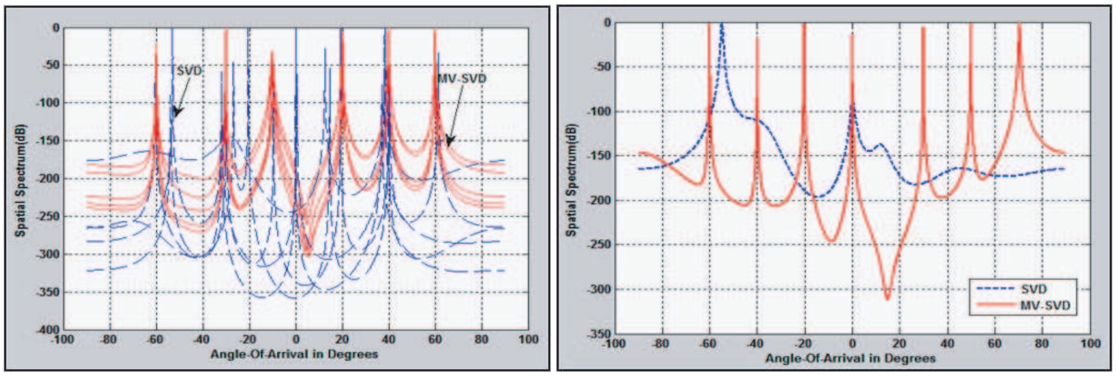
   where
   a) $U_n = [e_{N+1}, e_{N+2}, \ldots, e_M]$ from decomposition of the correlation matrix $Y_1$ as SVD $(Y_1) =$ U.S.$V^H$.
   b) $RA$ is calculated as $RA = U_s B U_s^H$ where $U_s = [e_1, e_2, \ldots, e_N]$ is the signal subspace.
   c) $B$ is obtained by dividing the diagonal of matrix $S$ into two arrays $SS$ and $SN$ where $B = \text{diagonal}(\frac{1}{SS} - \text{sigma} \times I_N)$, $SS = \text{diagonal}(S_s)$ is the signal Eigenvalues, and $SN = \text{diagonal}(S_n)$ is the noise Eigenvalues.
   d) $\text{sigma} = \frac{\text{trce}(S_n)}{M-N}$.

## 4. SIMULATION RESULTS

In this section, the proposed algorithm is verified for different DOA estimation experiments of coherent and non-coherent signals. The performance of the proposed MV-SVD algorithm is compared to different coherent DOA estimation algorithms such as SVD, FBSS, Toepltiz, and VSS, and non-coherent algorithms such as MUSIC and ESPRIT. It is required to estimate the directions of arrival of the coherent signals impinging on a ULA consisting of $M = 9$ antenna elements with uniform element spacing $d = \lambda/2$. $K = 100$ snapshots are used. The noise is a white Gaussian noise with zero mean.

**Experiment 1:**

In this simulation, a comparison is performed between the proposed MV-SVD algorithm and the SVD algorithm. Consider six coherent signals impinging on the aforementioned ULA from the directions $-60°$, $-30°$, $-10°$, $20°$, $40°$, and $60°$ at SNR $= 0$ dB. Applying SVD algorithm to detect the maximum allowed number of signals ($N = 2M/3 = 6$ signals), the SVD requires $m > N$. In this case, $m = 7$ and $p = 3$. Figure 2(a) shows the angular spectrum of the MV-SVD algorithm compared to the angular spectrum of the SVD algorithm at SNR $= 0$ dB. It is clear that the MVSVD algorithm is significantly better than SVD at the same conditions. The MV-SVD provides much higher resolution than the SVD. In addition, when the experiment is performed for five independent runs as shown in Figure 2(a), the proposed algorithm exhibits highly stable results, as opposed to SVD algorithm which exhibits poor stability.




(a) (b)

**Figure 2.** (a) Angular spectrums of the MV-SVD algorithm compared to the angular spectrums of the SVD algorithm for N=6 signals at SNR = 0 dB. (b) Angular spectrum of the MV-SVD algorithm compared to the angular spectrum of the SVD algorithm for $N = 7$ signals at SNR = 0 dB.

If the number of the received signals is increased above $(2M/3 = 6)$, assuming $N = 7$ signals impinging from the directions $-60°$, $-40°$, $-20°$, $0°$, $30°$, $50°$, and $70°$ respectively at the same SNR = 0 dB. The SVD requires $m = 8$ and $p = 2$. The resulting spectrum shown in Figure 2(b) indicates that SVD completely failed to detect number of signals greater than $2M/3$ while the proposed algorithm accurately detected all the signals. It can detect up to $M - 1$ sources.

**Experiment 2:**

In this simulation, the performance of the proposed MV-SVD algorithm is compared to the performances of SVD, VSS, and FBSS algorithms at low SNR. Consider four coherent signals $s_1(t)$, $s_2(t)$, $s_3(t)$ and $s_4(t)$ impinging on the aforementioned ULA from the directions $-40°$, $-20°$, $20°$, and $60°$ respectively at SNR = $-5$ dB. Figure 3 shows the angular spectrum of the proposed MV-SVD algorithm compared to the angular spectrums of the SVD, VSS, and FBSS algorithms. The estimated angles of arrivals of the four signals applying these algorithms are listed in Table 1. By comparison, it is clear that the proposed algorithm provides more accurate results and higher resolution than the other algorithms at low SNR. Both VSS and SVD provide small spectrum peaks with relatively small drifts in the estimated angles form the incidence angles. In contrast, the FBSS provides sharp and high peaks but with large drifts in the estimated angles.

**Table 1.** Estimated angels of arrival applying the proposed MV-SVD algorithm compared to the SVD, VSS, and FBSS algorithms at low SNR = $-5$ dB.

| **Angles of arrivals in degrees** | **−40** | **−20** | **20** | **60** |
|---|---|---|---|---|
| **MV-SVD** | −40 | −20 | 20.7 | 60.1 |
| **SVD** | −40.9 | −19.1 | 20.9 | 615 |
| **VSS** | −40 | −20 | 21 | 62.2 |
| **FBSS** | −39.3 | −21 | 13.1 | 73.5 |

**Experiment 3:**

In this simulation, the performance of the proposed MV-SVD algorithm is compared to the performance of SVD, VSS, FBSS, and Toeplitz algorithms for large number of signals at low SNR. Consider eight completely coherent signalscoming from angles of arrival $-60°$, $-40°$, $-20°$, $0°$, $10°$, $30°$, $50°$ and $70°$ respectively at SNR = $-5$ dB. Figure 4 shows only the resulting angular spectrums of the MV-SVD, VSS, and Toeplitzas both SVD and FBSS completely failed to detect the signals. The estimated angles are listed in Table 2. The proposed algorithmachieves higher resolution than VSS, and Toeplitz at low SNR.

**Table 2.** Estimated angels of arrivals applying the proposed MV-SVD algorithm compared to the SVD, VSS, FBSS, and Toeplitz algorithms for large number of signals at low SNR = $-5$ dB.

| Angles of arrivals in degrees | $-60$ | $-40$ | $-20$ | 0 | 10 | 30 | 50 | 70 |
|---|---|---|---|---|---|---|---|---|
| **MV-SVD** | $-60$ | $-40$ | $-21.2$ | 0.5 | 9.9 | 29.8 | 50.3 | 70.4 |
| **VSS** | $-60.4$ | $-40.2$ | $-22.8$ | 2.5 | 12.7 | 29.5 | 44.5 | 68.3 |
| **Toeplitz** | $-59.7$ | $-39.8$ | $-21.2$ | 0.5 | 9.8 | 30 | 50.9 | 71.2 |
| **SVD** | Not detected | Not detected | Not detected | Not detected | Not detected | Not detected | Not detected | Not detected |
| **FBSS** | Not detected | Not detected | Not detected | Not detected | Not detected | Not detected | Not detected | Not detected |

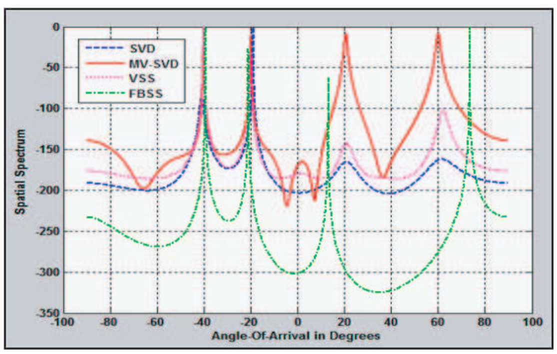



**Figure 3.** The angular spectrum of the proposed MV-SVD algorithm compared to the angular spectrums of the SVD, VSS, and FBSS algorithms at low SNR = $-5$ dB.

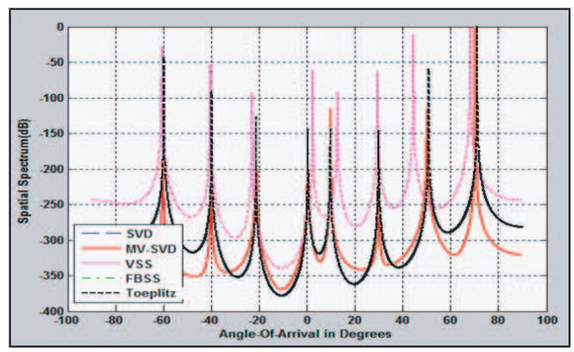



**Figure 4.** The angular spectrum of the proposed MV-SVD algorithm compared to the angular spectrums of the SVD, VSS, FBSS, and Toeplitz algorithms for large number of signals at low SNR = $-5$ dB.

**Experiment 4:**

The root mean square error (RMSE) at different SNR values is investigated in this simulation as a comparison criterion between the MV-SVD, SVD, VSS, FBSS, and Toeplitz algorithms. The RMSE is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N}\left(\theta_{Ei} - \theta_i\right)^2} \tag{11}$$

where $\theta_{Ei}$ is the estimated angle of the $i$th source and $\theta_i$ the true angle of arrival of the $i$th source. In this experiment, the RMSE versus SNR at different numbers of coherent signals is estimated as shown in Figure 5. Obviously, the proposed algorithm provides higher resolution capability and lower RMSE than SVD, VSS, FBSS, and Toeplitz algorithms especially at low SNR. Consider a ULA consisting of $M = 9$ antenna elements with uniform element spacing $d = \lambda/2$. The simulation is performed for two different cases:

***Case (1), if the number of coherent signals is $N \leq 2M/3$***

Figures 5(a), (b), and (c) show that the MV-SVD, SVD, VSS, FBSS, and Toeplitz algorithms can detect the received signals, but the proposed algorithm provides higher resolution and lower RMSE than the other algorithms, especially at low SNR values.

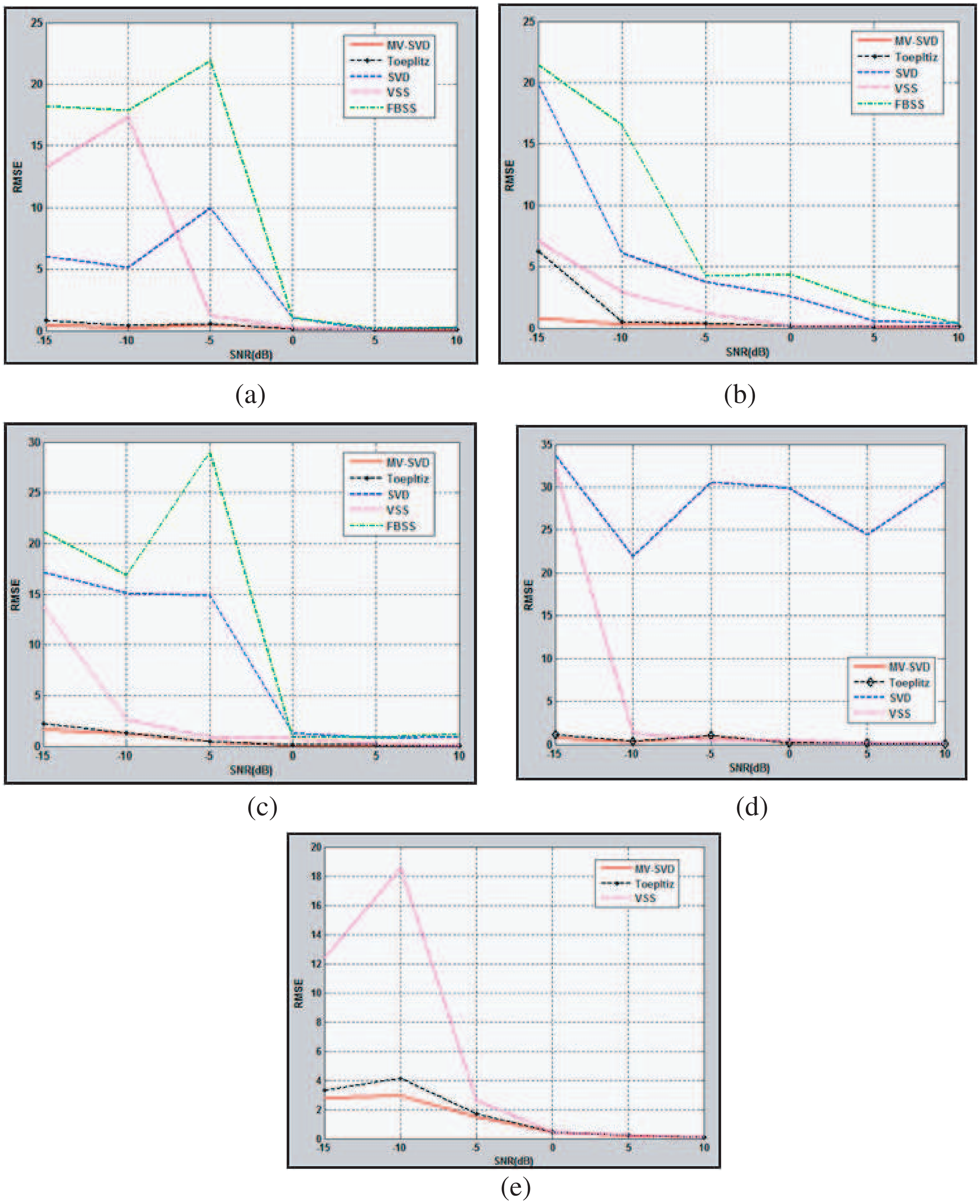



**Figure 5.** (a) Estimated RMSE at different SNR valuesfor $N = 4$ coherent signals impinging from angels of arrivals $-60°$, $-40°$, $-20°$, and $0°$. (b) Estimated RMSE at different SNR values for $N = 5$ coherent signals impinging from angels of arrivals $-60°$, $-40°$, $-20°$, $0°$, and $10°$. (c) Estimated RMSE at different SNR values for $N = 6$ coherent signals impinging from angels of arrivals $-60°$, $-40°$, $-20°$, $0°$, $10°$, and $30°$. (d) Estimated RMSE at different SNR values for $N = 7$ coherent signals impinging from angels of arrivals $-60°$, $-40°$, $-20°$, $0°$, $10°$, $30°$ and $50°$. (e) Estimated RMSE at different SNR values for $N = 8$ coherent signals impinging from angels of arrivals $-60°$, $-40°$, $-20°$, $0°$, $10°$, $30°$, $50°$ and $70°$.

***Case (2), if the number of coherent signals is*** $N > 2M/3$

If the number of received signals increased more than $2M/3$, only the VSS, Toeplitz and MV-SVD algorithms can detect signals up to $M-1$ signals. Figure 5(d) shows that the SVD detected the $N = 7$ incident signals but with very large RMSE compared to MV-SVD, VSS, and Toeplitz while the FBSS completely failed to detect the signals. Figure 5(e) shows that for $N = 8$ incident signals only the MV-SVD, VSS, and Toeplitz algorithms can detect the signals.

From the two cases, it is concluded that the proposed MV-SVD algorithm provides higher resolution, higher accuracy, and the lowest RMSE amongst these algorithms especially for large number of signals and at low SNR values.

**Experiment 5: (Detection of Non-Coherent Signals)**

In this section, the performance of the proposed algorithm in detecting non-coherent signals is compared to the performance of non-coherent algorithms such as MUSIC and ESPRIT. Consider a ULA consisting of $M = 10$ antenna elements with uniform element spacing $d = \lambda/2$. $K = 300$ snapshots are used. The noise is a white Gaussian noise with zero mean. Consider four non-coherent signals impinging on the array from the directions $-5°$, $10°$, $20°$, and $30°$ at low SNR $= -5$ dB. Figure 6 shows the angular spectrum of the proposed algorithm compared to the angular spectrums of the MUSIC and ESPRIT algorithms. The estimated angles of arrivals and the RMSE applying these algorithms are listed in Table 3. By comparison, it is clear that the proposed algorithm provides more accurate results and higher resolution than the other algorithms at low SNR.

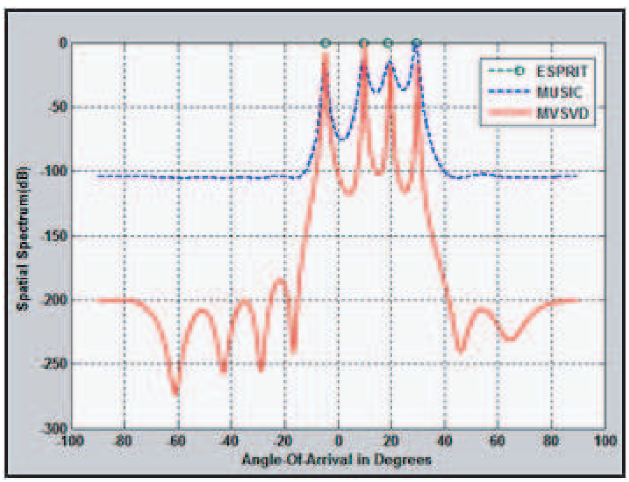



**Figure 6.** The angular spectrum of the proposed MV-SVD algorithm compared to the angular spectrums of the MUSIC and ESPRIT algorithms for non-coherent signals detection at low SNR $= -5$ dB.

**Table 3.** Estimated angels of arrivals applying the proposed MV-SVD algorithm compared to the MUSIC and ESPRIT algorithms for non-coherent signals detection at low SNR $= -5$ dB.

| **Angles of arrivals in degrees** | **−5** | **10** | **20** | **30** | **RMSE** |
|---|---|---|---|---|---|
| **MV-SVD** | −4.7 | 10 | 19.8 | 30 | **0.1803** |
| **MUSIC** | −4.8 | 9.8 | 19.6 | 29.3 | **0.4272** |
| **ESPRIT** | −4.6403 | 9.8136 | 18.6423 | 29.4522 | **0.7595** |

## 5. CONCLUSION

In this paper, a modified virtual SVD (MVSVD) algorithm is proposed. It is based on applying the virtual array extension to extend the original receiving data array from actual $M$ sensors to virtual $2M-1$ sensors then use the SVD method to construct the covariance matrix and estimate the angels of signal source using a modified MUSIC algorithm. The performance of the proposed MV-SVD algorithm is compared to those of the SVD, VSS, FBSS, and Toeplitz algorithms at different SNR values. The simulation results indicate that the proposed algorithm's resolution, stability and robustness are significantly better, especially at low SNR cases, and it can detect up to $M-1$ signal sources. In addition, it provides the lowest RMSE amongst these algorithms especially for large number of signals and at low SNR values. Furthermore, from the experimental results, the proposed algorithm achieves superior performance in detecting both coherent and non-coherent signals to the aforementioned algorithms.

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