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LOCALIZED MONOCHROMATIC AND PULSED WAVES IN HYPERBOLIC METAMATERIALS

Ioannis M. Besieris[1, *] **and Amr M. Shaarawi**[2]

[1]The Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24060, USA

[2]Department of Physics, The American University of Cairo, P. O. Box 74, New Cairo 11835, Egypt

Abstract—It is established in this article that a special class of metamaterials known as hyperbolic media allow the propagation of large classes of novel monochromatic and pulsed localized waves. Illustrative explicit solutions are given of "accelerating" oblique Airy beams, as well as nondiffracting and nondispersive spatiotemporally localized "all-speed" X-shaped and MacKinnon-type waves.

1. INTRODUCTION

Consider the time-harmonic Maxwell equations in a source-free region of a canonical uniaxially anisotropic nonmagnetic medium, viz.,

$$\begin{aligned}
\nabla \times \tilde{\vec{E}}\left(\vec{r},\omega\right) &= -i\omega\mu_0\tilde{\vec{H}}\left(\vec{r},\omega\right), \\
\nabla \times \tilde{\vec{H}}\left(\vec{r},\omega\right) &= i\omega\varepsilon_0\bar{\varepsilon}_r\left(\omega\right)\cdot\tilde{\vec{E}}\left(\vec{r},\omega\right), \\
\nabla \cdot \left[\bar{\varepsilon}_r\left(\omega\right)\cdot\tilde{\vec{E}}\left(\vec{r},\omega\right)\right] &= 0, \\
\nabla \cdot \left[\tilde{\vec{H}}\left(\vec{r},\omega\right)\right] &= 0.
\end{aligned} \tag{1}$$

Here, ε_0 and μ_0 denote the permittivity and permeability of vacuum, respectively, and $\bar{\varepsilon}_r(\omega) = \vec{a}_x\vec{a}_x\varepsilon_{rx}(\omega) + \vec{a}_y\vec{a}_y\varepsilon_{ry}(\omega) + \vec{a}_z\vec{a}_z\varepsilon_{rz}(\omega)$ is the relative permittivity dyadic, which is characterized by the constraint that the "transverse" permittivity elements $\varepsilon_{rx}(\omega)$ and $\varepsilon_{ry}(\omega)$ are equal

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* Corresponding author: Ioannis M. Besieris (besieris@vt.edu).
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but distinct from the element $\varepsilon_{rz}(\omega)$ describing the properties of the medium along the axis of symmetry.

In order to examine transverse-magnetic (TM) waves in the uniaxial medium, an electric Hertz vector potential is introduced as $\tilde{\vec{H}}(\vec{r},\omega) = i\omega\varepsilon_0\nabla\times\tilde{\vec{\Pi}}_e(\vec{r},\omega)$, with the additional restriction $\tilde{\vec{\Pi}}_e(\vec{r},\omega) = \tilde{\Pi}_e(\vec{r},\omega)\vec{a}_z$. From the first Maxwell curl equation, it follows, then, that

$$\nabla\times\tilde{\vec{E}}(\vec{r},\omega) = -i\omega\mu_0\left[i\omega\varepsilon_0\nabla\times\tilde{\vec{\Pi}}_e(\vec{r},\omega)\right] = k^2\nabla\times\tilde{\vec{\Pi}}_e(\vec{r},\omega), \quad (2)$$

where $k = \omega/c$, c being the speed of light in vacuum. Eq. (2) suggests the relationship $\tilde{\vec{E}}(\vec{r},\omega) = k^2\tilde{\vec{\Pi}}_e(\vec{r},\omega) + \nabla\Phi(\vec{r},\omega)$, where the scalar potential function $\Phi(\vec{r},\omega)$ is to be determined. From the second Maxwell curl equation it follows that

$$\nabla\times\tilde{\vec{H}}(\vec{r},\omega) = i\omega\varepsilon_0\nabla\times\nabla\times\tilde{\vec{\Pi}}_e(\vec{r},\omega) = i\omega\varepsilon_0\bar{\varepsilon}_r(\omega)\cdot\tilde{\vec{E}}(\vec{r},\omega). \quad (3)$$

It should be noted, however, that the last term on the right hand side can be rewritten as

$$\begin{aligned}
i\omega\varepsilon_0\bar{\varepsilon}_r\cdot\tilde{\vec{E}} &= i\omega\varepsilon_0\left(\varepsilon_{rx}\tilde{E}_x\vec{a}_x + \varepsilon_{rx}\tilde{E}_y\vec{a}_y + \varepsilon_{rz}\tilde{E}_z\vec{a}_z\right)\\
&= i\omega\varepsilon_0\left[\varepsilon_{rx}\tilde{\vec{E}} + (\varepsilon_{rz}-\varepsilon_{rx})\tilde{E}_z\vec{a}_z\right]\\
&= i\omega\varepsilon_0\left[\varepsilon_{rx}\left(k^2\tilde{\vec{\Pi}}_e+\nabla\Phi\right)+(\varepsilon_{rz}-\varepsilon_{rx})\left(k^2\tilde{\Pi}_e+\frac{\partial}{\partial z}\Phi\right)\vec{a}_z\right]. \quad (4)
\end{aligned}$$

Upon introduction of this form into Eq. (3), one obtains

$$\begin{aligned}
\nabla\times\nabla\times\tilde{\vec{\Pi}}_e &= \nabla\nabla\cdot\tilde{\vec{\Pi}}_e - \nabla^2\tilde{\vec{\Pi}}_e\\
&= \varepsilon_{rx}\left(k^2\tilde{\vec{\Pi}}_e+\nabla\Phi\right)+(\varepsilon_{rz}-\varepsilon_{rx})\left(k^2\tilde{\Pi}_e+\frac{\partial}{\partial z}\Phi\right)\vec{a}_z. \quad (5)
\end{aligned}$$

The divergence of the vector Hertz potential $\tilde{\vec{\Pi}}_e$ as well as the scalar potential Φ have not been specified up to this point. Benefiting from this freedom, the following relationship is introduced: $\nabla\cdot\tilde{\vec{\Pi}}_e = \varepsilon_{rx}\Phi$, or, equivalently, $\Phi = \varepsilon_{rx}^{-1}(\partial\tilde{\Pi}_e/\partial z)$. Based on this constraint, the monochromatic electric and magnetic fields corresponding to the extraordinary mode are given by

$$\begin{aligned}
\tilde{\vec{E}}(\vec{r},\omega) &= k^2\tilde{\vec{\Pi}}_e + \frac{1}{\varepsilon_{xr}(\omega)}\nabla\nabla\cdot\tilde{\vec{\Pi}}_e(\vec{r},\omega),\\
\tilde{\vec{H}}(\vec{r},\omega) &= i\omega\varepsilon_0\nabla\times\tilde{\vec{\Pi}}_e(\vec{r},\omega),
\end{aligned} \quad (6)$$

and the scalar Hertz potential component $\tilde{\Pi}_e(\vec{r},\,\omega)$ satisfies the equation

$$\left[\nabla_t^2 + \frac{\varepsilon_{zr}(\omega)}{\varepsilon_{xr}(\omega)}\frac{\partial^2}{\partial z^2} + \varepsilon_{zr}(\omega)\,k^2\right]\tilde{\Pi}_e(\vec{r},\omega) = 0, \tag{7}$$

where ∇_t^2 denotes the transverse (with respect to the symmetry axis z) Laplacian operator.

2. HYPERBOLIC MEDIUM

The dispersion relation corresponding to the expression governing $\tilde{\Pi}_e(\vec{r},\,\omega)$ in Eq. (7) is given by

$$\left(-k_x^2 - k_y^2 - \frac{\varepsilon_{zr}}{\varepsilon_{xr}}k_z^2 + \varepsilon_{zr}k^2\right) = 0. \tag{8}$$

The iso-frequency topology in wavenumber space embodied in this relation depends on the properties of the transverse and longitudinal relative permittivity elements. If both permittivity elements are positive, the wavenumber surface associated with the dispersion relation of the extraordinary mode is an ellipsoid. However, it may turn out that within a certain frequency band one diagonal permittivity element is positive and the other negative. Then, the dispersion relation is described by a hyperboloid (see Fig. 1). Under these conditions, the material is referred to as a *hyperbolic medium*. The expression in Eq. (7) is a de Broglie-like equation for $\varepsilon_{xr}(\omega) < 0$ and $\varepsilon_{zr}(\omega) > 0$, and a Klein-Gordon (Fock)-like equation for $\varepsilon_{xr}(\omega) > 0$ and $\varepsilon_{zr}(\omega) < 0$. The coordinate z is *timelike* in both cases.

The physical importance of hyperbolic wave dispersion was first recognized in the 50's in connection with electromagnetic

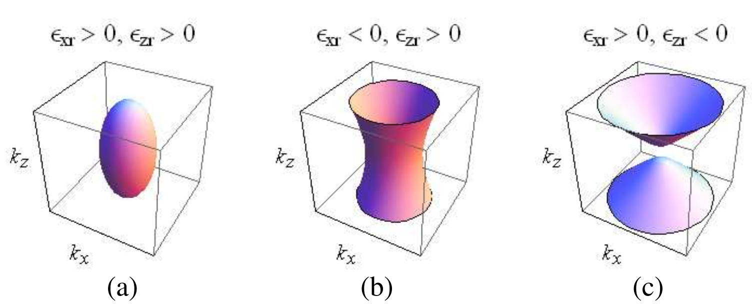


Figure 1. Topology in wavenumber space for different properties of the transverse and longitudinal relative permittivities.

wave propagation in the ionosphere and in stratified artificial materials. Recently, however, hyperbolic anisotropic metamaterials characterized by dielectric permittivities of different signs in orthogonal directions have attracted significant attention due to their particular physical properties, e.g., negative refraction, and potential physical applications, such as subwavelength imaging (enhanced hyperlensing). Hyperbolic media in the visible and near-infrared frequency regime can physically be realized with metal (plasmonic)-dielectric nanolayers or nanowire composites [1–3]. Extensive studies have been carried out of monochromatic plane wave and beam propagation in hyperbolic media [4, 5]; also, of the reflection and refraction of plane waves incident on the interface of an isotropic and a hyperbolic metamaterial [6]. It has been established that a uniaxial anisotropic material with $\varepsilon_{xr}(\omega) < 0$ and $\varepsilon_{zr}(\omega) > 0$ exhibits negative refraction behavior for TM polarization for all incident angles, but the TE polarization behaves altogether differently [7, 8]. Conditions have been derived for a uniaxial anisotropic plasma metamaterial to support a Faraday effect [9]. Monochromatic radiation in an unbounded hyperbolic material has been studied analytically [10, 11].

Since z is a *timelike* coordinate in Eq. (7) for both cases of hyperbolic behavior, one has a $(2+1)$-dimensional Lorentz symmetry with variable metric [12]. One can use well-known solutions to the quantum mechanical Klein-Gordon and de Broglie equations in order to establish monochromatic solutions describing wave propagation in a hyperbolic medium governed by Eq. (7). Our specific aim in this exposition is to explore the feasibility of novel monochromatic and pulsed localized waves in hyperbolic media. One class of the former is studied in the next section. Several classes of the latter are examined in Section 4.

3. MONOCHROMATIC LOCALIZED WAVES

We consider a paraxial approximation of Eq. (7) along the y direction; specifically [13, 14],

$$\begin{aligned}
&\tilde{\Pi}_e\left(\vec{r},\omega\right) \approx \psi\left(\vec{r},\omega\right)\exp\left[ik\sqrt{\varepsilon_{zr}}y\right];\\
&2ik\sqrt{\varepsilon_{zr}}\frac{\partial}{\partial y}\psi\left(\vec{r},\omega\right)+\frac{\partial^2}{\partial x^2}\psi\left(\vec{r},\omega\right)+\frac{\varepsilon_{zr}}{\varepsilon_{xr}}\frac{\partial^2}{\partial z^2}\psi\left(\vec{r},\omega\right)=0.
\end{aligned} \tag{9}$$

Under the assumptions that $\varepsilon_{xr}(\omega) < 0$ and $\varepsilon_{zr}(\omega) > 0$, the equation for the slowly varying envelope function $\psi(\vec{r},\,\omega)$ can be brought into the following nondimensional form:

$$i\frac{\partial}{\partial Y}\psi\left(\vec{R},\omega\right)+\frac{\partial^2}{\partial X^2}\psi\left(\vec{R},\omega\right)-\frac{\partial^2}{\partial Z^2}\psi\left(\vec{R},\omega\right)=0;\ \vec{R}=\{X,Y,Z\};$$
$$X=\frac{x}{x_0},\ \ Z=\frac{\bar{z}}{x_0},\ \ Y=\frac{\bar{y}}{2kx_0^2};\ \ \bar{y}=\frac{y}{\sqrt{\varepsilon_{zr}}},\ \ \bar{z}=z\sqrt{\frac{|\varepsilon_{xr}|}{\varepsilon_{zr}}}. \tag{10}$$

Thus, a parabolic approximation of the de Broglie-like equation along the y direction yields a *hyperbolic Schrödinger-like equation* analogous to that arising in the study of normal temporal dispersion or bidispersion. Using the hyperbolic rotation $\varsigma = X\cosh\phi + Z\sinh\phi$, $\xi = X\sinh\phi + Z\cosh\phi$, a broad class of skewed, nonspreading, "accelerating" Airy solutions can be obtained [15]. Specifically,

$$\begin{aligned}\psi(X,Y,Z) &= Ai\left[\frac{\zeta(X,Z)}{\sqrt{2}}-\frac{Y^2}{4}\right]Ai\left[\frac{\xi(X,Z)}{\sqrt{2}}-\frac{Y^2}{4}\right]\\ &\quad\times\exp\left(i\frac{Y}{2\sqrt{2}}\left[\zeta(X,Z)-\xi(X,Z)\right]\right).\end{aligned} \tag{11}$$

The new coordinates are no longer mutually orthogonal, but instead intersect at *the obliquity angle* θ defined by the relation $\phi = -(1/2)\tanh^{-1}(\cos\theta)$. Fig. 2 shows $|\psi(X,Y,0)|$ for (a) $\theta = 90°$, (b) $\theta = 45°$ and (c) $\theta = 135°$.

Finite-energy, slowly diffracting oblique Airy solutions to Eq. (10) can be obtained by analogy to those in the case of bidispersive media given in [15]. Fig. 3 shows the propagation behavior of a finite-energy Airy beam when $\theta = 45°$ on the planes $Z = 0$ and $Z = 3$.

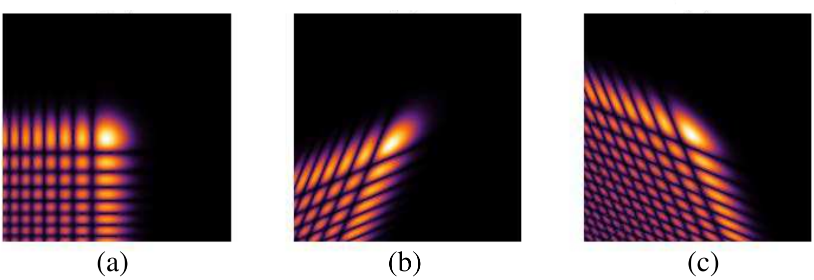
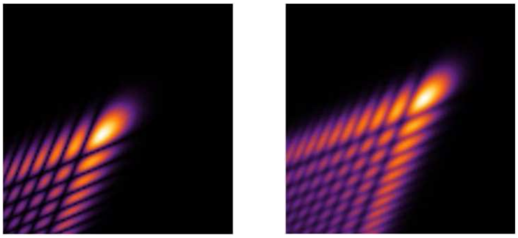


Figure 2. Airy beam intensity profiles for (a) $\theta = 90°$, (b) $\theta = 45°$ and (c) $\theta = 135°$ on the plane $Z = 0$.



(a) (b)

Figure 3. $|\psi(X, Y, Z)|$ for $\theta = 45°$ on the planes (a) $Z = 0$ and (b) $Z = 3$.

4. PULSED LOCALIZED WAVES

The study of propagation of localized pulsed signals in hyperbolic media is complicated, in general, due to the frequency dependence of the permittivity matrix elements. Consider, however, a *canonical* situation whereby the permittivity matrix elements are constant within a certain frequency regime. Then, approximately, one has

$$\left(\nabla_t^2 - \frac{|\varepsilon_{zr}|}{\varepsilon_{xr}}\frac{\partial^2}{\partial z^2} + |\varepsilon_{zr}|\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Pi_e(\vec{r}, t) = 0 \tag{12}$$

in the time domain for $\varepsilon_{xr} > 0$ and $\varepsilon_{zr} < 0$. A large class of spatiotemporally nonsingular localized luminal, subluminal and superluminal pulsed solutions to this equation can be derived. These solutions differ substantially from analogous ones in isotropic free space. In the case of propagation along the z direction, the roles of subluminality and superluminality are interchanged by comparison to the propagation of the same structures in free space. A subluminal wave packet is X shaped whereas a superluminal one has the form of a sinc function.

More interesting forms of spatiotemporally localized waves in hyperbolic media arise from specific choices of frequency dependence of the permittivity matrix elements. Consider the case where [16]

$$\varepsilon_{xr} = -\frac{\alpha + 1}{\alpha - 1}, \quad \varepsilon_{zr} = \alpha\left(1 - \frac{\omega_p^2}{\omega^2}\right); \quad \alpha > 1, \quad \omega > \omega_p. \tag{13}$$

These two expressions for the relative permittivities are introduced next into the dispersion relation given in Eq. (8) and, furthermore, the constraint $k_z = \omega/v$, where v is a free parameter with units of speed, is used. As a consequence, an exact solution for the space-time

Hertz potential wave function $\Pi_e(\vec{r}, t)$ can be obtained by means of the Fourier-Hankel spectral superposition

$$\Pi_e(\rho, \phi, \tau) = e^{im\phi} \int_{\omega_p}^{\infty} d\omega\, e^{i\omega\tau} J_m\left(\rho q\sqrt{\omega^2 - \omega_p^2}\right) \tilde{F}(\omega); \qquad \tau \equiv t - \frac{z}{v},\ q \equiv \sqrt{\frac{\alpha}{v^2}\left(\frac{v^2}{c^2} + \frac{\alpha - 1}{\alpha + 1}\right)} \tag{14}$$

in cylindrical coordinates. The simplest exact solution can be obtained for $m = 0$ and by choosing the temporal spectrum $\tilde{F}(\omega) = \exp(-a_1\omega)$, where a_1 is a positive parameter. It is given explicitly as

$$\Pi_e(\rho, \tau) = \frac{e^{-\omega_p\sqrt{(\rho q)^2 + (a_1 - i\tau)^2}}}{\sqrt{(\rho q)^2 + (a_1 - i\tau)^2}}. \tag{15}$$

It represents a nonsingular wave function propagating in the z direction, with a constant speed $0 < v < \infty$, without sustaining any spreading due to diffraction or dispersion. The modulus of this expression versus τ and ρ is shown in Fig. 4. The form of the solution

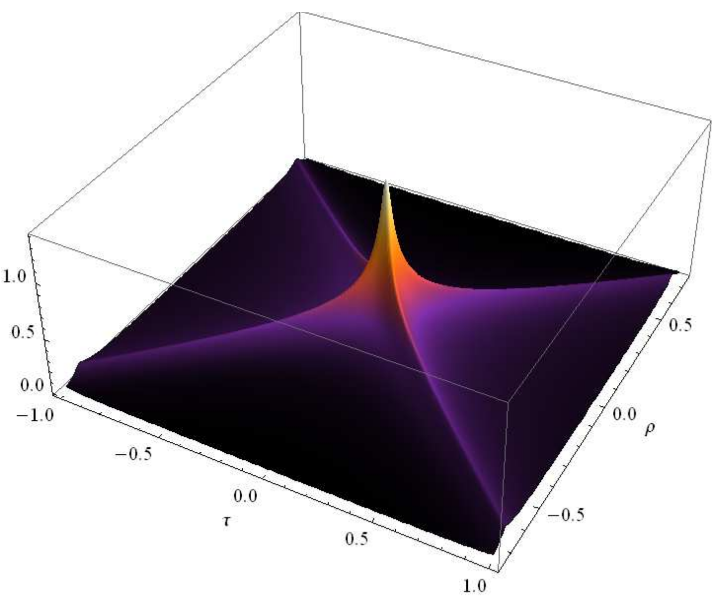


Figure 4. $|\Pi_e(\rho, \tau)|$ versus τ and ρ for the parameter values $a_1 = 10^{-8}$ s, $v = 2c$, $\omega_p = 10^7$ rad/s and $\alpha = 10^6$.

in Eq. (15) as well as its shape closely resemble those of the X wave solution to the Klein-Gordon equation, and for $\omega_p = 0$ the X wave solution of the scalar wave equation in free space. The latter two solutions are restricted to superluminal speeds $v > c$ [17–19], whereas the one given in Eq. (15) for a hyperbolic medium is an "all-speed" X-shaped solution.

The reason for this significant difference is that the Hertz potential $\Pi_e(\vec{r}, t)$ corresponding to the two permittivity matrix elements in Eq. (13) is an exact solution to the equation

$$\left(-\frac{\partial^2}{\partial t^2}\nabla_t^2 + \alpha\frac{\alpha-1}{\alpha+1}\frac{\partial^4}{\partial t^2 \partial z^2} + \alpha\frac{\alpha-1}{\alpha+1}\omega_p^2\frac{\partial^2}{\partial z^2} + \frac{\alpha}{c^2}\frac{\partial^4}{\partial t^4} + \frac{\alpha}{c^2}\omega_p^2\frac{\partial^2}{\partial t^2}\right)\Pi_e(\vec{r}, t) = 0. \tag{16}$$

Consider, next, the situation where

$$\varepsilon_{xr} = -\frac{\alpha-1}{\alpha+1}, \quad \varepsilon_{zr} = -\alpha\left(1 - \frac{\omega_p^2}{\omega^2}\right); \quad \alpha > 1, \quad \omega < \omega_p. \tag{17}$$

Proceeding as in the previous case and using the constraint $k_z = \omega/v$ leads to the Fourier-Hankel synthesis

$$\begin{gathered}\Pi_e(\rho, \phi, \tau) = e^{im\phi}\int_0^{\omega_p} d\omega e^{i\omega\tau} J_m\left(\rho q\sqrt{\omega_p^2 - \omega^2}\right)\tilde{F}(\omega); \\ \tau \equiv t - \frac{z}{v}, \quad q = \sqrt{\frac{\alpha}{v^2}\left(\frac{v^2}{c^2} + \frac{\alpha-1}{\alpha+1}\right)}.\end{gathered} \tag{18}$$

The simplest explicit exact solution is given by

$$\Pi_e(\rho, \tau) = \frac{\sin\left(\omega_p\sqrt{(\rho q)^2 + (a_1 - i\tau)^2}\right)}{\sqrt{(\rho q)^2 + (a_1 - i\tau)^2}}. \tag{19}$$

Again, this represents a nonsingular wave function propagating in the z direction, with a constant speed $0 < v < \infty$, without sustaining any spreading due to diffraction or dispersion. The modulus of this expression versus τ and ρ is shown in Fig. 5. The form of the solution in Eq. (19) as well as its shape closely resemble those of the MacKinnon wave solutions to the Klein-Gordon and the scalar wave equations. The latter two solutions are restricted to subluminal speeds $v < c$ and, furthermore, are modulated by a plane wave propagating in the z direction with the superluminal speed c^2/v [17–19]. In contradistinction, the expression given in Eq. (19) for a hyperbolic medium is an "all-speed" envelope MacKinnon-type solution.

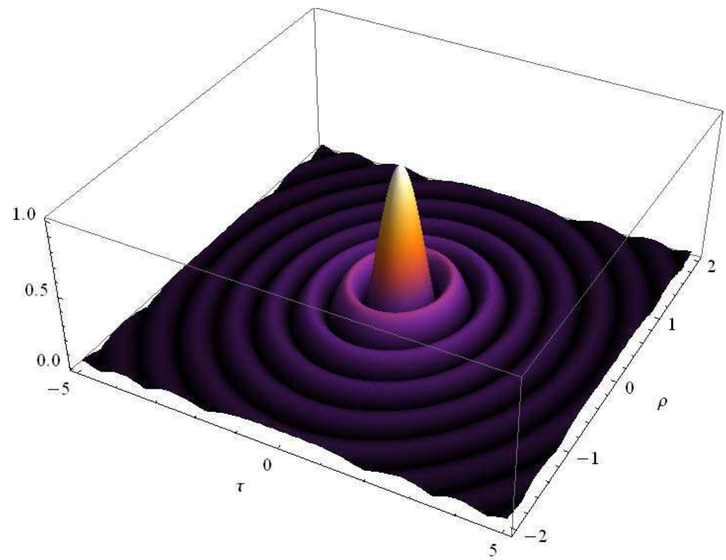


Figure 5. $|\Pi_e(\rho,\,\tau)|$ versus τ and ρ for the parameter values $v = 2c$, $\omega_p = 10^7\,\text{rad/s}$ and $\alpha = 4 \times 10^8$.

5. CONCLUDING REMARKS

The feasibility of monochromatic and pulsed localized waves in hyperbolic media has been explored in this exposition. A novel class of monochromatic localized "accelerating" oblique Airy beams has been derived in Section 3. In Section 4, it has be shown that it is possible to derive purely superluminal and purely subluminal spatiotemporally localized waves in hyperbolic media. The "all-speed" solutions derived in this article are meant to emphasize the large disparities that may exist between pulsed localized waves in hyperbolic media and free space. The X-shaped solution in Eq. (15) and the MacKinnon-like solution in Eq. (19) are nondiffracting and nondispersive due to the infinite energy they contain. The corresponding transverse magnetic electromagnetic fields can be derived from a temporal Fourier inversion of the time-harmonic fields in Eq. (6), viz.,

$$\begin{aligned} \vec{E}\left(\vec{r},t\right) &= -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{\Pi}_e\left(\vec{r},t\right) + \frac{1}{\varepsilon_{xr}}\nabla\nabla\cdot\vec{\Pi}_e\left(\vec{r},t\right); \\ \vec{H}\left(\vec{r},t\right) &= \varepsilon_0\frac{\partial}{\partial t}\nabla\times\vec{\Pi}_e\left(\vec{r},t\right). \end{aligned} \tag{20}$$

Finite energy solutions can be achieved by techniques analogous to those used to launch causally pulsed localized waves in free space, e.g., from finite apertures constructed on the basis of the Huygens principle [20, 21].

The discussion in this article has been restricted to idealized lossless hyperbolic metamaterials. However, dissipation is present in such media and must be accounted for. This task is relatively easy in the case of the monochromatic localized beams discussed in Section 3. The incorporation of dissipation in the study of pulsed localized waves in hyperbolic media is a much more difficult problem. One possible approach is to begin with the dispersion relation in Eq. (8), incorporate physically meaningful models of the complex relative permittivities $\varepsilon_{rx}(\omega)$ and $\varepsilon_{rz}(\omega)$, and finally resort to a slowly varying envelope approximation.

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