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Influence of SOLT Calibration Standards on Multiport VNA S-Parameter Measurements

Wei Zhao*, Jiankang Xiao, and Hongbo Qin

Abstract—For the GSOLT calibration algorithm of n-port vector network analyzers (VNA), the sensitivity coefficients for the S-parameters of the n-port device under test (DUT) are developed as functions of the S-parameter deviations of SOLT standards. By introducing the generalized flow graph of the 3n-term error model, analytic formulas for the S-parameter deviations of the n-port DUT with respect to the error terms have been deduced. In addition, expressions for the deviations of the error terms in regard to the nonideal calibration elements are given by a series of matrix operations. Finally, the analytic expressions of the sensitivity coefficients are concluded, which can be used for establishing the type-B uncertainty budget for S-parameter measurements.

1. INTRODUCTION

With the development of microwave technology, multiport devices, whose S-parameters are typically measured by multiport vector network analyzers (VNA), are becoming more widespread in RF systems [1]. To achieve high precision, the calibration procedure should be implemented before the measurement [2, 3]. For the n-port VNA with n+1 measurement channels, the general short-open-load-thru (GSOLT) procedure is now widely used [4–6]. In practical application, the S-parameters associated with the calibration standards short (S), open (O), load (L) and thru (T) are not ideal as expected. Through the GSOLT calibration based on such non-ideal standards, the error terms and, consequently, the S-parameters of the DUT will deviate from their true values, so it is necessary to investigate the impact that the S-parameter deviations of SOLT standards have on the uncertainty of the S_{ij}. Although some techniques have been developed for the estimation of uncertainties in two-port VNA measurements, they are inapplicable to the multiport VNA due to the lack of use of matrix formalisms [7–9]. To solve this, the sensitivity coefficients for the S-parameters of n-port DUT have been deduced in matrix form by using the concept of general node equation for the GSOLT calibration [10]. However, it is still unclear how the nonideal SOLT standards affect the calibrated S_{ij}.

In this paper, the generalized flow graph of the 3n-term error model, where nodes and branch gains are expressed by column vectors and square matrixes respectively, is proposed for the GSOLT calibration of the n-port VNA. Based on this flow graph, the dependence of the S-parameter deviations on the error term deviations is solved in the error correction procedure. Then the error term deviations associated with the non-ideal standards are calculated in the process of error calibration. Finally, the analytic expressions for the sensitivity coefficients, which use the S-parameters of SOLT standards as input quantities and the S-parameters of the n-port DUT as output quantities, can be further concluded.

2. THEORY

The 3n-term error model of the n-port VNA with n+1 measurement channels is shown in Fig. 1, where the number "1" represents the source port [6]. In Fig. 1, S_{mj1} ($j = 1 \sim n$) are defined as the raw

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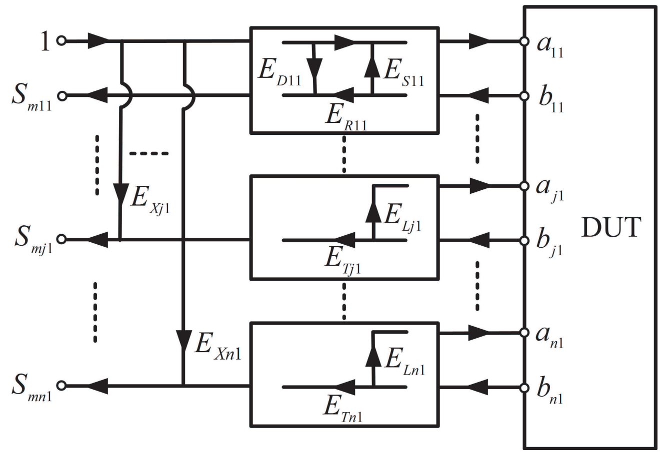


Figure 1. Error model of the n-port VNA with the resource connected to port 1.

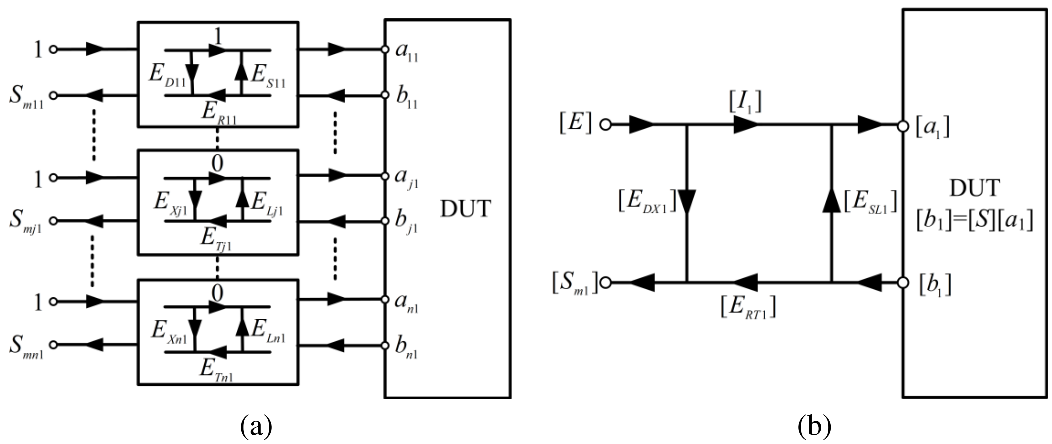


Figure 2. Transformations of the flow graph of the 3n-term error model. (a) Use of the node splitting rule. (b) Use of the GSM concept.

scattering parameters, meanwhile, the power waves at port j of the DUT are defined as a_{j1} and b_{j1}. As compared with the model given in [4], this model also includes n-1 leakage errors between the excited port and the unexcited ports.

Before the error correction procedure, all the error coefficients E_{Dii}, E_{Sii}, E_{Rii}, E_{Xji}, E_{Lji} and E_{Tji} (i, $j = 1 \sim n$, $i \neq j$) are solved by the GSOLT procedure [4]. Then when the S-parameter measurements of an n-port DUT are performed, the error correction will be applied to compute the actual S-parameters [6]. To further simplify the error correction algorithm, the following transformations are made for the flow graph of the 3n-term error model in Fig. 1.

The flow graph (Fig. 2(a)) is constructed from Fig. 1 by using the node splitting rule and then further converted into the generalized flow graph (Fig. 2(b)) by the concept of the generalized scattering matrix (GSM) [11–13]. In Fig. 2(b), the generalized nodes and branch gains are defined as

$$[E] = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad [S_{m1}] = \begin{bmatrix} S_{m11} \\ \vdots \\ S_{mj1} \\ \vdots \\ S_{mn1} \end{bmatrix}, \quad [a_1] = \begin{bmatrix} a_{11} \\ \vdots \\ a_{j1} \\ \vdots \\ a_{n1} \end{bmatrix}, \quad [b_1] = \begin{bmatrix} b_{11} \\ \vdots \\ b_{j1} \\ \vdots \\ b_{n1} \end{bmatrix}, \tag{1}$$

$$[E_{DX1}] = \begin{bmatrix} E_{D11} & & & & \\ & \ddots & & & \\ & & E_{Xi1} & & \\ & & & \ddots & \\ & & & & E_{Xn1} \end{bmatrix}, \quad [I_1] = \begin{bmatrix} 1 & & & & \\ & \ddots & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{bmatrix},$$

$$[E_{SL1}] = \begin{bmatrix} E_{S11} & & & & \\ & \ddots & & & \\ & & E_{Li1} & & \\ & & & \ddots & \\ & & & & E_{Ln1} \end{bmatrix}, \quad [E_{RT1}] = \begin{bmatrix} E_{R11} & & & & \\ & \ddots & & & \\ & & E_{Ti1} & & \\ & & & \ddots & \\ & & & & E_{Tn1} \end{bmatrix}.$$

Finally, the generalized flow graph of the 3n-term error model for port i ($i = 1$ in Fig. 1) excitation can be summarized below:

Table 1. Relationship between error terms and error matrixes.

Error Matrix	Error Term	Source
$[E_{DXi}]$	E_{Dii}	Directivity
	E_{Xji}	Leakage
$[E_{SLi}]$	E_{Sii}	Source match
	E_{Lji}	Load match
$[E_{RTi}]$	E_{Rii}	Reflection tracking
	E_{Tji}	Transmission tracking

The relationship between error terms and error matrixes is described in Table 1. For the error matrix $[E_{UVi}]$, the subscript "UVi" means that the (i, i)th element is E_{Uii} and the (j, i)th element is E_{Vji} $(i \neq j)$. The generalized branch gain $[I_i]$ has only one nonzero element "1" at the (i, i)th position. According to the generalized 3-term error model, we can obtain the matrix equation as

$$\left\{ \begin{matrix} [S_{mi}] \\ [a_i] \end{matrix} \right\} = \left\{ \begin{matrix} [E_{DXi}] & [E_{RTi}] \\ [I_i] & [E_{SLi}] \end{matrix} \right\} \left\{ \begin{matrix} [E] \\ [b_i] \end{matrix} \right\} \tag{2}$$

where the generalized power waves $[E] = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix}$, $[S_{mi}] = \begin{bmatrix} S_{m1i} \\ \vdots \\ S_{mii} \\ \vdots \\ S_{mni} \end{bmatrix}$ on the VNA side and $[a_i] = \begin{bmatrix} a_{1i} \\ \vdots \\ a_{ii} \\ \vdots \\ a_{ni} \end{bmatrix}$,

$[b_i] = \begin{bmatrix} b_{1i} \\ \vdots \\ b_{ii} \\ \vdots \\ b_{ni} \end{bmatrix}$ on the DUT side. From Eq. (1), the reflected wave vector $[b_i]$ and the incident wave vector $[a_i]$ can be deduced as follows:

$$[b_i] = [E_{RTi}]^{-1}\{[S_{mi}] - [E_{DXi}][E]\} \tag{3}$$

$$[a_i] = [I_i][E] + [E_{SLi}][b_i] \tag{4}$$

Once the reflected wave vectors $[b_i]$ ($i = 1 \sim n$) and the incident wave vectors $[a_i]$ ($i = 1 \sim n$) are calculated by Eq. (2) and Eq. (3), the S-matrix of the n-port DUT can be concluded as

$$[S] = [B][A]^{-1} \tag{5}$$

where $[A] = \{[a_1], \ldots [a_i], \ldots [a_n]\}$ and $[B] = \{[b_1], \ldots [b_i], \ldots [b_n]\}$. However, nonideal calibration elements will cause the deviation of $[S]$ from its true value. In this paper, the effect of nonideal calibration elements (S, O, L and T) on the S-parameter measurements is investigated by the following steps:

2.1. Deviations of S-parameters

In the first step, the dependence of $[\delta S]$ of a DUT on the deviations of the error matrixes $[E_{DXi}]$, $[E_{RTi}]$ and $[E_{SLi}]$ is derived. According to the self loop rule [12], the generalized flow graph in Fig. 3 can be simplified into that in Fig. 4(a). Then, the series rule and the parallel rule are successively used to obtain the ratio between $[E]$ and $[S_{mi}]$ in Fig. 4(b) [12].

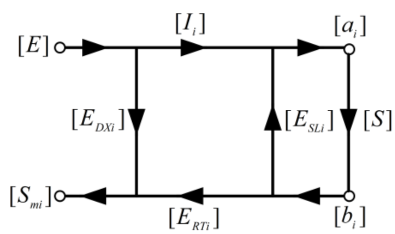


Figure 3. Generalized flow graph of the 3n-term error model.

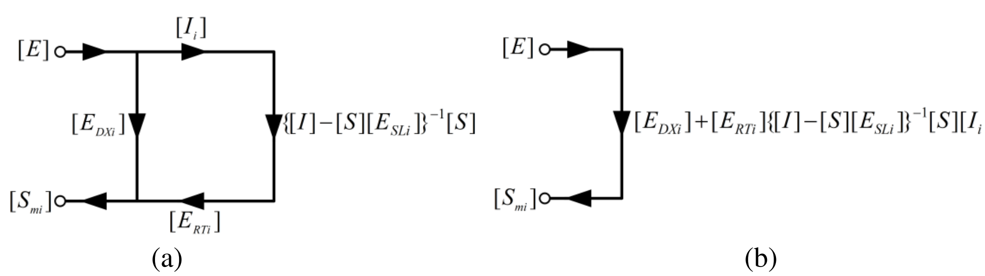


Figure 4. Simplification of the generalized flow graph. (a) Removal of self-loop. (b) Combination of paths..

$[I]$ is the identity matrix and the branch gain in Fig. 4(a) can be also expressed as

$$\{[I] - [S][E_{SLi}]\}^{-1}[S] = [S]\{[I] - [E_{SLi}][S]\}^{-1} \tag{6}$$

From Fig. 4(b), we can get the following equation.

$$[E_{DXi}][E] + [E_{RTi}]\{[I] - [S][E_{SLi}]\}^{-1}[S][I_i][E] = [S_{mi}] \tag{7}$$

Assumed that the deviation $[\delta S]$ is not affected by the variations of the raw values $[S_{mi}]$, the incorrectly defined error terms will lead to the incorrect $[S] + [\delta S]$.

$$[E_{DXi} + \delta E_{DXi}][E] + [E_{RTi} + \delta E_{RTi}]\{[I] - [S + \delta S][E_{SLi} + \delta E_{SLi}]\}^{-1}[S + \delta S][I_i][E] = [S_{mi}] \tag{8}$$

To solve the deviation matrix $[\delta S]$, we need find the inverse of matrix in Eq. (7). Let $[D + \delta D]^{-1} = [D]^{-1} + [\delta X]$ where the deviation $[\delta X]$ is underdetermined, when the deviation $[\delta D]$ of a random non-singular matrix $[D]$ is very close to zero. By using the definition of the inverse matrix $[D+\delta D]\{[D]^{-1}+[\delta X]\} = [I]$, and omitting the product of small terms $[\delta D][\delta X]$, we can get $[\delta X] = -[D]^{-1}[\delta D][D]^{-1}$ and the equation given below [14, 15]:

$$\{[D] + [\delta D]\}^{-1} = [D]^{-1} - [D]^{-1}[\delta D][D]^{-1} \tag{9}$$

Using Eq. (8) and omitting the smaller term $[\delta S][\delta E_{SLi}]$ can obtain

$$\begin{aligned}\{[I]-[S+\delta S][E_{SLi}+\delta E_{SLi}]\}^{-1} &= \{[I]-[S][E_{SLi}]\}^{-1}+\{[I]-[S][E_{SLi}]\}^{-1}\{[\delta S][E_{SLi}]\\ &\quad +[S][\delta E_{SLi}]\}\{[I]-[S][E_{SLi}]\}^{-1}\end{aligned} \tag{10}$$

Substituting Eq. (9) into Eq. (7) and subtracting Eq. (6) form the result, we can deduce the following equation by omitting the products of deviations.

$$\begin{aligned}&-\{[I]-[S][E_{SLi}]\}[E_{RTi}]^{-1}[\delta E_{DXi}][E]-\{[I]-[S][E_{SLi}]\}[E_{RTi}]^{-1}[\delta E_{RTi}][S]\{[I]-[E_{SLi}][S]\}^{-1}[I_i][E]\\ &-[S][\delta E_{SLi}][S]\{[I]-[E_{SLi}][S]\}^{-1}[I_i][E]=[\delta S]\{[I]-[E_{SLi}][S]\}^{-1}[I_i][E]\end{aligned} \tag{11}$$

Because of the dependence among error terms, raw measured S-parameters and actual S-parameters, we can rewrite vectors $[b_i]$ and $[a_i]$ by substituting Eq. (5) and Eq. (6) into Eq. (2) and Eq. (3).

$$[b_i] = [S]\{[I]-[E_{SLi}][S]\}^{-1}[I_i][E] \tag{12}$$

$$[a_i] = \{[I]-[E_{SLi}][S]\}^{-1}[I_i][E] \tag{13}$$

Substituting Eq. (11) and Eq. (12) into Eq. (10) can obtain the equation given below:

$$[\delta S][a_i]=-[C_i][\delta E_{DXi}][E]-[S][\delta E_{SLi}][b_i]-[C_i][\delta E_{RTi}][b_i] \tag{14}$$

where the matrix $[C_i]=\{[I]-[S][E_{SLi}]\}[E_{RTi}]^{-1}$. With the subscript i in Eq. (13) varied from 1 to n, we can establish an equation for the deviation $[\delta S]$ and consequently derive

$$\begin{aligned}[\delta S] &= -\{[C_1][\delta E_{DX1}][E],\dots[C_i][\delta E_{DXi}][E],\dots[C_n][\delta E_{DXn}][E]\}[A]^{-1}\\ &\quad -\{[S][\delta E_{SL1}][b_1],\dots[S][\delta E_{SLi}][b_i],\dots[S][\delta E_{SLn}][b_n]\}[A]^{-1}\\ &\quad -\{[C_1][\delta E_{RT1}][b_1],\dots[C_i][\delta E_{RTi}][b_i],\dots[C_n][\delta E_{RTn}][b_n]\}[A]^{-1}\end{aligned} \tag{15}$$

It is observed that the deviation matrix $[\delta S]$ is developed as a function of $[\delta E_{DXi}]$, $[\delta E_{SLi}]$ and $[\delta E_{RTi}]$. Finally, the scattering parameter deviation δS_{kl} can be concluded as

$$\delta S_{kl}=-\sum_{i=1\dots n}d_{il}[c_{ik}^T][\delta e_{DXi}]-\sum_{i=1\dots n}d_{il}[S_{k1},\dots S_{kn}][\mathrm{diag}(b_i)][\delta e_{SLi}]-\sum_{i=1\dots n}d_{il}[c_{ik}^T][\mathrm{diag}(b_i)][\delta e_{RTi}] \tag{16}$$

where $[c_{ik}^T]$ is the kth row of $[C_i]$ and d_{il} is the $(i,\, l)$th element of $[A]^{-1}$. For the diagonal matrix $[\mathrm{diag}(b_i)]$ the diagonal elements are composed of $[b_i]$. Meanwhile, the deviations $[\delta e_{DXi}]$, $[\delta e_{SLi}]$ and $[\delta e_{RTi}]$ are defined as $n\times 1$ column vectors:

$$[\delta e_{DXi}]=\begin{bmatrix}\delta E_{X1i}\\ \vdots\\ \delta E_{Dii}\\ \vdots\\ \delta E_{Xni}\end{bmatrix},\quad [\delta e_{SLi}]=\begin{bmatrix}\delta E_{L1i}\\ \vdots\\ \delta E_{Sii}\\ \vdots\\ \delta E_{Lni}\end{bmatrix},\quad [\delta e_{RTi}]=\begin{bmatrix}\delta E_{T1i}\\ \vdots\\ \delta E_{Rii}\\ \vdots\\ \delta E_{Tni}\end{bmatrix}$$

To explore how the deviations of errors $[\delta e_{DXi}]$, $[\delta e_{SLi}]$ and $[\delta e_{RTi}]$ affect the S-parameter deviation, we need further deduce the analytical expressions for $[c_{ik}^T]$, d_{il} and $[\mathrm{diag}(b_i)]$. Because the error terms are $|E_{Dii}|\approx 0$, $|E_{Sii}|\approx 0$, $|E_{Xji}|\approx 0$, $|E_{Lji}|\approx 0$, $|E_{Rii}|\approx 1$ and $|E_{Tji}|\approx 1$ in the high-performance VNA, E_{Dii}, E_{Sii}, E_{Xji} and E_{Lji} can be regarded as small terms and their products will be neglected in the following calculation.

As mentioned before, the matrix $[C_i]=\{[I]-[S][E_{SLi}]\}[E_{RTi}]^{-1}$ and so we can get

$$[C_i]=\begin{bmatrix}\frac{1-E_{L1i}S_{11}}{E_{T1i}} & \dots & \frac{-E_{Sii}S_{1i}}{E_{Rii}} & \dots & \frac{-E_{Lni}S_{1n}}{E_{Tni}}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \frac{-E_{L1i}S_{i1}}{E_{T1i}} & \dots & \frac{1-E_{Sii}S_{ii}}{E_{Rii}} & \dots & \frac{-E_{Lni}S_{in}}{E_{Tni}}\\ \vdots & \ddots & \vdots & \ddots & \vdots\\ \frac{-E_{L1i}S_{n1}}{E_{T1i}} & \dots & \frac{-E_{Sii}S_{ni}}{E_{Rii}} & \dots & \frac{1-E_{Lni}S_{nn}}{E_{Tni}}\end{bmatrix} \tag{17}$$

As the kth row of $[C_i]$, the vector $[c_{ik}^T]$ is given below:

$$[c_{ik}^T] = \left[\frac{\varepsilon_{1-k} - E_{L1i}S_{k1}}{E_{T1i}}, \ldots \frac{\varepsilon_{i-k} - E_{Sii}S_{ki}}{E_{Rii}}, \ldots \frac{\varepsilon_{n-k} - E_{Lni}S_{kn}}{E_{Tni}}\right] \tag{18}$$

where $\varepsilon_{i-k} = 1$ for $i = k$ and $\varepsilon_{i-k} = 0$ for $i \neq k$.

To deduce the variable d_{il} defined as the $(i,\ l)$th element of $[A]^{-1}$, we first represent $[a_i]$ with $\{[I] + [E_{SLi}][S]\}[I_i][E]$ by using Eq. (8) and Eq. (12), and thus obtain an approximate expression for $[A]$.

$$[A] = [I] + \begin{bmatrix} E_{S11}S_{11} & \ldots & E_{L1i}S_{1i} & \ldots & E_{L1n}S_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{Li1}S_{i1} & \ldots & E_{Sii}S_{ii} & \ldots & E_{Lin}S_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{Ln1}S_{n1} & \ldots & E_{Lni}S_{ni} & \ldots & E_{Snn}S_{nn} \end{bmatrix} \tag{19}$$

Then inverting the matrix $[A]$ with Eq. (8), we can deduce

$$[A]^{-1} = [I] - \begin{bmatrix} E_{S11}S_{11} & \ldots & E_{L1i}S_{1i} & \ldots & E_{L1n}S_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{Li1}S_{i1} & \ldots & E_{Sii}S_{ii} & \ldots & E_{Lin}S_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ E_{Ln1}S_{n1} & \ldots & E_{Lni}S_{ni} & \ldots & E_{Snn}S_{nn} \end{bmatrix} \tag{20}$$

From Eq. (19), the variable d_{il} is finally described as

$$d_{il} = \begin{cases} 1 - E_{Sll}S_{ll} & i = l \\ -E_{Lil}S_{il} & i \neq l \end{cases} \tag{21}$$

Based on the expression for $[a_i]$ in Eq. (18) and the S-matrix definition $[b_i] = [S][a_i]$, we can have

$$[b_i] = \begin{bmatrix} S_{1i} + E_{Sii}S_{1i}S_{ii} + \sum\limits_{j=1\ldots n}^{j\neq i} E_{Lji}S_{1j}S_{ji} \\ \vdots \\ S_{ii} + E_{Sii}S_{ii}S_{ii} + \sum\limits_{j=1\ldots n}^{j\neq i} E_{Lji}S_{ij}S_{ji} \\ \vdots \\ S_{ni} + E_{Sii}S_{ni}S_{ii} + \sum\limits_{j=1\ldots n}^{j\neq i} E_{Lji}S_{nj}S_{ji} \end{bmatrix} \tag{22}$$

Based on Eq. (21), the diagonal matrix $[\mathrm{diag}(b_i)]$ can be easily obtained. Form the above analysis, the coefficients $[c_{ik}^T]$, d_{il} and $[\mathrm{diag}(b_i)]$ are solved, and then substituting them into Eq. (15) can derive the analytical expression for δS_{kl} in the following two cases:

1) When $k = l$, the deviation δS_{kl} is expressed as

$$\begin{aligned} \delta S_{ll} \ = \ & \frac{2E_{Sll}S_{ll} - 1}{E_{Rll}}\delta E_{Dll} + \sum_{i=1\ldots n}^{i\neq l} \frac{E_{Lil}S_{li}}{E_{Til}}\delta E_{Xil} + \sum_{i=1\ldots n}^{i\neq l} \frac{E_{Lil}S_{il}}{E_{Tli}}\delta E_{Xli} \\ & -S_{ll}\left(S_{ll} + \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{li}S_{il}\right)\delta E_{Sll} + \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{li}S_{ii}S_{il}\delta E_{Sii} \end{aligned}$$

$$-\sum_{i=1\dots n}^{i\neq l} S_{li}\left(S_{il}+\sum_{j=1\dots n}^{j\neq l} E_{Ljl}S_{ij}S_{jl}\right)\delta E_{Lil}+\sum_{i=1\dots n}^{i\neq l}\sum_{j=1\dots n}^{j\neq i} E_{Lil}S_{lj}S_{ji}S_{il}\delta E_{Lji}$$
$$-\frac{S_{ll}-E_{Sll}S_{ll}^2+\sum\limits_{i=1\dots n}^{i\neq l} E_{Lil}S_{li}S_{il}}{E_{Rll}}\delta E_{Rll}+\sum_{i=1\dots n}^{i\neq l}\frac{E_{Lil}S_{li}S_{il}}{E_{Til}}\delta E_{Til}+\sum_{i=1\dots n}^{i\neq l}\frac{E_{Lil}S_{li}S_{il}}{E_{Tli}}\delta E_{Tli} \quad (23)$$

2) When $k \neq l$, the deviation δS_{kl} is expressed as

$$\begin{aligned}\delta S_{kl} =& \frac{E_{Sll}S_{kl}}{E_{Rll}}\delta E_{Dll}+\frac{E_{Lkl}S_{kl}}{E_{Rkk}}\delta E_{Dkk}+\frac{E_{Sll}S_{ll}+E_{Lkl}S_{kk}-1}{E_{Tkl}}\delta E_{Xkl}+\sum_{i=1\dots n}^{i\neq k,l}\frac{E_{Lil}S_{ki}}{E_{Til}}\delta E_{Xil}\\ &+\sum_{i=1\dots n}^{i\neq k,l}\frac{E_{Lil}S_{il}}{E_{Tki}}\delta E_{Xki}-S_{kl}\left(S_{ll}+\sum_{i=1\dots n}^{i\neq l}E_{Lil}S_{li}S_{il}\right)\delta E_{Sll}+\sum_{i=1\dots n}^{i\neq l}E_{Lil}S_{ki}S_{ii}S_{il}\delta E_{Sii}\\ &-\sum_{i=1\dots n}^{i\neq l}S_{ki}\left(S_{il}+\sum_{j=1\dots n}^{j\neq l}E_{Ljl}S_{ij}S_{jl}\right)\delta E_{Lil}\\ &+\sum_{i=1\dots n}^{i\neq l}\sum_{j=1\dots n}^{j\neq i}E_{Lil}S_{kj}S_{ji}S_{il}\delta E_{Lji}+\frac{E_{Sll}S_{kl}S_{ll}}{E_{Rll}}\delta E_{Rll}+\frac{E_{Lkl}S_{kk}S_{kl}}{E_{Rkk}}\delta E_{Rkk}\\ &-\frac{S_{kl}+\sum\limits_{i=1\dots n}^{i\neq k,l}E_{Lil}S_{ki}S_{il}}{E_{Tkl}}\delta E_{Tkl}+\sum_{i=1\dots n}^{i\neq k,l}\frac{E_{Lil}S_{ki}S_{il}}{E_{Til}}\delta E_{Til}+\sum_{i=1\dots n}^{i\neq k,l}\frac{E_{Lil}S_{ki}S_{il}}{E_{Tki}}\delta E_{Tki}\end{aligned} \quad (24)$$

From Eq. (22) and Eq. (23), we can calculate the sensitivity coefficients with respect to the error terms as input quantities and the S-parameters of the DUT as output quantities.

2.2. Deviations of Error Terms

In the second step, the deviations of error terms associated with SOLT calibration standards are solved. By measuring standards SOL at port i, we can use the 3-term error model in Fig. 5 to deduce the system errors E_{Dii}, E_{Sii} and E_{Rii} by the linear equations below:

$$\begin{bmatrix}1 & S_{mii}(\mathrm{S})\Gamma_{Si} & \Gamma_{Si}\\ 1 & S_{mii}(\mathrm{O})\Gamma_{Oi} & \Gamma_{Oi}\\ 1 & S_{mii}(\mathrm{L})\Gamma_{Li} & \Gamma_{Li}\end{bmatrix}\begin{bmatrix}E_{Dii}\\ E_{Sii}\\ E_{Rii}-E_{Dii}E_{Sii}\end{bmatrix}=\begin{bmatrix}S_{mii}(\mathrm{S})\\ S_{mii}(\mathrm{O})\\ S_{mii}(\mathrm{L})\end{bmatrix} \quad (25)$$

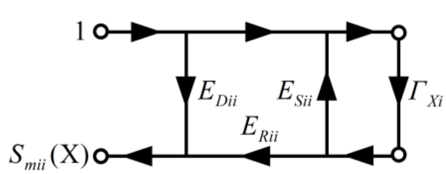


Figure 5. Error model of SOL calibration.

where $S_{mii}(\mathrm{X})$ is the raw reflection coefficient of the standard X at port i. The S-parameter deviations of standards SOL ($\delta\Gamma_{Si}$, $\delta\Gamma_{Oi}$ and $\delta\Gamma_{Li}$) will cause the error deviations (δE_{Dii}, δE_{Sii} and δE_{Rii}).

$$\left\{\begin{bmatrix} 1 & S_{mii}(\mathrm{S})\Gamma_{Si} & \Gamma_{Si} \\ 1 & S_{mii}(\mathrm{O})\Gamma_{Oi} & \Gamma_{Oi} \\ 1 & S_{mii}(\mathrm{L})\Gamma_{Li} & \Gamma_{Li} \end{bmatrix} + \begin{bmatrix} 0 & S_{mii}(\mathrm{S})\delta\Gamma_{Si} & \delta\Gamma_{Si} \\ 0 & S_{mii}(\mathrm{O})\delta\Gamma_{Oi} & \delta\Gamma_{Oi} \\ 0 & S_{mii}(\mathrm{L})\delta\Gamma_{Li} & \delta\Gamma_{Li} \end{bmatrix}\right\}$$
$$\left\{\begin{bmatrix} E_{Dii} \\ E_{Sii} \\ E_{Rii} - E_{Dii}E_{Sii} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -E_{Sii} & -E_{Dii} & 1 \end{bmatrix}\begin{bmatrix} \delta E_{Dii} \\ \delta E_{Sii} \\ \delta E_{Rii} \end{bmatrix}\right\} = \begin{bmatrix} S_{mii}(\mathrm{S}) \\ S_{mii}(\mathrm{O}) \\ S_{mii}(\mathrm{L}) \end{bmatrix} \tag{26}$$

Subtracting (25) from (26) and substituting $S_{mii}(X) = E_{Dii} + E_{Rii}\Gamma_{Xi}/(1 - E_{Sii}\Gamma_{Xi})$ into the result, we can get the expression for the deviation vector $[\delta E_{Dii},\ \delta E_{Sii},\ \delta E_{Rii}]^T$ via a series of matrix operations [10]. Finally, we can deduce

$$\delta E_{Dii} = E_{Rii}\left[\frac{\Gamma_{Oi}\Gamma_{Li}}{(\Gamma_{Oi}-\Gamma_{Si})(\Gamma_{Si}-\Gamma_{Li})}\delta\Gamma_{Si} + \frac{\Gamma_{Li}\Gamma_{Si}}{(\Gamma_{Li}-\Gamma_{Oi})(\Gamma_{Oi}-\Gamma_{Si})}\delta\Gamma_{Oi} + \frac{\Gamma_{Si}\Gamma_{Oi}}{(\Gamma_{Si}-\Gamma_{Li})(\Gamma_{Li}-\Gamma_{Oi})}\delta\Gamma_{Li}\right] \tag{27}$$

$$\delta E_{Sii} = \frac{(1-E_{Sii}\Gamma_{Oi})(1-E_{Sii}\Gamma_{Li})}{(\Gamma_{Oi}-\Gamma_{Si})(\Gamma_{Si}-\Gamma_{Li})}\delta\Gamma_{Si} + \frac{(1-E_{Sii}\Gamma_{Li})(1-E_{Sii}\Gamma_{Si})}{(\Gamma_{Li}-\Gamma_{Oi})(\Gamma_{Oi}-\Gamma_{Si})}\delta\Gamma_{Oi} + \frac{(1-E_{Sii}\Gamma_{Si})(1-E_{Sii}\Gamma_{Oi})}{(\Gamma_{Si}-\Gamma_{Li})(\Gamma_{Li}-\Gamma_{Oi})}\delta\Gamma_{Li} \tag{28}$$

$$\delta E_{Rii} = E_{Rii}\left[\frac{2E_{Sii}\Gamma_{Oi}\Gamma_{Li}-\Gamma_{Oi}-\Gamma_{Li}}{(\Gamma_{Oi}-\Gamma_{Si})(\Gamma_{Si}-\Gamma_{Li})}\delta\Gamma_{Si} + \frac{2E_{Sii}\Gamma_{Li}\Gamma_{Si}-\Gamma_{Li}-\Gamma_{Si}}{(\Gamma_{Li}-\Gamma_{Oi})(\Gamma_{Oi}-\Gamma_{Si})}\delta\Gamma_{Oi} + \frac{2E_{Sii}\Gamma_{Si}\Gamma_{Oi}-\Gamma_{Si}-\Gamma_{Oi}}{(\Gamma_{Si}-\Gamma_{Li})(\Gamma_{Li}-\Gamma_{Oi})}\delta\Gamma_{Li}\right] \tag{29}$$

It is worth noting that δE_{Xji} is independent on the S-parameter deviations of SOLT standards, that is to say $\delta E_{Xji} = 0$.

Measuring standard T between port i and port j, we can use the flow graph combined with the transmission matrix (T-matrix) to represent the measurements $S_{mii}(T)$ and $S_{mji}(T)$ in Fig. 6.

The T-matrix of the error box at port i and that of the 2-port network of standard T, both selected by dash in Fig. 6, are defined as $[T_i]$ and $[T_{ij}]$ respectively. Cascading the T-matrixes $[T_i]$ and $[T_{ij}]$ can deduce the equation as follows:

$$\begin{bmatrix} \dfrac{S_{mii}(T)}{S_{mji}(T) - E_{Xji}} \\ \dfrac{1}{S_{mji}(T) - E_{Xji}} \end{bmatrix} = [T_i][T_{tij}]\begin{bmatrix} \dfrac{E_{Lji}}{E_{Tji}} \\ \dfrac{1}{E_{Tji}} \end{bmatrix} \tag{30}$$

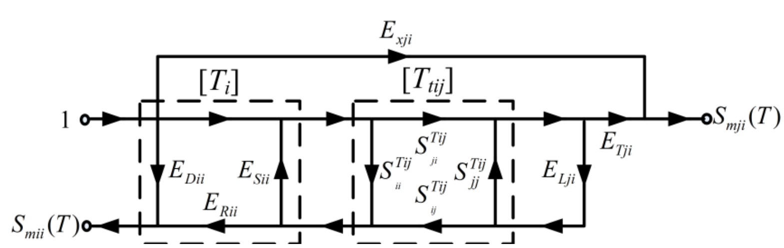


Figure 6. Error model of T calibration.

If the left hand of (30) is invariable, the deviations $[\delta T_i]$, $[\delta T_{tij}]$, δE_{Lji} and δE_{Tji} occur.

$$\begin{bmatrix} \dfrac{S_{mii}(T)}{S_{mji}(T)-E_{Xji}} \\ \dfrac{1}{S_{mji}(T)-E_{Xji}} \end{bmatrix} = \{[T_i]+[\delta T_i]\}\{[T_{tij}]+[\delta T_{tij}]\}\left\{\begin{bmatrix} \dfrac{E_{Lji}}{E_{Tji}} \\ \dfrac{1}{E_{Tji}} \end{bmatrix} + \begin{bmatrix} \delta\dfrac{E_{Lji}}{E_{Tji}} \\ \delta\dfrac{1}{E_{Tji}} \end{bmatrix}\right\} \tag{31}$$

Subtracting (30) from (31) and omitting the product of small terms, we can deduce

$$\begin{bmatrix} \delta E_{Lji} \\ \delta E_{Tji} \end{bmatrix} = \begin{bmatrix} -1 & E_{Lji} \\ 0 & E_{Tji} \end{bmatrix} [T_{tij}]^{-1}[T_i]^{-1}[\delta T_i][T_{tij}] \begin{bmatrix} E_{Lji} \\ 1 \end{bmatrix} + \begin{bmatrix} -1 & E_{Lji} \\ 0 & E_{Tji} \end{bmatrix} [T_{tij}]^{-1}[\delta T_{tij}] \begin{bmatrix} E_{Lji} \\ 1 \end{bmatrix} \tag{32}$$

The right hand of (32) is composed of two terms. The first term contains the deviation $[\delta T_i]$, which associates with the deviations $\delta\Gamma_{Si}$, $\delta\Gamma_{Oi}$ and $\delta\Gamma_{Li}$ of standards SOL. The second term contains the deviation $[\delta T_{tij}]$, which associates with the deviations δS_{ii}^{Tij}, δS_{ij}^{Tij}, δS_{ji}^{Tij} and δS_{jj}^{Tij} of the 2-port network of T connection. From Fig. 6, we can get the expressions for $[T_i]$ and $[T_{ij}]$ given below:

$$[T_i] = \begin{bmatrix} E_{Rii} - E_{Dii}E_{Sii} & E_{Dii} \\ -E_{Sii} & 1 \end{bmatrix} \tag{33}$$

$$[T_{ij}] = -\frac{1}{S_{ji}^{Tij}} \begin{bmatrix} S_{ii}^{Tij}S_{jj}^{Tij} - S_{ji}^{Tij}S_{ij}^{Tij} & -S_{ii}^{Tij} \\ S_{jj}^{Tij} & -1 \end{bmatrix} \tag{34}$$

By taking the total differential of Eq. (32) and Eq. (33), we can get

$$[\delta T_i] = \begin{bmatrix} -E_{Sii}\delta E_{Dii} - E_{Dii}\delta E_{Sii} + \delta E_{Rii} & \delta E_{Dii} \\ -\delta E_{Sii} & -\delta E_{Rii} \end{bmatrix} \tag{35}$$

$$[\delta T_{ij}] = \frac{1}{(S_{ji}^{Tij})^2} \begin{bmatrix} -S_{ji}^{Tij}S_{jj}^{Tij}\delta S_{ii}^{Tij} + S_{ii}^{Tij}S_{jj}^{Tij}\delta S_{ji}^{Tij} + \left(S_{ji}^{Tij}\right)^2\delta S_{ij}^{Tij} - S_{ii}^{Tij}S_{ji}^{Tij}\delta S_{jj}^{Tij} & S_{ji}^{Tij}\delta S_{ii}^{Tij} - S_{ii}^{Tij}\delta S_{ji}^{Tij} \\ S_{jj}^{Tij}\delta S_{ji}^{Tij} - S_{ji}^{Tij}\delta S_{jj}^{Tij} & -\delta S_{ji}^{Tij} \end{bmatrix} \tag{36}$$

Substituting Eqs. (32) to (35) into Eq. (31) can determine the deviations δE_{Lji} and δE_{Tji} as follows:

$$\begin{aligned} \delta E_{Lji} = \; & \frac{M^2 + N^2\Gamma_{Oi}\Gamma_{Li} - MN(\Gamma_{Oi}+\Gamma_{Li})}{S_{ji}^{Tij}S_{ij}^{Tij}(\Gamma_{Si}-\Gamma_{Oi})(\Gamma_{Si}-\Gamma_{Li})}\delta\Gamma_{Si} + \frac{M^2 + N^2\Gamma_{Li}\Gamma_{Si} - MN(\Gamma_{Li}+\Gamma_{Si})}{S_{ji}^{Tij}S_{ij}^{Tij}(\Gamma_{Oi}-\Gamma_{Li})(\Gamma_{Oi}-\Gamma_{Si})}\delta\Gamma_{Oi} \\ & + \frac{M^2 + N^2\Gamma_{Si}\Gamma_{Oi} - MN(\Gamma_{Si}+\Gamma_{Oi})}{S_{ji}^{Tij}S_{ij}^{Tij}(\Gamma_{Li}-\Gamma_{Si})(\Gamma_{Li}-\Gamma_{Oi})}\delta\Gamma_{Li} \\ & - \frac{N^2}{S_{ji}^{Tij}S_{ij}^{Tij}}\delta S_{ii}^{Tij} - \frac{NE_{Lji}}{S_{ji}^{Tij}}\delta S_{ji}^{Tij} - \frac{NE_{Lji}}{S_{ij}^{Tij}}\delta S_{ij}^{Tij} - E_{Lji}^2\delta S_{jj}^{Tij} \end{aligned} \tag{37}$$

$$\begin{aligned} \delta E_{Tji} = \; & \frac{ME_{Tji}S_{jj}^{Tij}(\Gamma_{Oi}+\Gamma_{Li}) - PE_{Tji}\Gamma_{Oi}\Gamma_{Li} - MQE_{Tji}}{S_{ji}^{Tij}S_{ij}^{Tij}(\Gamma_{Si}-\Gamma_{Oi})(\Gamma_{Si}-\Gamma_{Li})}\delta\Gamma_{Si} \\ & + \frac{ME_{Tji}S_{jj}^{Tij}(\Gamma_{Li}+\Gamma_{Si}) - PE_{Tji}\Gamma_{Li}\Gamma_{Si} - MQE_{Tji}}{S_{ji}^{Tij}S_{ij}^{Tij}(\Gamma_{Oi}-\Gamma_{Li})(\Gamma_{Oi}-\Gamma_{Si})}\delta\Gamma_{Oi} \\ & + \frac{ME_{Tji}S_{jj}^{Tij}(\Gamma_{Si}+\Gamma_{Oi}) - PE_{Tji}\Gamma_{Si}\Gamma_{Oi} - MQE_{Tji}}{S_{ji}^{Tij}S_{ij}^{Tij}(\Gamma_{Li}-\Gamma_{Si})(\Gamma_{Li}-\Gamma_{Oi})}\delta\Gamma_{Li} \\ & + \frac{NE_{Tji}S_{jj}^{Tij}}{S_{ji}^{Tij}S_{ij}^{Tij}}\delta S_{ii}^{Tij} - \frac{NE_{Tji}}{S_{ji}^{Tij}}\delta S_{ji}^{Tij} + \frac{E_{Tji}E_{Lji}S_{jj}^{Tij}}{S_{ij}^{Tij}}\delta S_{ij}^{Tij} - E_{Tji}E_{Lji}\delta S_{jj}^{Tij} \end{aligned} \tag{38}$$

where the variables $M = S_{ii}^{Tij} - E_{Lji}Q$, $N = 1 - E_{Lji}S_{jj}^{Tij}$, $P = S_{jj}^{Tij} + E_{Sii}S_{ji}^{Tij}S_{ij}^{Tij} - E_{Lji}S_{jj}^{Tij}S_{jj}^{Tij}$ and $Q = S_{ii}^{Tij}S_{jj}^{Tij} - S_{ji}^{Tij}S_{ij}^{Tij}$. From the above analysis in part B, the deviations of error terms are expressed by the S-parameter deviations of SOLT standards, which implies that the sensitivity coefficients with respect to the S-parameters of SOLT standards as input quantities and the error terms as output quantities can be determined.

2.3. Combination of Part A and Part B

In the last step, combining the results of part A and B can obtain the sensitivity coefficients of S-parameters for arbitrary n-port DUT with respect to the nonideal calibration standards. In general, the system errors are solved in the ideal condition ($\Gamma_{Si} = -1$, $\Gamma_{Oi} = 1$, $\Gamma_{Li} = 0$, $S_{ii}^{Tij} = S_{jj}^{Tij} = 0$, $S_{ji}^{Tij} = S_{ij}^{Tij} = 1$), therefore the results in Part B can be simplified as

$$\delta E_{Dii} = -E_{Rii}\delta\Gamma_{Li} \tag{39}$$

$$\delta E_{Sii} = \frac{E_{Sii}-1}{2}\delta\Gamma_{Si} - \frac{1+E_{Sii}}{2}\delta\Gamma_{Oi} + \delta\Gamma_{Li} \tag{40}$$

$$\delta E_{Rii} = \frac{E_{Rii}}{2}\delta\Gamma_{Si} - \frac{E_{Rii}}{2}\delta\Gamma_{Oi} - 2E_{Rii}E_{Sii}\delta\Gamma_{Li} \tag{41}$$

$$\delta E_{Lji} = -\frac{E_{Lji}}{2}\delta\Gamma_{Si} + \frac{E_{Lji}}{2}\delta\Gamma_{Oi} + \delta\Gamma_{Li} - \delta S_{ii}^{Tij} - E_{Lji}\delta S_{ji}^{Tij} - E_{Lji}\delta S_{ij}^{Tij} \tag{42}$$

$$\delta E_{Tji} = \frac{E_{Tji}E_{Lji}}{2}\delta\Gamma_{Si} + \frac{E_{Tji}E_{Lji}}{2}\delta\Gamma_{Oi} - E_{Tji}(E_{Sii}+E_{Lji})\delta\Gamma_{Li} - E_{Tji}\delta S_{ji}^{Tij} - E_{Tji}E_{Lji}\delta S_{jj}^{Tij} \tag{43}$$

Substituting Eqs. (38)–(42) into Eq. (22) and Eq. (23), we can finally conclude the sensitivity coefficients for the S-parameters as follows:

1) When $k = l$, the deviation δS_{kl} is expressed as

$$\begin{aligned}\delta S_{ll} = & -\frac{S_{ll}\left(1 - S_{ll} - \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{li}S_{il}\right)}{2}\delta\Gamma_{Sl} - \sum_{i=1\ldots n}^{i\neq l}\frac{E_{Lil}S_{li}S_{ii}S_{il}}{2}\delta\Gamma_{Si} \\ & + \frac{S_{ll}\left(1 + S_{ll} + \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{li}S_{il}\right)}{2}\delta\Gamma_{Ol} - \sum_{i=1\ldots n}^{i\neq l}\frac{E_{Lil}S_{li}S_{ii}S_{il}}{2}\delta\Gamma_{Oi} \\ & + \delta\Gamma_{Ll} - \sum_{i=1\ldots n} S_{li}\left(S_{il} + \sum_{j=1\ldots n}^{j\neq l} E_{Ljl}S_{ij}S_{jl}\right)\delta\Gamma_{Ll} + \sum_{i=1\ldots n}^{i\neq l}\sum_{j=1\ldots n} E_{Lil}S_{lj}S_{ji}S_{il}\delta\Gamma_{Li} \\ & + \sum_{i=1\ldots n}^{i\neq l} S_{li}\left(S_{il} + \sum_{j=1\ldots n}^{j\neq l} E_{Ljl}S_{ij}S_{jl}\right)\delta S_{ll}^{Til} - \sum_{i=1\ldots n}^{i\neq l}\sum_{j=1\ldots n}^{j\neq i} E_{Lil}S_{lj}S_{ji}S_{il}\delta S_{ii}^{Tij}\end{aligned} \tag{44}$$

2) When $k \neq l$, the deviation δS_{kl} is expressed as

$$\begin{aligned}\delta S_{kl} = & \frac{S_{kl}\left(S_{ll} - E_{Lkl} + \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{li}S_{il}\right) + \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{ki}S_{il}}{2}\delta\Gamma_{Sl} + \frac{E_{Lkl}S_{kk}S_{kl}}{2}\delta\Gamma_{Sk} \\ & - \sum_{i=1\ldots n}^{i\neq l}\frac{E_{Lil}S_{ki}S_{ii}S_{il}}{2}\delta\Gamma_{Si} + \frac{S_{kl}\left(S_{ll} - E_{Lkl} + \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{li}S_{il}\right) - \sum_{i=1\ldots n}^{i\neq l} E_{Lil}S_{ki}S_{il}}{2}\delta\Gamma_{Ol}\end{aligned}$$

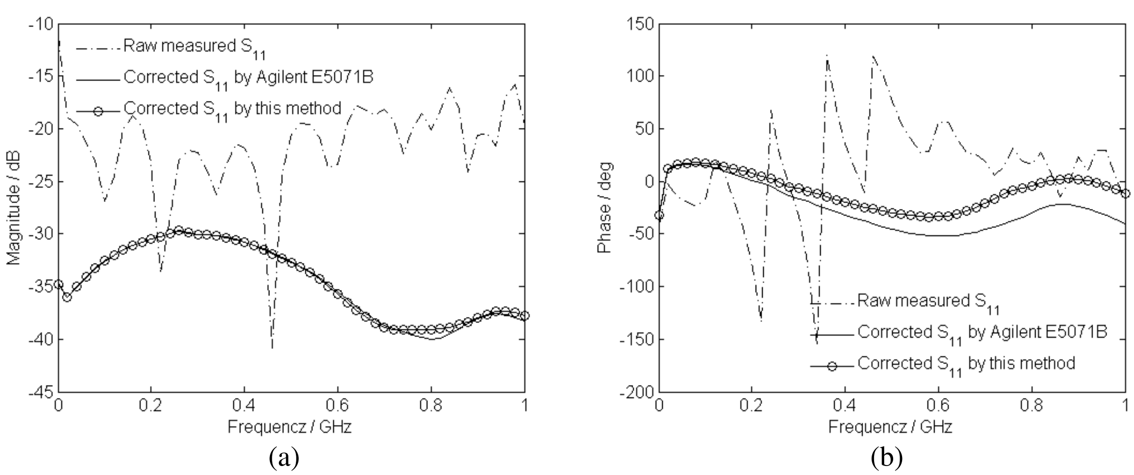
$$-\frac{E_{Lkl}S_{kk}S_{kl}}{2}\delta\Gamma_{Ok}-\sum_{i=1\ldots n}^{i\neq l}\frac{E_{Lil}S_{ki}S_{ii}S_{il}}{2}\delta\Gamma_{Oi}+E_{Lkl}S_{kl}\delta\Gamma_{Ll}$$
$$-\sum_{i=1\ldots n}S_{ki}\left(S_{il}+\sum_{j=1\ldots n}^{j\neq l}E_{Ljl}S_{ij}S_{jl}\right)\delta\Gamma_{Ll}-E_{Lkl}S_{kl}\delta\Gamma_{Lk}+\sum_{i=1\ldots n}^{i\neq l}\sum_{j=1\ldots n}E_{Lil}S_{kj}S_{ji}S_{il}\delta\Gamma_{Li}$$
$$+\sum_{i=1\ldots n}^{i\neq l}S_{ki}\left(S_{il}+\sum_{j=1\ldots n}^{j\neq l}E_{Ljl}S_{ij}S_{jl}\right)\delta S_{ll}^{Til}+E_{Lkl}S_{kl}\delta S_{kk}^{Tkl}-\sum_{i=1\ldots n}^{i\neq l}\sum_{j=1\ldots n}^{j\neq i}E_{Lil}S_{kj}S_{ji}S_{il}\delta S_{ii}^{Tij}$$
$$+\left(S_{kl}+\sum_{i=1\ldots n}^{i\neq l}E_{Lil}S_{ki}S_{il}\right)\delta S_{kl}^{Tkl}+\sum_{i=1\ldots n}^{i\neq l}E_{Lil}S_{ki}S_{il}\delta S_{li}^{Til}-\sum_{i=1\ldots n}^{i\neq k,l}E_{Lil}S_{ki}S_{il}\delta S_{ki}^{Tik} \quad (45)$$

From Eq. (43) and Eq. (44), we can find out how the S-parameter deviations of SOLT standards affect the uncertainty of S_{ij}.

3. MEASUREMENT RESULTS

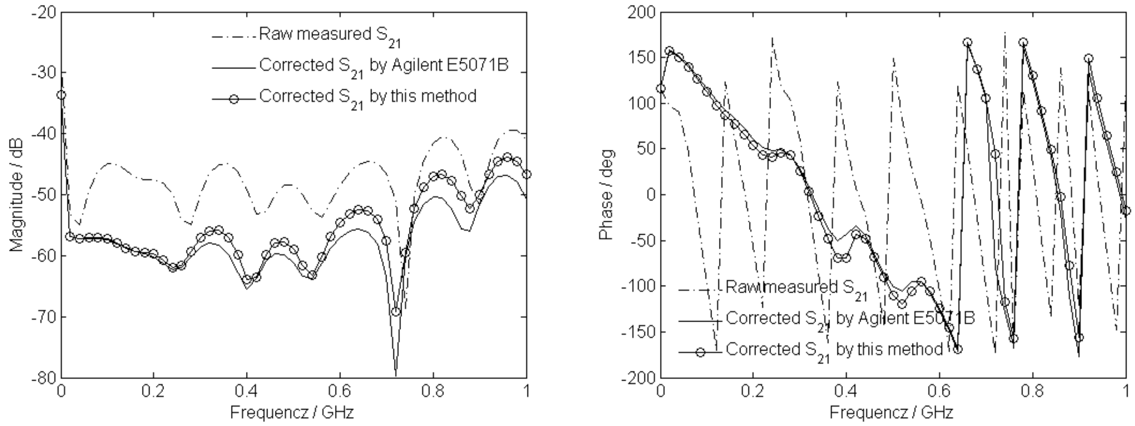
To verify the proposed method, three steps are needed: first solving the system errors by GSOLT calibration, then determining the corrected S-parameters of the multiport DUT, and finally calculating the sensitivity coefficients of the S-parameters. During the whole analysis process, because the test port connectors are of the same sex, the reflection coefficients satisfy the relationship of $\Gamma_{Si}=\Gamma_S$, $\Gamma_{Oi}=\Gamma_O$ and $\Gamma_{Li}=\Gamma_L$. Meanwhile, it is established that $S_{ii}^{Tij}=S_{11}^T$, $S_{ij}^{Tij}=S_{12}^T$, $S_{ji}^{Tij}=S_{21}^T$ and $S_{jj}^{Tij}=S_{22}^T$ with the port number "j" larger than the port number "i". In these cases, there are totally 7 uncertainty sources associated with the nonideal SOLT standards: Γ_S, Γ_O, Γ_L, S_{11}^T, S_{12}^T, S_{21}^T and S_{22}^T. Based on the above and by applying the GSOLT calibration (Fig. 5 and Fig. 6), all the errors in Table 1 can be deduced from Eq. (24) and Eq. (29). After that, measurements are performed on a four-port dual directional coupler using the commercial four-port VNA (Agilent E5071B). Once the system errors and the raw measured data are obtained in the ideal condition ($\Gamma_S=-1$, $\Gamma_O=1$, $\Gamma_L=0$, $S_{11}^T=S_{22}^T=0$, $S_{21}^T=S_{12}^T=1$), the S-parameters of the four-port dual directional coupler are calculated by Eq. (4) and compared with the Agilent E5071B results in Figs. 7–10.

Experiment results in Figs. 7–10 can attest the good performance of the calibration method based on the generalized flow graph of the 3n-term error model. However, because the results by this method



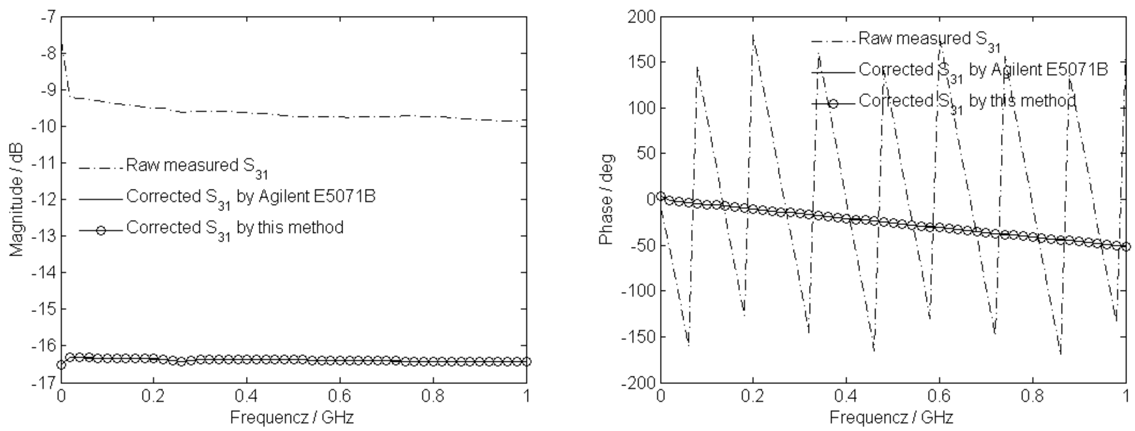
(a) (b)

Figure 7. Comparison of S_{11}. (a) Magnitude. (b) Phase.



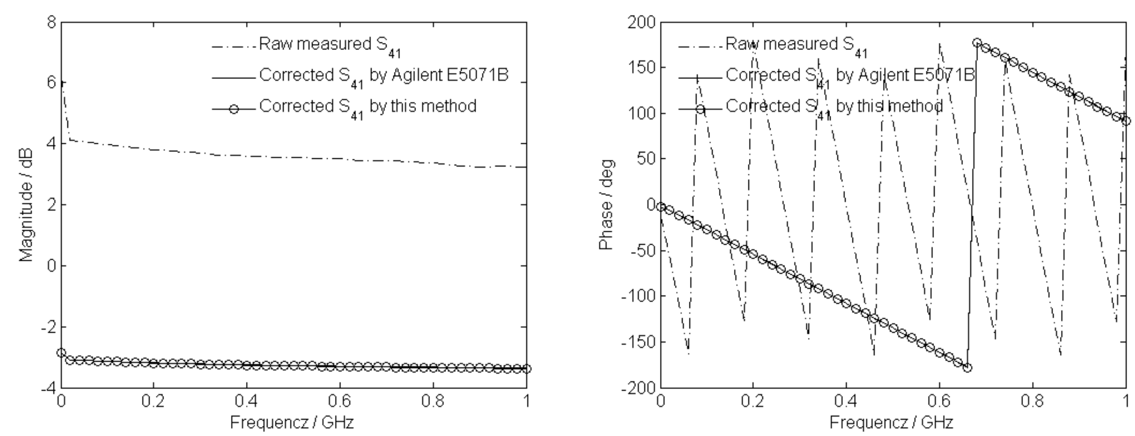
(a) (b)

Figure 8. Comparison of S_{21}. (a) Magnitude. (b) Phase.



(a) (b)

Figure 9. Comparison of S_{31}. (a) Magnitude. (b) Phase.



(a) (b)

Figure 10. Comparison of S_{41}. (a) Magnitude. (b) Phase.

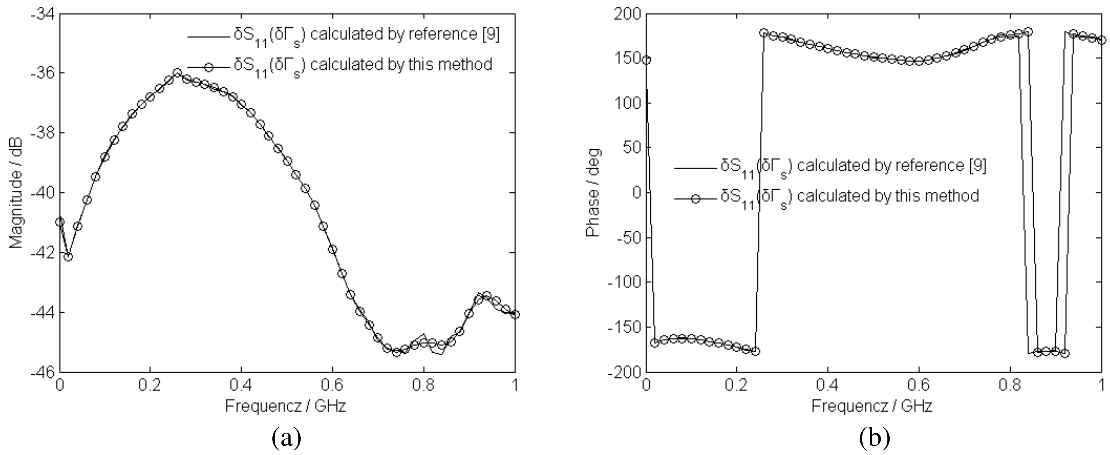


Figure 11. Comparison of $\delta S_{11}(\delta\Gamma_S)$. (a) Magnitude. (b) Phase.

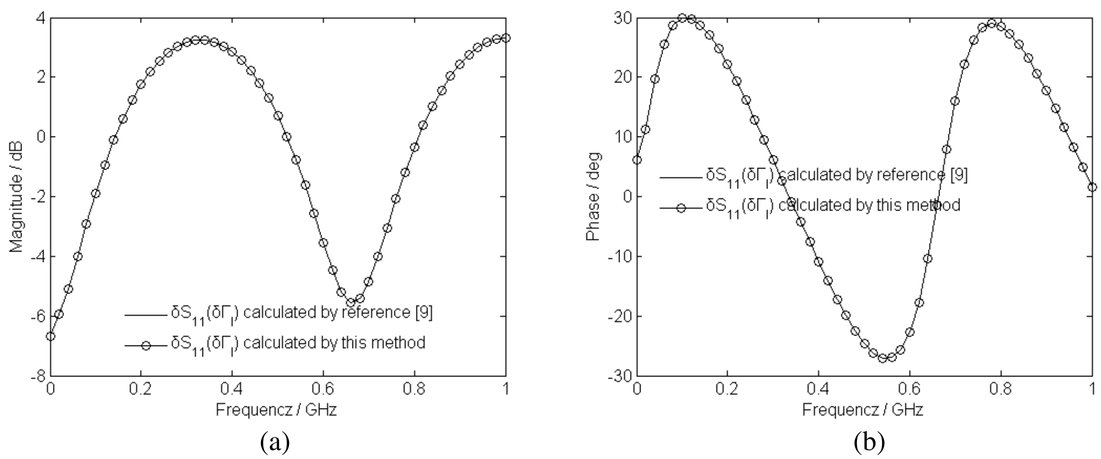
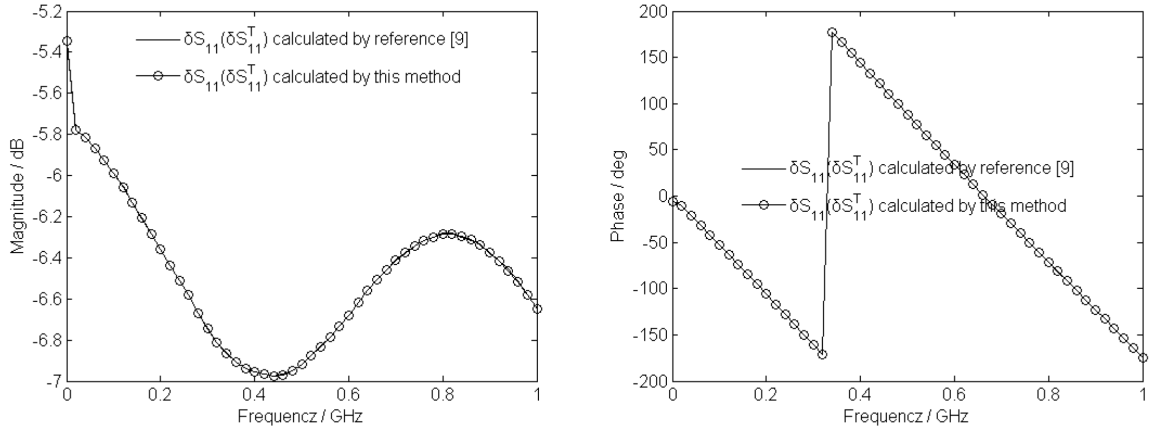


Figure 12. Comparison of $\delta S_{11}(\delta\Gamma_l)$. (a) Magnitude. (b) Phase.

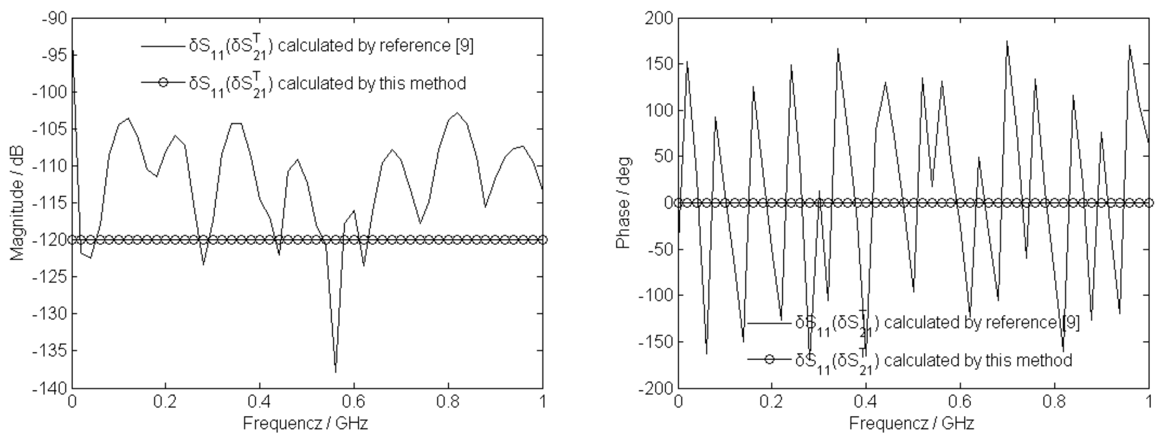
is based on the assumption that $\Gamma_S = -1$, $\Gamma_O = 1$, $\Gamma_L = 0$, $S_{11}^T = S_{22}^T = 0$ and $S_{21}^T = S_{12}^T = 1$, there will be the S-parameter deviations of the DUT caused by the nonideal standards, such as the phase deviation of corrected S_{11} in Fig. 7(b). To explore how the SOLT standards affect the S-parameter deviations of the DUT, the sensitivity coefficients are calculated by Eq. (43) and Eq. (44). The following results (Fig. 11–Fig. 18) show that the sensitivity coefficients calculated by this paper are in very good agreement with those given in [10], demonstrating the correctness of proposed method.

The sensitivity coefficients $\delta S_{ll}(\delta S_{21}^T)$ and $\delta S_{ll}(S_{12}^T)$, which are ignored in Eq. (43) due to the approximation, are assigned a value of 1.0e-6 (e.g., $\delta S_{1l}(\delta S_{21}^T)$ in Fig. 14). In fact, the modulus of the actual sensitivity coefficients $\delta S_{ll}(\delta S_{21}^T)$ and $\delta S_{ll}(S_{12}^T)$ are mostly less than 1.0e-5 for this four-port dual directional coupler. In addition, because the sensitivity coefficient $\delta S_{ll}(\delta\Gamma_l)$ contains a constant term "1" in Eq. (43), the modulus of $\delta S_{ll}(\delta\Gamma_l)$ generally has a large value, which can explain the phase deviation of S_{11} in Fig. 7(b). The great deviation of S_{21} in Fig. 8 is also caused by the nonideal L standard, whose impact can be estimated by the expression $\delta S_{kl}(\delta\Gamma_L) = -\sum\limits_{i=1\ldots n} S_{ki}S_{il}$ in this experiment. In conclusion, these results demonstrated that the analytical expressions for the sensitivity coefficients are correct and useful in the multiport DUT measurement.



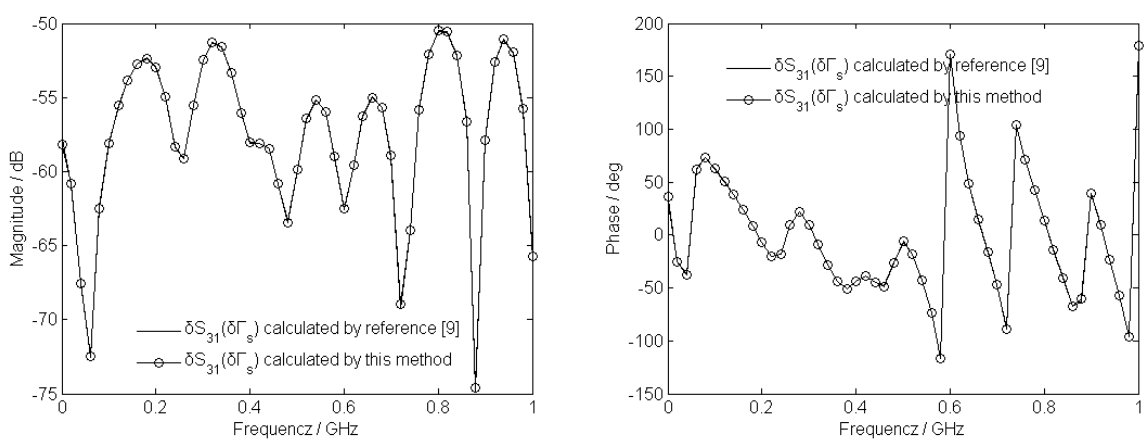
(a) (b)

Figure 13. Comparison of $\delta S_{11}(\delta S_{11}^T)$. (a) Magnitude. (b) Phase.



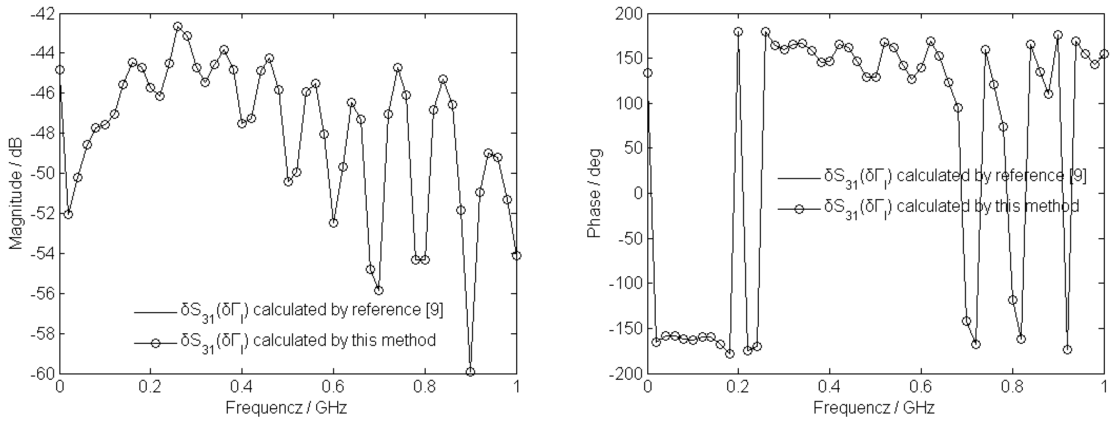
(a) (b)

Figure 14. Comparison of $\delta S_{11}(\delta S_{21}^T)$. (a) Magnitude. (b) Phase.



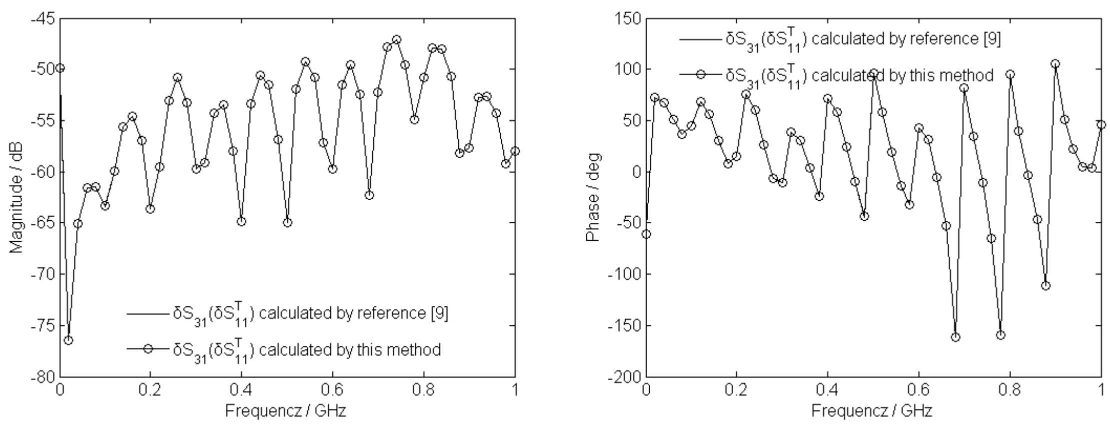
(a) (b)

Figure 15. Comparison of $\delta S_{11}(\delta \Gamma_S)$. (a) Magnitude. (b) Phase.



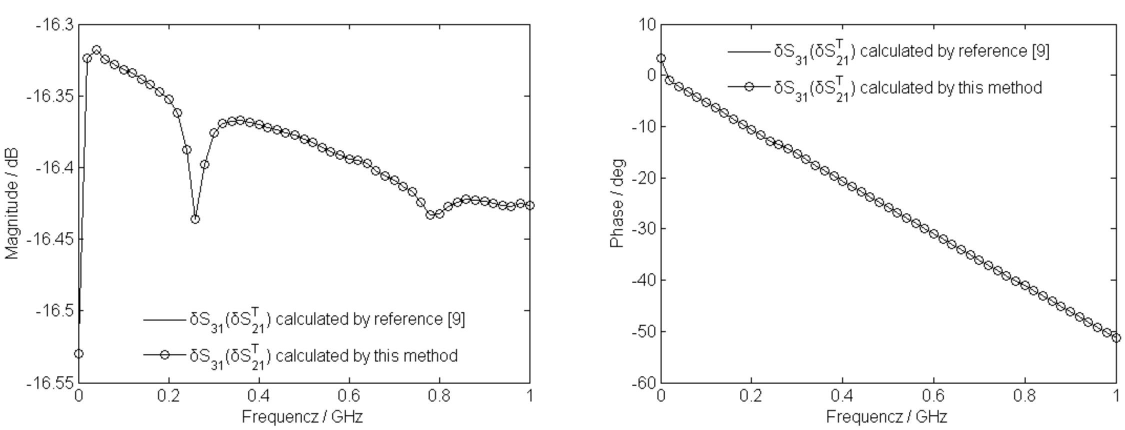
(a) (b)

Figure 16. Comparison of $\delta S_{31}(\delta\Gamma_l)$. (a) Magnitude. (b) Phase.



(a) (b)

Figure 17. Comparison of $\delta S_{31}(\delta S_{11}^T)$. (a) Magnitude. (b) Phase.



(a) (b)

Figure 18. Comparison of $\delta S_{31}(\delta S_{21}^T)$. (a) Magnitude. (b) Phase.

4. CONCLUSION

In this paper, the analytical expressions for the sensitivity coefficients of the S-parameters are developed for the GSOLT calibration of the n-port VNA. Using the generalized flow graph of the 3n-term error model can conveniently deduce the S-parameters of the n-port DUT. To investigate the influence of SOLT standards on the S_{ij}, the following steps are involved: Firstly, the S-parameter deviations of the n-port DUT with respect to the error terms are solved during the error correction procedure. Then, expressions representing the deviations of the error terms in regard to the nonideal SOLT calibration standards are determined in the process of error calibration. Finally, the sensitivity coefficients, with respect to the S-parameters of SOLT standards as input quantities and the S-parameters of the n-port DUT as output quantities, are derived and can be used for establishing the type-B uncertainty budget for S-parameter measurements.

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