Analysis of the Generalized Case of Scattering from a Ferromagnetic Microwire Grid

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Abstract—This paper investigates the generalized case of scattering from a planar grid, containing infinite numbers of axially magnetized ferromagnetic microwires placed parallel to each other in free space. A semi-analytical solution is obtained by calculating the local field at the surface of the reference microwire which is the sum of the scattered field from the other microwires as well as the incident field. Graf's theorem is used to transform the scattered field from one coordinate system to the other. Scattering field coefficients for the reference microwire are obtained by matching the tangential field components at the surface of the reference microwire. Simulated results are expressed in terms of the Reflection, Transmission and Absorption Coefficients for the TM_z and TE_z polarizations. For validation, results of the proposed analysis specialized to the case of normal incidence with TM_z polarization are compared with the results available in the literature.

1. INTRODUCTION

Ferrite materials have been in use for a long time in nonreciprocal microwave passive devices such as isolators and circulators [13]. There has of late been a renewed interest in Ferrites among the microwave research community in view of their potential application in wire-based metamaterials (MTMs) [1–3]. The property of ferrite that has been found to be useful for designing wire-based double negative (DNG) metamaterials using only a single type of element is the occurrence of ferromagnetic resonance (FMR) inside the ferrite medium due to which the real part of permeability of the ferrite medium becomes negative beyond FMR frequency [4–7]. Ferromagnetic resonance occurs inside the ferrite medium when a uniform plane wave propagates inside the ferrite medium with a component of **H** vector lying in a plane orthogonal to the direction of applied internal magnetization **H₀**. As a result, the permeability of ferrite medium becomes a tensor and an extraordinary wave propagation takes place inside the ferrite medium which leads to ferromagnetic resonance (FMR) at FMR frequency. Consequently, the real part of effective permeability $\text{Re}[\mu_e]$ becomes negative beyond FMR frequency (see Fig. 1) [4–7].

Electromagnetic scattering from a ferromagnetic microwire for the normal incidence case as well as for the generalized case has been derived by many authors [4–6]. In order to design a metamaterial with ferromagnetic microwires, one needs to analyze electromagnetic scattering from a well arranged structure of microwires or nanowires (e.g., wire grid). The problem of two dimensional scattering from an array of ferrite,conducting and dielectric cylinders has been discussed by many authors in [8–12]. Liberal et al., in [8] discusses the 2-dimensional scattering problem for a ferrite planar grid containing an infinite number of microwires by using local field method and impedance loaded surface approach. The solution obtained in [8] is restricted to the far field analysis for a case of normal incidence and transverse magnetic (TM_z) polarization only. In [9], Polewski and Mazur have discussed the 2-dimensional scattering problem by using iterative scattering procedure for finite number of cylinders for both open
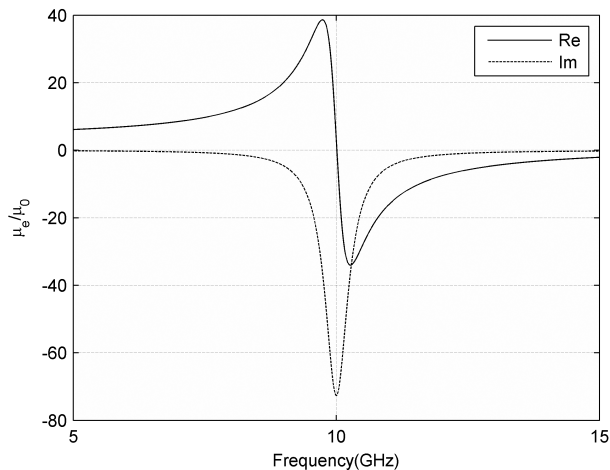
Figure 1. Real and imaginary parts of the effective permeability for considered ferromagnetic microwire under consideration (Liberal et al.) [8].

and close problem. In actual practice, the direction and the polarization of the incident wave can be arbitrary. In order to gain a better insight into the problem, it is required to investigate the generalized case of scattering for an arbitrary polarization. In this paper, a generalized case of scattering from a planar ferromagnetic microwire grid is investigated by satisfying the boundary condition at the surface of the reference microwire which is assumed to be placed along the z-axis. In the proposed analysis, the sample results are obtained for a microwire grid similar to that considered by Liberal et al. in [8] and field coefficients for the reference wire are obtained by satisfying the continuity of the tangential field components at the surface of the microwire. The total incident field (Local field) at the surface of the reference microwire is the sum of the scattered field components due to the other microwires and the incident field at the surface of the microwire itself. By using Graf's theorm, we can easily transform the scattered field components from the other microwires in terms of the coordinates of the reference microwire [15–17]. As the grid contains an infinite number of microwires, the summation series will contain an infinite number of terms in the form of Hankel function of second kind and nth order. The summation of the series of the Hankel function can be obtained by using Poisson's summation rule with the singularity cancellation [12, 14, 18]. By satisfying the tangential boundary condition at the surface of the reference microwire, field coefficients for the reference microwire are obtained. The field coefficients obtained in this manner will be the same for each microwire of the grid, irrespective of its position because each wire is characterized by the same parameters. Once the field coefficients are obtained, the scattered field in the far zone is calculated in terms of the zero-order propagating floquet mode and finally, the Reflection, Transmission and Absorption Coefficients are calculated for TM_z and TE_z polarizations.

2. FORMULATION OF THE SCATTERED FIELD

The tensor permeability for axially (z-axis) biased ferrite microwire can be represented in matrix form as [4–6, 13]

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}, \tag{1}$$

where

$$\mu = \mu_0 \left(1 + \chi_p - j\chi_s \right), \tag{2}$$
$$\kappa = \mu_0 \left(K_p - jK_s \right), \tag{3}$$

$$\chi_p = \frac{\omega_0\omega_m\left(\omega_0^2 - \omega^2\right) + \omega_0\omega_m\omega^2\delta^2}{\left[\omega_0^2 - \omega^2\left(1 + \delta^2\right)\right]^2 + 4\omega_0^2\omega^2\delta^2}, \tag{4}$$

$$\chi_s = \frac{\omega_0\omega_m\delta\left[\omega_0^2 + \omega^2\left(1 + \delta^2\right)\right]}{\left[\omega_0^2 - \omega^2\left(1 + \delta^2\right)\right]^2 + 4\omega_0^2\omega^2\delta^2}, \tag{5}$$

$$K_p = \frac{\omega_0\omega_m\delta\left[\omega_0^2 - \omega^2\left(1 + \delta^2\right)\right]}{\left[\omega_0^2 - \omega^2\left(1 + \delta^2\right)\right]^2 + 4\omega_0^2\omega^2\delta^2}, \tag{6}$$

$$K_s = \frac{2\omega_0\omega_m\omega^2\delta}{\left[\omega_0^2 - \omega^2\left(1 + \delta^2\right)\right]^2 + 4\omega_0^2\omega^2\delta^2}, \tag{7}$$

where ω_0 is the Larmor resonant frequency, ω_m the resonant frequency at the saturation limit, δ the loss factor, and ω the operating frequency. The complex permittivity and effective permebility of the ferrite medium are respectively given by

$$\epsilon_c = \epsilon_0 - j\frac{\sigma}{\omega}, \tag{8}$$

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu}. \tag{9}$$

As proposed in [8], the microwires of infinite length, each with radius 'a' and having applied internal axial magnetization H_0, are placed parallel to each other in y-z plane with the uniform spacing d as shown in Fig. 2. The reference microwire is assumed to be placed along the z-axis and impinged by uniform plane wave with polarization angle α_0 and incident angle θ_0. The z-components of the incident and scattered fields in context to the reference microwire are given in cylindrical coordinates ρ, ϕ and z, respectively by

$$E_{z0}^{inc}(\rho,\phi,z) = E_0\sin\theta_0\cos\alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \tag{10}$$

$$H_{z0}^{inc}(\rho,\phi,z) = \frac{E_0}{\eta_0}\sin\theta_0\sin\alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \tag{11}$$

$$E_{z0}^{s}(\rho,\phi,z) = E_0\sin\theta_0 \sum_{n=-\infty}^{+\infty} C_n H_n^{(2)}(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \tag{12}$$

$$H_{z0}^{s}(\rho,\phi,z) = \frac{E_0}{\eta_0}\sin\theta_0 \sum_{n=-\infty}^{+\infty} D_n H_n^{(2)}(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}. \tag{13}$$

Here $\beta_{\rho_0} = \beta_0\sin\theta_0$, $\beta_z = \beta_0\cos\theta_0$, $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$, is the free space propagation constant, and $\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the intrinsic impedance of free space. The superscripts '*inc*' and '*s*' denote the incident and the scattered fields respectively. J_n is the nth order Bessel's function of the first kind and $H_n^{(2)}$ the nth order Hankel's function of the second kind.

The ϕ-components for the incident and scattered fields in context to the reference microwire may be easily deduced from Maxwell's equations to be

$$E_{\phi0}^{inc}(\rho,\phi,z) = -E_0\frac{n\cos\theta_0\cos\alpha_0}{\beta_0\rho\sin\theta_0} \sum_{n=-\infty}^{+\infty} j^n J_n(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi} + jE_0\sin\alpha \sum_{n=-\infty}^{+\infty} j^n J_n'(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \tag{14}$$

$$H_{\phi0}^{inc}(\rho,\phi,z) = -\frac{jE_0}{\eta_0}\cos\alpha_0 \sum_{n=-\infty}^{+\infty} j^n J_n'(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi} - \frac{E_0}{\eta_0}\frac{n\cos\theta_0\sin\alpha_0}{\beta_0\rho\sin\theta_0} \sum_{n=-\infty}^{+\infty} j^n J_n(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \tag{15}$$

$$E_{\phi0}^{s}(\rho,\phi,z) = -E_0\frac{n\cos\theta_0}{\beta_0\rho\sin\theta_0} \sum_{n=-\infty}^{+\infty} C_n H_n^{(2)}(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi} + jE_0 \sum_{n=-\infty}^{+\infty} D_n H_n^{(2)'}(\beta_{\rho_0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \tag{16}$$
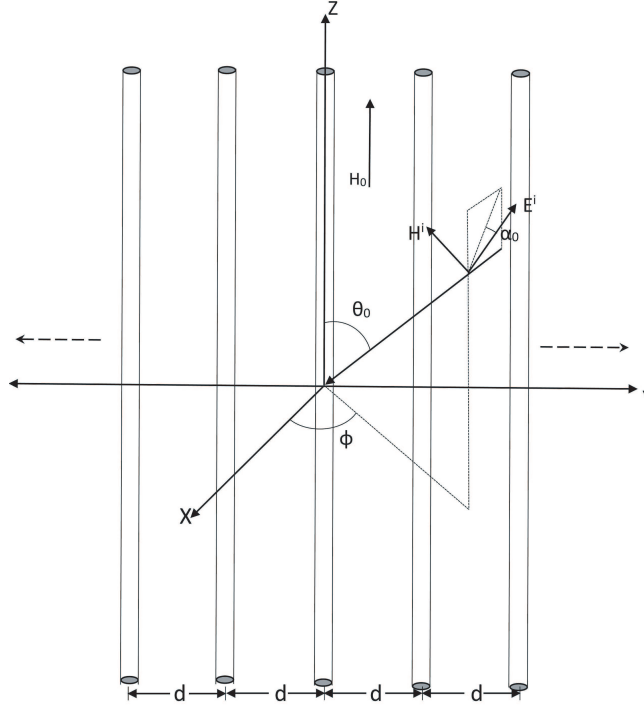
Figure 2. Geometry of the scattering problem.

$$H_{\phi 0}^s(\rho, \phi, z) = -\frac{jE_0}{\eta_0}\sin\alpha_0\sum_{n=-\infty}^{+\infty}C_n H_n^{(2)'}(\beta_{\rho 0}\rho)e^{-\beta_z z}e^{-jn\phi} - \frac{E_0}{\eta_0}\frac{n\cos\theta_0}{\beta_0\rho\sin\theta_0}\sum_{n=-\infty}^{+\infty}D_n H_n^{(2)}(\beta_{\rho 0}\rho)e^{-j\beta_z z}e^{-jn\phi}, \quad (17)$$

where $'$ denotes the first derivative with respect to the argument.

The z-components of the inside field for the reference microwire in cylindrical coordinates ρ, ϕ and z are given by [6, 7]

$$E_{z0}^d(\rho, \phi, z) = E_0\sum_{n=-\infty}^{+\infty}\left[A_n J_n(\gamma_{\rho 1}\rho) + B_n J_n(\gamma_{\rho 2}\rho)\right]e^{-j\beta_z z}e^{-jn\phi}, \quad (18)$$

$$H_{z0}^d(\rho, \phi, z) = E_0\sum_{n=-\infty}^{+\infty}\left[\eta_1 A_n J_n(\gamma_{\rho 1}\rho) + \eta_2 B_n J_n(\gamma_{\rho 2}\rho)\right]e^{-j\beta_z z}e^{-jn\phi}, \quad (19)$$

$$E_{\phi 0}^d(\rho, \phi, z) = E_0\sum_{n=-\infty}^{+\infty}\left[A_n X_{1n}(\rho) + B_n X_{2n}(\rho)\right]e^{-j\beta_z z}e^{-jn\phi}, \quad (20)$$

$$H_{\phi 0}^d(\rho, \phi, z) = E_0\sum_{n=-\infty}^{+\infty}\left[A_n\Lambda_{1n}(\rho) + B_n\Lambda_{2n}(\rho)\right]e^{-j\beta_z z}e^{-jn\phi}, \quad (21)$$

where

$$X_{in}(\rho) = \frac{1}{D}\left(d\eta_i\gamma_{\rho_i} - b\gamma_{\rho_i}\right)J_n'(\gamma_{\rho_i}\rho) + \frac{1}{D}\frac{jn(e\eta_i - a)}{\rho}J_n(\gamma_{\rho_i}\rho), \quad (22)$$

$$\Lambda_{in}(\rho) = \frac{1}{D}\left(a\gamma_{\rho_i}\frac{\omega\epsilon_c}{\beta_z} - b\gamma_{\rho_i}\eta_i\right)J_n'(\gamma_{\rho_i}\rho) - j\frac{n}{D\rho}\left(a\eta_i + \frac{b\omega\epsilon_c}{\beta_z}\right)J_n(\gamma_{\rho_i}\rho), \quad (23)$$

$$a = j\beta_z\beta_\rho^2, \quad (24)$$

$$b = \omega^2 \kappa \beta_z \epsilon_c, \tag{25}$$

$$c_1 = \left(\beta_\rho^2 - \frac{\omega^2 \kappa^2 \epsilon_c}{\mu} \right), \tag{26}$$

$$d = -j\omega\mu c_1, \tag{27}$$

$$e = \omega\kappa\beta_z^2, \tag{28}$$

$$D = \left(\omega^2 \kappa \epsilon_c \right)^2 - \beta_\rho^4, \tag{29}$$

$$\eta_i = \frac{-jg_1}{\left(\gamma_{\rho_i}^2 - f_1 \right)}, \tag{30}$$

$$g_1 = \frac{\omega\kappa\beta_z\epsilon_c}{\mu}, \tag{31}$$

$$f_1 = \frac{\mu_0 \beta_\rho^2}{\mu}, \tag{32}$$

$$\gamma_{\rho_i} = \sqrt{\frac{1}{2} \left((f_1 + c_1) \pm \sqrt{(f_1 - c_1)^2 + 4d_1 g_1} \right)}, \tag{33}$$

$$d_1 = \frac{\mu_0 \omega \kappa \beta_z}{\mu}, \tag{34}$$

$$\beta_\rho = \sqrt{(\omega^2 \mu \epsilon_c - \beta_z^2)}. \tag{35}$$

Here, i takes the suffix '1' or '2' according to the '+' or '−' sign taken inside the square root in (33), respectively. In order to calculate the contribution of the other microwires to the local field at the surface of the reference microwire, Graf's theorem is used to transform the scattered field components from one set of coordinates to another [15–17]. With the help of this theorem, the scattered field of each microwire placed in the vicinity of the reference microwire is transformed in terms of the coordinates of the reference microwire. For example, the scattered field from gth microwire in terms of the ith microwire can be represented as [11, 15]:

$$H_n^{(2)}(\beta_{\rho_0}\rho)e^{jn\phi_g} = \sum_{m=-\infty}^{+\infty} J_m \left(\beta_{\rho_0}\rho \right) H_{m-n}^{(2)}(\beta_{\rho_0} d_{ig}) e^{jm\phi_i} e^{j(m-n)\phi_{ig}}, \tag{36}$$

where in case of a planar grid, $\phi_{ig} = \pm\pi$ and $d_{ig} = ld$, where d is the uniform spacing among the microwires and l the index for the microwires which is an integer. For the reference microwire, $l = 0$. The continuity of tangential components of fields at the surface of the reference microwire placed along the z-axis $(\rho = a)$ translates to

$$E_{z_0}^{loc} + E_{z_0}^{s} = E_{z_0}^{d}, \tag{37}$$

$$H_{z_0}^{loc} + H_{z_0}^{s} = H_{z_0}^{d}, \tag{38}$$

$$E_{\phi_0}^{loc} + E_{\phi_0}^{s} = E_{\phi_0}^{d}, \tag{39}$$

$$H_{\phi_0}^{loc} + H_{\phi_0}^{s} = H_{\phi_0}^{d}. \tag{40}$$

where $E_{z_0}^{loc}, H_{z_0}^{loc}, E_{\phi_0}^{loc}$ and $E_{\phi_0}^{loc}$ are the local field components at the surface of the reference microwire. The local field components can be calculated by adding the incident field to the scattered field from the other microwires at the surface of the reference microwires. For example, The $E_{z_0}^{loc}$ components can be represented as

$$E_{z_0}^{loc} = E_{z_0}^{inc} + \sum_{l=-\infty}^{+\infty} E_{z_l}^{s}; \quad l \neq 0. \tag{41}$$

As the reference microwire divides the complete space into two semi infinite regions, we can change the

limits of the summation suitably from $-\infty \le l \le +\infty$ to $1 \le l \le +\infty$ as follows

$$E_{z_0}^{loc} = E_{z_0}^{inc} + 2\sum_{l=1}^{+\infty} E_{z_l}^s. \tag{42}$$

Substituting the values of the field components given by (10)–(21) in (37)–(40) and solving further leads to the following matrix equation. With the help of this matrix equation, the unknown field coefficients A_n, B_n, C_n and D_n can be obtained.

$$\begin{bmatrix} J_n(\gamma_{\rho 1}a) & J_n(\gamma_{\rho 2}a) & -\sin\theta_0 S_{l0} & 0 \\ \eta_0\eta_1 J_n(\gamma_{\rho 1}a) & \eta_0\eta_2 J_n(\gamma_{\rho 2}a) & 0 & -\sin\theta_0 S_{l0} \\ \beta_{\rho 0}aX_{1n}(a) & \beta_{\rho 0}aX_{2n}(a) & n\cos\theta_0 S_{l0} & -j\beta_{\rho 0}aS_{l0}' \\ \eta_0\beta_{\rho 0}a\Lambda_{1n}(a) & \eta_0\beta_{\rho 0}a\Lambda_{2n}(a) & j\beta_{\rho 0}aS_{l0}' & n\cos\theta_0 S_{l0} \end{bmatrix}$$

$$\times \begin{bmatrix} A_n \\ B_n \\ C_n \\ D_n \end{bmatrix} = \begin{bmatrix} j^n\sin\theta_0\cos\alpha J_n(\beta_{\rho 0}a) \\ j^n\sin\theta_0\sin\alpha J_n(\beta_{\rho 0}a) \\ -j^n n\cos\theta_0\cos\alpha J_n(\beta_{\rho 0}a) + j^{n+1}\beta_{\rho 0}a\sin\alpha J_n'(\beta_{\rho 0}a) \\ -j^{n+1}\beta_{\rho 0}a\cos\alpha J_n'(\beta_{\rho 0}a) - j^n n\cos\theta_0\sin\alpha J_n(\beta_{\rho 0}a) \end{bmatrix}, \tag{43}$$

where

$$S_{l0} = H_n^{(2)}(\beta_{\rho 0}\rho) + 2\sum_{l=1}^{+\infty} J_m(\beta_{\rho 0}\rho) H_{m-n}^{(2)}(\beta_{\rho 0}d_{ig}), \tag{44}$$

and

$$S_{l0}' = H_n'^{(2)}(\beta_{\rho 0}\rho) + 2\sum_{l=1}^{+\infty} J_m'(\beta_{\rho 0}\rho) H_{m-n}^{(2)}(\beta_{\rho 0}d_{ig}). \tag{45}$$

Once the field coefficients are obtained, $E_{z_0}^{loc}$ can be calculated with the help of (42). The result will appear in terms of the summation of the series of Hankel function which can be obtained by using Poission's summation rule with the singularity cancellation [12, 14, 18]. Further, if it is assumed that $a \ll d \ll \lambda$, the total scattered field of the grid in the far zone can be represented as a propagating zero order floquet mode [8]. Then the scattered field in the far zone is a plane wave given by

$$E_z^s(x) = \frac{2}{\beta_{\rho 0}d} e^{-j\beta_{\rho 0}|x|} \sum_{n=-\infty}^{+\infty} j^n a_n^s E_{z_0}^{loc}, \tag{46}$$

where, a_n^s is the scattering field coefficient for the single ferromagnetic microwire given in [6]. After substituting $E_{z_0}^{loc}$ from (42) in (46), the scattered field and hence the Power Reflection, Transmission and Absorption Coefficients for TM_z-polarization can be obtained by

$$R_{TM} = \left| \frac{E^s}{E^{inc}} \right|^2, \tag{47}$$

$$T_{TM} = \left| 1 + \frac{E^s}{E^{inc}} \right|^2, \tag{48}$$

$$A_{TM} = 1 - R_{TM} - T_{TM}. \tag{49}$$

Similarly, the Power Reflection, Transmission and Absorption Coefficients for TE_z-polarization can be obtained by proceeding with the $H_{z_0}^{loc}$ component.

3. NUMERICAL RESULTS

Since the radius-to-wavelength ratio at the maximum operating frequency (15 GHz) for the microwire under consideration is only 1.5×10^{-4}, the azimuthal dependence of scattered field may be neglected without any significant loss in accuracy. Thus only the term, $n = 0$, in the expansions for the

inside and the scattered field makes the significant contribution. The sample results are obtained for a planar grid containing 'Co' based ferrite microwires of the following specifications as considered in [8]: radius, $a = 1\,\mu\text{m}$, spacing, $d = 3\,\text{mm}$, conductivity, $\sigma = 6.7 \times 10^5\,\text{S/m}$, gyromagnetic ratio, $\gamma = 2 \times 10^{11}\,\text{T}^{-1}\text{s}^{-1}$, saturation magnetization, $\mu_0 M_s = 0.55\,\text{T}$, loss factor, $\alpha = 0.02$, internal magnetization, $H_0 = 113.45\,\text{kA/m}$ along the z-coordinate and an operating frequency band of 5–15 GHz is assumed. Simulation results are plotted for the Reflection, Transmission and Absorption Coefficient against the operating frequency and the incident angle θ_0 for two different polarization angles $\alpha_0 = 0°$ and $90°$ (i.e., TM_z and TE_z polarizations respectively).

3.1. TM_z Polarization ($\alpha_0 = 0°$)

Figure 3 shows the simulation results plotted for the magnitudes of the Reflection, Transmission and Absorption Coefficients for a polarization angle of $\alpha_0 = 0°$. In this case, the incident wave is TM_z polarized, and the **H** vector of the incident wave is in a plane normal to the axis of the wire (i.e., z-axis). It results in an extraordinary wave propagation inside the ferrite medium, and thus there is a Ferromagnetic resonance (FMR) inside the ferrite medium at the FMR frequency. The scattering behavior of the grid may conveniently be explained with the help of the scattering behavior of the
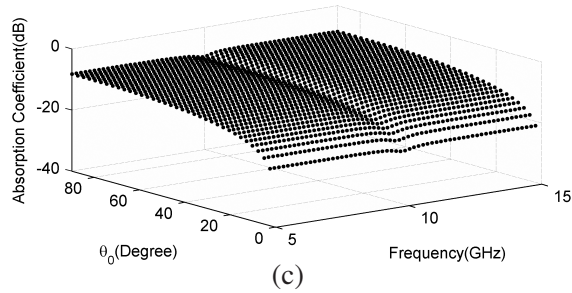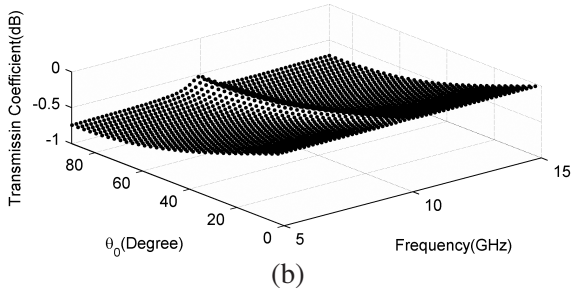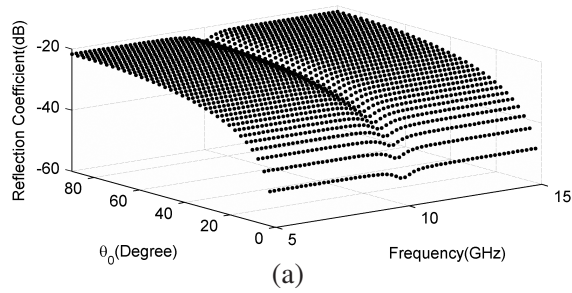

Figure 3. (a) Magnitude of the Reflection Coefficient, (b) magnitude of the Transmission Coefficient, (c) magnitude of the Absorption Coefficient, for a polarization angle of $\alpha_0 = 0°$ (i.e., TM_z polarization).
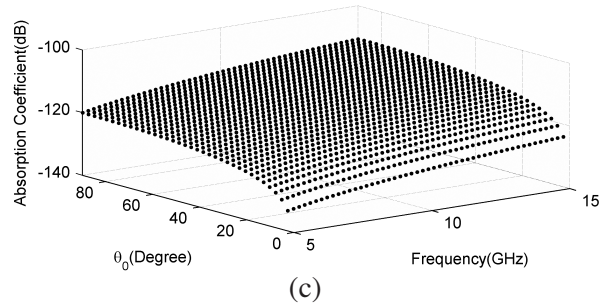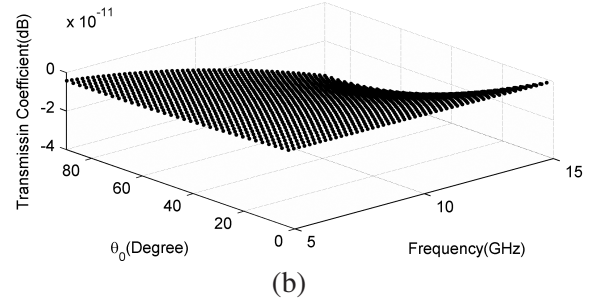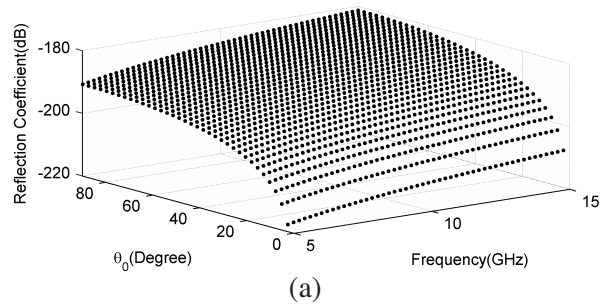
Figure 4. (a) Magnitude of the Reflection Coefficient, (b) magnitude of the Transmission Coefficient, (c) magnitude of the Absorption Coefficient, for a polarization angle of $\alpha_0 = 90°$ (i.e., TE_z polarization).

single microwire as explained in [4] in terms of two frequency-ranges containing frequencies below and above FMR where $\text{Re}[\mu_e] > 0$ and $\text{Re}[\mu_e] < 0$, respectively. For frequencies below FMR, $\text{Re}[\mu_e] > 0$ as shown in Fig. 1, the medium inside the microwire behaves similar to lossy dielectric and thus the scattering is weak. However, for frequencies above FMR, $\text{Re}[\mu_e] < 0$. As a result, the imaginary part of the propagation constant (phase constant) of ferrite medium becomes negative. Consequently, the microwire supports only evanescent field inside, and the microwire essentially behaves like a plasma region giving rise to increased scattering. In other words, there is a remarkable difference in the scattering behavior of the single microwire for the frequencies below and above FMR. Now, in case of a microwire grid, the difference in the magnitudes of the scattered fields for the frequencies below and above FMR is not so much well pronounced as in the case of a single microwire. This is due to the contribution to the scattered field made by the microwires other than the reference microwire. The magnitude of the Reflection and Absorption coefficient for small angle of incidence (say $\theta_0 \rightarrow 10°$) turns out to be very small because of the low values of the tangential field components (Fig. 3(a) and Fig. 3(c)). As a result, plasma-like behavior of ferrite microwire beyond FMR is compensated by the low values of the tangential field components. On account of the decreased magnitude of the Reflection and Absorption coefficients, the magnitude of the Transmission coefficient is increased.
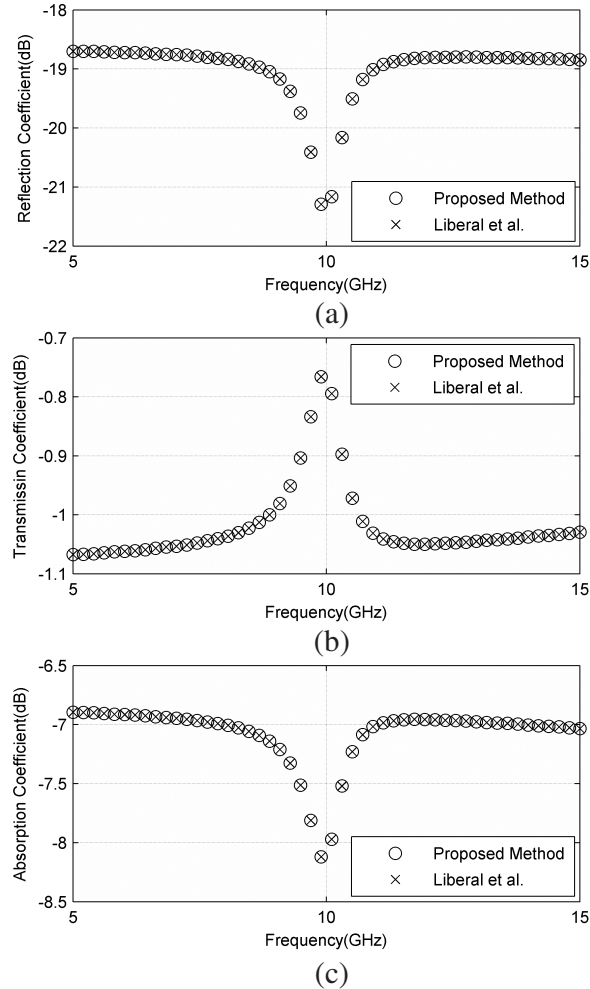

Figure 5. Comparison of the results for normal incidence and TM_z Polarization; (a) Magnitude of the Reflection Coefficient. (b) Magnitude of the Transmission Coefficient. (c) Magnitude of the Absorption Coefficient, with the results obtained by Liberal et al. [8].

3.2. TE_z Polarization ($\alpha_0 = 90°$)

Figure 4 shows the simulation results plotted for the magnitudes of the Reflection, Transmission and Absorption coefficients for a polarization angle of $\alpha_0 = 90°$, i.e., TE_z polarization. In this case, the **H** vector of the incident wave is in a plane along the wire (i.e., z-axis), i.e., parallel to the internal magnetization. Hence, an ordinary wave propagation takes place inside the ferrite medium. In this case, the ferrite medium behaves like a lossy dielectric medium which results in a very weak scattered field. Due to the ordinary wave propagation inside the ferrite medium, there is no effect of ferromagnetic resonance observed on the results of Reflection, Transmission and Absorption Coefficients (Fig. 4). Further, it is to be noticed from Fig. 4 that the magnitude of the Reflection and Absorption Coefficients increases with frequency. This is due to the fact that the skin depth decreases with an increment in frequency and it results in the decrement of the Transmitted field, which is further compensated by the increment in the magnitude of the Reflection and Absorption Coefficients.

4. COMPARISON OF THE RESULTS

In this section, a comparative remark is made upon the results available in [8] and the results obtained by the proposed analysis specialized to the case of normal incidence and TM_z Polarization (i.e., $\alpha_0 = 0°$). Fig. 5 shows the comparison of the results in terms of the Reflection, Transmission and Absorption Coefficients for the microwire of radius, $a = 1\,\mu\text{m}$ and spacing, $d = 3\,\text{mm}$. It is to be noticed from Fig. 5 that the results of the proposed analysis specialized to the case of normal incidence and TM_z polarization perfectly reduces to the results obtained by Liberal et al. in [8]. This proves that the analysis given in [8] is a special case of the proposed analysis.

5. CONCLUSION

A generalized approach for the analysis of electromagnetic scattering from a ferromagnetic microwire grid is presented for both TM_z and TE_z polarizations in this paper. The derivation of the field coefficients and hence the magnitude of the scattered fields is carried out by using the tangential boundary condition at the surface of the reference microwire. Simulated results are presented for two different polarization angles, $\alpha = 0°$ and $90°$ (i.e., TM_z and TE_z polarizations respectively). The results of the proposed analysis specialized to the case of normal incidence and TM_z polarization reduces to the results obtained by Liberal et al. in [8]. Hence, it is shown that the proposed analysis is the most generalized case of the scattering from a ferromagnetic microwire grid containing an infinite numbers of microwires. The next step in this approach could be the case of a grid containing finite number of wires. It is expected that the present analysis may find an application in the design of wire based metamaterials.

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