Progress In Electromagnetics Research B, Vol. 59, 19–29, 2014

Diffraction from a Grating on a Chiral Medium: Application of Analytical Regularization Method

Sergey B. Panin[1, *], Bahattin Turetken[1], Anatoly Y. Poyedinchuk[2], and Yury A. Tuchkin[3]

Abstract—Theoretical results on the electromagnetic wave diffraction from a periodic strip grating placed on a chiral medium are obtained. Analytical Regularization Method based on the solution to the vector Riemann-Hilbert boundary value problem was used to get robust numerical results in the resonant domain, where direct solution methods typically fail. It was shown that in the case of normal incidence of linearly polarized wave the cross-polarized field appears in the reflected field. For elliptically polarized incident wave the diffraction character essentially depends on the polarization direction of the incident wave. These diffraction peculiarities are more pronounced in the resonant domain. Influence of the dichroism caused by chiral medium losses is thoroughly studied. The combination of a chiral medium and a grating can be effectively used for a frequency and polarization selection and for a mode conversion.

1. INTRODUCTION

Interest in microwave and millimeter wave devices involving artificial composite materials has increased, with emphasis on chiral composites possessing spatial dispersion and offering unusual electromagnetic characteristics, for one, optical activity and circular dichroism [1]. Recent advances in the technologies used to make chiral materials propose new fields of their applications. Among them are chiral-filled waveguides, effective microwave absorbers, microstrip substrates, and antenna radomes.

It is remarkable that a chiral insertion can do more than varying one or another characteristic of the system, which includes it in. It imparts novel properties even to well-known structures, for instance a cross-polarized component in the reflected field for a linearly polarized wave incident normally on an ordinary strip grating placed in close vicinity of the isotropic chiral half space [2]. The appearance of new properties is caused by that the chiral material is essentially richer and complicated medium in comparison with conventional dielectric. Indeed, for one, eigenwaves of a homogeneous isotropic chiral medium consist of decoupled left and right circularly polarized waves. Any inhomogeneity makes eigenwaves of a chiral medium coupled so that the boundary value vector problem can be no longer reduced to the scalar problems and requires vector analysis capability.

Nonetheless, the diffraction gratings are widely used for a long time, and their resonant response in combination with artificial materials is rather novel and theoretically interesting as well as exciting for technological developments [3, 4]. The nearly total resonance transformation of an elliptically polarized incident wave into a specularly reflected linear polarized wave was found in a layered structure comprising a strip grating, magnetodielectric layer, and screened chiral layer [5]. Such a structure supports the resonance polarization transformation regimes of essential auto collimation reflection and near total nonspecular reflection with a high telescopicity factor [6].

Received 1 December 2013, Accepted 25 February 2014, Scheduled 7 March 2014

* Corresponding author: Sergey B. Panin (Sergiy.Panin@tubitak.gov.tr).
[1] MILTAL, Material Institute, TUBITAK-MRC, 1 Dr. Zeki Acar Str., Gebze/Kocaeli 41470, Turkey. [2] Department of Diffraction Theory and Diffraction Electronics, Institute for Radiophysics and Electronics of NAS of Ukraine, 12, Academic Proskura Str., Kharkov 61085, Ukraine. [3] Department of Electronic Engineering, Gebze Institute of Technology, PK 141, 41400, Turkey.

The analysis of electromagnetic structures incorporated with chiral materials has been an important topic in computational electromagnetics. Numerical analysis was usually carried out using a variety of numerical methods, such as the method of moments [7], finite-difference time-domain (frequency-domain) methods [8, 9], and boundary value solutions [10]. The method combining the building block approach of multimode network theory with a rigorous mode-matching procedure was proposed in [11] for scattering from chiral periodic structure. Using the conventional coupled-mode theory and mode transformation method, a unified local mode approach was proposed [12] to analyze the chiral fiber long-period gratings formed by twisting a high-birefringence fiber.

The customary numerical approaches usually lead to the operator equations that conceptually may be regarded as explicit (or implicit) "algebraization" of the initial integral (or integro-differential) equations of the fist kind by some means of projection and discretization. Such equations are typically ill-posed, their solution in such a case exhibits unpredictable inaccuracy and possible instability with an increase in the size of the algebraic system [13, 14]. These faults are more pronounced in the resonant domain, which is of great interest to technical applications. The idea of Analytical Regularization Method (ARM) can be utilized for mathematically equivalent transformation of a first-kind operator equation to a second-kind Fredholm equation, which allows efficient numerical solution with preassigned accuracy [3, 15–17]. The brief explanation of the principal ideas of ARM is presented in Appendix.

In this paper the technique, described in [2] for the case when the magnetodielectric layer of finite thickness separates the strip grating and chiral medium, is generalized to diffraction by the grating placed directly on a chiral medium. In the case considered herein, the 2D diffraction problem cannot be reduced to the two operator equations independent correspondingly to the principal part of the problem singularity. This reflects the essential vector nature of the problem and significantly complicates it mathematically. The obtained efficient and robust problem solution is based on the version of analytical regularization method proposed by Anatoly Ye. Poyedinchuk [3] for the vector Riemann-Hilbert boundary value problem.

The diffraction features of the structure under consideration are numerically and physically investigated in the resonant domain for excitation by linearly and elliptically polarized waves.

2. PROBLEM FORMULATION

The problem geometry is shown in Fig. 1, where a periodic grating of infinitely thin and perfectly conducting strips parallel to the OX axis lies on the plane interface ($z = 0$) between magnetodielectric and chiral half-spaces. The grating period is l, and the slot width is d. Let the inequality $|y| < d/2$ indicate the grating slot and $d/2 < |y| < l/2$ refer to the grating strip. Relative permittivities and permeabilities of magnetodielectric ($z > 0$) and chiral ($z < 0$) half-spaces are scalar complex ε_1, ε_2 and μ_1, μ_2, correspondingly; γ is the chirality parameter of the lower half-space. The frequency dispersion of the constitutive parameters is inherent in the chiral medium; therefore ε_2, μ_2 and γ are in general the functions of frequency.

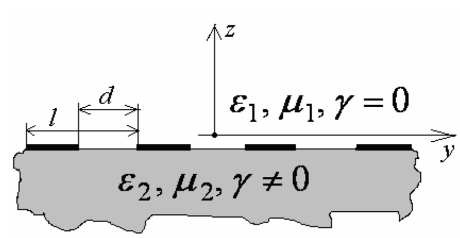

Figure 1. The problem geometry.

The grating is excited with monochromatic elliptically polarized plane wave: $\mathbf{E}^{in} = \mathbf{E}_0 \exp[-i(k_1 z - \omega\, t)]$, $\mathbf{H}^{in} = \mathbf{H}_0 \exp[-i(k_1 z - \omega t)]$, that is normally incident from the upper half-space such that

$$\mathbf{E}_0 = \left(\tilde{e}, \rho_1 \tilde{h}, 0\right), \qquad \mathbf{H}_0 = \left(\tilde{h}, -\frac{\tilde{e}}{\rho_1}, 0\right), \tag{1}$$

where ω is a frequency, and $\tilde{e}$, $\tilde{h}$ are complex field amplitudes and $k_j = \omega\sqrt{\varepsilon_0\mu_0\varepsilon_j\mu_j}$, $\rho_j = \sqrt{\mu_0\mu_j/\varepsilon_0\varepsilon_j}$, $j = 1,\ 2$. The incident wave (1) can be considered as a superposition of two plane waves: E-polarized ($\mathbf{E}||OX$) and H-polarized ($\mathbf{E}||OY$) ones.

For the chosen time dependence $\exp(-i\omega t)$ the absence of losses in the chiral medium is provided by conditions: $\mathrm{Im}\varepsilon_2 = 0$, $\mathrm{Im}\mu_2 = 0$, $\mathrm{Im}\gamma = 0$ and $\gamma^2 < \varepsilon_2\mu_2$. Chiral medium with absorption should satisfy the following conditions: $\mathrm{Im}\varepsilon_2 > 0$, $\mathrm{Im}\mu_2 > 0$ and $(\mathrm{Im}\gamma)^2 < \mathrm{Im}\varepsilon_2\mathrm{Im}\mu_2$.

The diffracted field is to be found.

Because the problem constitutive parameters are scalars, and the incident field and the problem geometry are x-independent, the field is solved in two-dimensional terms ($\partial/\partial x \equiv 0$). A unique solution is sought to meet [3] Maxwell's equations, the radiation condition, boundary conditions, a quasi-periodicity condition, and a condition of the field energy finiteness in any bounded domain.

3. RAYLEIGH FIELD EXPANSION

For the two-dimensional problem, Maxwell's equations and chiral constitutive relations [1]

$$\mathbf{D} = \varepsilon_0\varepsilon_2\mathbf{E} + i\gamma\sqrt{\varepsilon_0\mu_0}\mathbf{H}, \qquad \mathbf{B} = \mu_0\mu_2\mathbf{H} - i\gamma\sqrt{\varepsilon_o\mu_o}\mathbf{E},$$

give the field decomposition in the isotropic chiral medium as a sum of two circularly polarized waves

$$\begin{gathered}\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^-, \qquad \mathbf{H} = \mathbf{H}^+ + \mathbf{H}^- = -\frac{i}{\rho_2}\left(\mathbf{E}^+ - \mathbf{E}^-\right), \\ \Delta_{yz}E_x^\pm + \left(k^\pm\right)^2 E_x^\pm = 0, \qquad E_y^\pm = \mp\frac{1}{k^\pm}\frac{\partial\, E_x^\pm}{\partial\, z}, \qquad E_z^\pm = \pm\frac{1}{k^\pm}\frac{\partial\, E_x^\pm}{\partial\, y}\end{gathered} \tag{2}$$

where $k^\pm = -k_2(1 \pm \eta)$, $\eta = \gamma/\sqrt{\varepsilon_2\mu_2}$ is the relative chirality parameter.

Thus, the eigenwaves (2) of a homogeneous chiral medium are the right- and left-handed circularly polarized plane waves $(\mathbf{E}^+, \mathbf{H}^+)$ and $(\mathbf{E}^-, \mathbf{H}^-)$ with propagation constants $k^\pm$.

Using the grating periodicity along the OY axis, the field is expanded into a Fourier series. The series substitution in the Helmholtz equation gives the field representation, which coincides with the Rayleigh expansion of the diffracted field in an infinite series of partial waves of the spatial spectrum:

$$\begin{gathered}\begin{pmatrix} E_1^x \\ H_1^x \end{pmatrix} = \begin{pmatrix} \tilde{e} \\ \tilde{h} \end{pmatrix}\exp\left(-ik_1z\right) + \sum_{n=-\infty}^{+\infty}\begin{pmatrix} a_n \\ b_n \end{pmatrix}\exp\left(i\zeta_n^1 z\right)\exp\left(i\,\xi_n y\right), \qquad 0 < z \\ \begin{pmatrix} E_2^x \\ H_2^x\, i\rho_2 \end{pmatrix} = \sum_{n=-\infty}^{+\infty}\left[\begin{pmatrix} x_n \\ x_n \end{pmatrix}\exp\left(-i\zeta_n^+ z\right) + \begin{pmatrix} y_n \\ -y_n \end{pmatrix}\exp\left(-i\zeta_n^- z\right)\right]\exp\left(i\,\xi_n y\right), \qquad z < 0\end{gathered} \tag{3}$$

where $\xi_n = \frac{2\pi}{l}n$, $\zeta_n^1 = \sqrt{k_1^2 - (\xi_n)^2}$, and $\zeta_n^\pm = \sqrt{(k^\pm)^2 - (\xi_n)^2}$ are the propagation constants of the n-th field harmonic. For ζ_n^1, $\zeta_n^\pm$, the root branches with the real and imaginary parts are chosen to be both positive to satisfy the radiation condition. With these field components known, the rest follow from Maxwell's equations.

The diffracted field (3) is the wave superposition, which consists of a finite number of plane uniform travelling waves (ζ_n^1, $\zeta_n^\pm \in R$) and an infinite number of slow nonuniform surface waves (ζ_n^1, $\zeta_n^\pm \in C$) localized near the inhomogeneity plane $z = 0$, i.e., exponentially decaying for $z \to \infty$.

The problem posing includes the condition of the field energy finiteness in any bounded domain (which leads to well-known Meixner's condition). When the diffracted field is expanded into a Fourier series, this condition determines the space of numerical sequences, to which unknown Fourier coefficients should belong. For the considered structure, which has infinitely thin edge of a grating strip, this condition demands $\{x_n\}_{n=-\infty}^{\infty}$, $\{y_n\}_{n=-\infty}^{\infty} \in l_2\ (1)$, where x_n and y_n are complex field amplitudes mentioned above, and $l_2(\mu)$ for any μ is Hilbert space of square summable sequences with correspondent weight: $l_2(\mu) = \{\{z_n\}_{n=-\infty}^{\infty}\colon\ \sum_{n=-\infty}^{\infty}(1 + |n|)^\mu |z_n|^2\}$.

4. DUAL SERIES EQUATIONS

The boundary condition, that the tangential components of the electric field are equal over the grating period (i.e., $E_1^x = E_2^x$, $E_1^y = E_2^y$ for $|y| < l/2$, $z = 0$), yields

$$a_n = x_n + y_n - \tilde{e}\delta_0^n, \quad b_n = -i\frac{\omega\varepsilon_0\varepsilon_1}{\zeta_n^1}\left[\frac{\zeta_n^+}{k^+}x_n - \frac{\zeta_n^-}{k^-}y_n\right] + \tilde{h}\delta_0^n,$$

where δ_m^n is Kronecker delta.

The boundary conditions, that the tangential components of the magnetic field are continuous over the grating slot (i.e., $H_1^x = H_2^x$, $H_1^y = H_2^y$ for $|y| < d/2$, $z = 0$) and the electric field tangential components vanish on the metal strip (i.e., $E_2^x = 0$, $E_2^y = 0$ for $d/2 < |y| \le l/2$, $z = 0$), result in the following coupled systems of dual series equations:

$$\begin{cases} \sum\limits_{n=-\infty}^{+\infty} \zeta_n^1\left\{x_n\left[1+S_n^{11}\right] + y_n\left[1+S_n^{12}\right]\right\}\exp\left(i\xi_n y\right) = 2k_1\tilde{e}, & |y| < \frac{d}{2} \\ \sum\limits_{n=-\infty}^{+\infty} \left\{x_n + y_n\right\}\exp\left(i\xi_n y\right) = 0, & \frac{d}{2} < |y| \le \frac{l}{2} \end{cases} \tag{4}$$

$$\begin{cases} \sum\limits_{n=-\infty}^{+\infty} \left\{x_n\left[S_n^{21}-1\right] + y_n\left[S_n^{22}+1\right]\right\}\exp\left(i\xi_n y\right) = -2i\rho_2\tilde{h}, & |y| < \frac{d}{2} \\ \sum\limits_{n=-\infty}^{+\infty} \zeta_n^1\left\{x_n S_n^{21} + y_n S_n^{22}\right\}\exp\left(i\xi_n y\right) = 0, & \frac{d}{2} < |y| \le \frac{l}{2} \end{cases} \tag{5}$$

where $S_n^{11} = \frac{\mu_1}{\mu_2(1+\eta)}\frac{\zeta_n^+}{\zeta_n^1}$, $S_n^{12} = \frac{\mu_1}{\mu_2(1-\eta)}\frac{\zeta_n^-}{\zeta_n^1}$, $S_n^{21} = -\frac{\varepsilon_1}{\varepsilon_2(1+\eta)}\frac{\zeta_n^+}{\zeta_n^1}$ and $S_n^{22} = \frac{\varepsilon_1}{\varepsilon_2(1-\eta)}\frac{\zeta_n^-}{\zeta_n^1}$. Coupling between systems (4) and (5) describes the vector nature of the problem. In the case of chirality absence ($\gamma = 0$), these systems become decoupled and vector problem is reducible to two scalar independent problems: for E-polarized field (4) and H-polarized field (5).

Presenting the propagation constant of n-th harmonic along the OZ axis in the upper half-space in the form $\zeta_n^1 = \frac{2\pi}{l}|n|i(1-o_n)$, where $o_n = 1 + i\sqrt{(\frac{\chi_1}{n})^2 - 1}$, $\chi_1 = \frac{l}{\lambda_0}\sqrt{\varepsilon_1\mu_1}$, λ_0 is wave length in free space, we conclude that $\zeta_n^1 \sim |n|$ and $o_n = O(n^{-2})$ as $|n| \to \infty$. Thus the series in equations of the systems have different convergence rate.

Our immediate goal is an extraction of the series with the slowest convergence rate. For this purpose, we introduce new unknowns

$$X_n = x_n\left[S_n^{21}-1\right] + y_n\left[S_n^{22}+1\right] + 2i\rho_2\tilde{h}\delta_0^n, \qquad Y_n = x_n + y_n,$$

and rewrite the systems (4) and (5) in the form

$$\begin{cases} \sum\limits_{n=-\infty}^{+\infty} X_n\exp\left(in\varphi\right) = 0, & |\varphi| < \varphi_0 \\ \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \zeta_n^1 q_n^{22}\left\{X_n + q_n^{11}Y_n\right\}\exp\left(in\varphi\right) + k_1\frac{\rho_2}{\rho_1+\rho_2}\left(X_0 - 2i\rho_2\tilde{h}\right) = 0, & \varphi_0 < |\varphi| < \pi \end{cases} \tag{6}$$

$$\begin{cases} \sum\limits_{\substack{n=-\infty \\ n\neq 0}}^{+\infty} \zeta_n^1 q_n^{21}\left\{q_n^{12}X_n + Y_n\right\}\exp\left(in\varphi\right) + k_1\left(\frac{\rho_1+\rho_2}{\rho_2}Y_0 - 2\tilde{e}\right) = 0, & |\varphi| < \varphi_0 \\ \sum\limits_{n=-\infty}^{+\infty} Y_n\exp\left(in\varphi\right) = 0, & \varphi_0 < |\varphi| < \pi \end{cases} \tag{7}$$

where $\varphi = 2\pi y/l$, $\varphi_0 = \pi d/l$ and the following notations are used

$$\begin{aligned} q_n^{11} &= \frac{S_n^{21} + S_n^{22}}{S_n^{22} - S_n^{21}}, \qquad q_n^{12} = \frac{S_n^{12} - S_n^{11}}{\left(1+S_n^{11}\right)\left(1+S_n^{22}\right) + \left(1+S_n^{12}\right)\left(1-S_n^{21}\right)}, \\ q_n^{21} &= \frac{\left(1+S_n^{11}\right)\left(1+S_n^{22}\right) + \left(1+S_n^{12}\right)\left(1-S_n^{21}\right)}{2 + S_n^{22} - S_n^{21}}, \qquad q_n^{22} = \frac{S_n^{22} - S_n^{21}}{2 + S_n^{22} - S_n^{21}}. \end{aligned} \tag{8}$$

On the base of the asymptotic estimates for (8) as $|n| \to \infty$ we can present the coefficients q_n^{ij} $(i, j = 1, 2)$ in the form

$$q_n^{ij} = q^{ij} + \tilde{q}_n^{ij}, \tag{9}$$

where $q^{11} = \eta$, $q^{12} = \frac{\varepsilon_2 \mu_1 \eta}{(\varepsilon_1+\varepsilon_2)(\mu_1+\mu_2)-\varepsilon_2\mu_2\eta^2}$, $q^{21} = \frac{(\varepsilon_1+\varepsilon_2)(\mu_1+\mu_2)-\varepsilon_2\mu_2\eta^2}{\mu_2(\varepsilon_1+\varepsilon_2(1-\eta^2))}$, $q^{22} = \frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2(1-\eta^2)}$ and $\tilde{q}_n^{ij} = O(n^{-2})$.

Now taking into account of presentation (9) we can rearrange the series with different convergence rates and rewrite systems (6) and (7) in the form

$$\begin{cases} \sum\limits_{n=-\infty}^{+\infty} X_n \exp(in\varphi) = 0, & |\varphi| < \varphi_0 \\ \sum\limits_{n=-\infty}^{+\infty} |n| \left\{X_n + q^{11} Y_n\right\} \exp(in\varphi) = F^1(\varphi), & \varphi_0 < |\varphi| < \pi \end{cases} \tag{10}$$

$$\begin{cases} \sum\limits_{n=-\infty}^{+\infty} |n| \left\{q^{12} X_n + Y_n\right\} \exp(in\varphi) = F^2(\varphi), & |\varphi| < \varphi_0 \\ \sum\limits_{n=-\infty}^{+\infty} Y_n \exp(in\varphi) = 0, & \varphi_0 < |\varphi| < \pi \end{cases} \tag{11}$$

where functions $F^{1,2}(\varphi) = \sum\limits_{n=-\infty}^{+\infty} \{V_n^{21,11} X_n + V_n^{22,12} Y_n + f_n^{2,1}\} \exp(in\varphi)$ and vector-columns:

$$V_n^{11} = |n| \left[\frac{q_n^{12}}{q^{21}}\left(o_n q_n^{21} - \tilde{q}_n^{21}\right) - \tilde{q}_n^{12}\right], \qquad V_n^{22} = |n| \left[\frac{q_n^{11}}{q^{22}}\left(o_n q_n^{22} - \tilde{q}_n^{22}\right) - \tilde{q}_n^{11}\right],$$

$$V_{n=0}^{12} = i\chi_1 \frac{\rho_1+\rho_2}{\rho_2} \frac{\mu_2\left(\varepsilon_1 + \varepsilon_2\left(1-\eta^2\right)\right)}{\left(\varepsilon_1+\varepsilon_2\right)\left(\mu_1+\mu_2\right) - \varepsilon_2\mu_2\eta^2}, \qquad V_{n\neq 0}^{12} = |n| \frac{1}{q^{21}} \left(o_n q_n^{21} - \tilde{q}_n^{21}\right),$$

$$V_{n=0}^{21} = i\chi_1 \frac{\rho_2}{\rho_1+\rho_2} \frac{\varepsilon_1 + \varepsilon_2\left(1-\eta^2\right)}{\varepsilon_1}, \qquad V_{n\neq 0}^{21} = |n| \frac{1}{q^{22}} \left(o_n q_n^{22} - \tilde{q}_n^{22}\right),$$

$$f_n^1 = -\tilde{e}\chi_1 \frac{2i\mu_2\left(\varepsilon_1+\varepsilon_2\left(1-\eta^2\right)\right)}{\left(\varepsilon_1+\varepsilon_2\right)\left(\mu_1+\mu_2\right) - \varepsilon_2\mu_2\eta^2} \delta_0^n, \qquad f_n^2 = \tilde{h}\chi_1 \frac{2\rho_2^2}{\rho_1+\rho_2} \frac{\varepsilon_1+\varepsilon_2\left(1-\eta^2\right)}{\varepsilon_1} \delta_0^n.$$

Values $\tilde{q}_n^{ij}$ $(i, j = 1, 2)$ should be calculated in accordance with (9) as $\tilde{q}_n^{ij} = q_n^{ij} - q^{ij}$.

One can see that $V_n^{ij} = O(|n|^{-1})$ as $|n| \to \infty$. Thus, we have rapidly and uniformly convergent series separated in the right-hand sides of the systems. The matrix-operators $V^{ij} = \{V_n^{ij} \delta_m^n\}_{m,n=-\infty}^{+\infty}$ specify compact operator in $l_2 = l_2(0)$ space and the vector-columns $f^{1,2} = \{f_n^{1,2}\}_{n=-\infty}^{+\infty}$ belong to space $l_2(-1)$.

Therefore, functions $F^{1,2}(\varphi)$ are sufficiently smooth periodic functions, continuously differentiable with respect to φ. These functions present (if one remembers that they involve unknown Fourier coefficients) the continuous part of the diffraction problem operator. The principal part of the problem singularity is separated in the left-hand sides of the systems as the slowly convergent series. The coefficients $q^{11,12}$ in (10), (11) are unequal to zero (for $\eta \neq 0$) that binds the systems in terms of singularity of the problem operator.

When the magnetodielectric layer of thickness h separates the strip grating and chiral medium, the coefficients $q^{11,12}$ become n-dependent and for $|n| \to \infty$ tends to zero like [2] $q_n^{11,\,12}|_{|n|\to\infty} = O[\exp(-\sigma_n h|n|)]$, where $\sigma_n = 4\pi\frac{1}{l}\{1 - \frac{1}{2}(\frac{1}{n}\frac{l}{\lambda'})^2\} > 0$, λ' is wave length in the magnetodielectric layer. In this case, the terms with $q_n^{11,\,12}$ can be included in the rapidly and uniformly convergent series in the right-hand sides of (10), (11). In such a way the diffraction problem is reduced to the two operator equations correspondingly independent of the problem singularity [2].

Coupled functional systems (10) and (11) correspond to an operator equation of the first kind in l_2. These equations are ill conditioned [13, 14], i.e., numerically unstable.

The solution of the systems obtained can be equivalently transformed to the Riemann-Hilbert vector problem that is the conjugation problem in theory of vector analytic functions. Namely, the problem consists in the determination of vector function, analytic inside and outside the circle in the complex plane, through its known value along the circle arc. The stages of the systems transformation to the conjugation problem and its solution were recently proposed by A. Ye. Poyedinchuk and described in detail in [3]. These stages with the preceding stage of problem singularity extraction in the form of systems (10) and (11) compose the analytical regularization core.

Applying the technique described in [3] yields the infinite system of linear algebraic equations. The system obtained looks formally like one of the second kind. Nonetheless, following from the above explanation, the system operator maps $l_2(1)$ to $l_2(-1)$. In order to arrive the space $l_2 = l_2(0)$ we introduce new unknowns $\hat{X}_n$ and $\hat{Y}_n$ from the relations $X_n = \tau_n \hat{X}_n$, $Y_n = \tau_n \hat{Y}_n$, where $\tau_n = \max(1, t|n|^{1/2})$, $n = 0, \pm1, \pm2, \ldots$. Using new unknowns and dividing each line of the system with the first index s by τ_s, $s = 0, \pm1, \pm2, \ldots$, one obtains the system of the second kind in the sense explained in Appendix:

$$[\mathbf{I} + \mathbf{H}]\mathbf{z} = \mathbf{b}. \tag{12}$$

The compactness of the operator $\mathbf{H}$ ensures that the condition numbers of the truncated matrices $[\mathbf{I} + \mathbf{H}]_N$ are uniformly bounded for appropriately large (and growing to infinity) truncation numbers N. Thus, this equation can be effectively solved with any preassigned accuracy by a standard truncation procedure. In particular, asymptotic behaviour of coefficients x_n and y_n can be derived analytically from the system. Summation of Fourier series with such asymptotical coefficient gives classic Meixner's condition for singularity of current near the edges of metal surfaces [18, 19].

5. NUMERICAL RESULTS

The solution obtained in form (12) allows the robust investigation of the diffraction peculiarities in the resonant domain. Introduce the values $a_0^x = a_0$, $a_0^y = -\rho_1 b_0$ and $b_0^x = x_0 + y_0$, $b_0^y = -i(x_0 - y_0)$ to name the reflection ($z > 0$) and transmission ($z < 0$) coefficients of the zero harmonics ($n = 0$ in (3)). They describe the x- and y-components of the electric field, respectively. The superscript x corresponds to the E-polarized field and the superscript y to the H-polarized field. These coefficients define the electric field averaged over the grating period. From the boundary conditions $a_0^x + \tilde{e} = b_0^x$, $a_0^y + \rho_1 \tilde{h} = b_0^y$. The introduced coefficients are interdependent since they are determined via x_0, y_0, which are, in their turn, functionally coupled through the boundary conditions. The relation between x- and y-components shows that the problem is vectorial.

In the case of only E- or only H-polarized wave incidence, we call co-polarization that coincides with incident wave polarization; the cross-polarization is normal to co-polarization.

At first, we consider the case of excitation by the linearly polarized waves of E- and H-polarizations. Suppose that the complex amplitudes of E-, H-polarized incident waves are $\tilde{e} = 1$, $\tilde{h} = 1/\rho_1$, correspondingly. Bellow, for simplicity reasons we will neglect the frequency dispersion of chiral media. The specified dispersion dependence can be directly introduced just by considering functional dependence of ε, μ and γ upon ω.

The behaviour of reflection coefficients of co-polarization as function of the non-dimensional frequency parameter $\chi = \frac{l}{\lambda} = \frac{l}{2\pi}\sqrt{\varepsilon_0\mu_0}\,\omega$ is shown in Fig. 2(a). The reflection coefficients have not only peculiarities of a type of the classical Wood anomalies observed at the grazing points $\chi_n^w = n/\sqrt{\varepsilon_1\mu_1}$, $n \neq 0$, but also anomalies in the vicinities of the points $\chi_n^{\pm} = n/\left(\sqrt{\varepsilon_2\mu_2} \pm \gamma\right)$, $n \neq 0$. From the definition of ζ_n^1 and $\zeta_n^{\pm}$ follows that introduced values χ_n^w and $\chi_n^{\pm}$ give the frequencies where the n-th surface harmonic of upper and chiral half-spaces, respectively, becomes propagating one. The energy redistribution between the propagating harmonics causes the diffraction peculiarities.

Without the grating, the linearly polarized wave normally incident on chiral half-space simultaneously produces the left- and right-hand circularly polarized waves in it. These waves propagate in the direction of the incident wave; their superposition is the plane wave, and its plane of polarization rotates as this wave propagates inside the chiral medium. In this case, the reflected field does not contain the cross component. For the oblique wave incidence, the circularly-polarized waves excited in the chiral medium have different propagation directions, and their superposition is not a linearly polarized wave anymore. Thus, in order to meet the boundary conditions the reflected field should have the cross component.

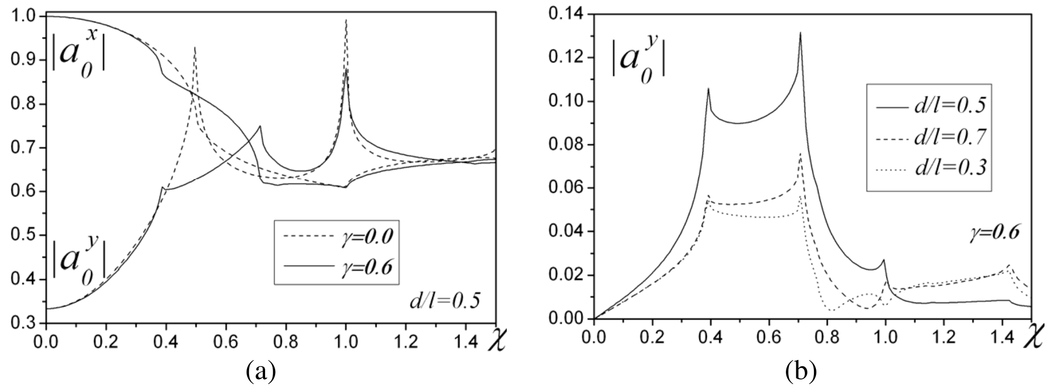

Figure 2. The reflection coefficients of the (a) co- and (b) cross-polarized waves as a function of the frequency parameter χ for E- and for H-incident waves ($\varepsilon_1 = 1,\ \mu_1 = 1,\ \varepsilon_2 = 4,\ \mu_2 = 1$).

On the other hand, the grating, being a periodical inhomogeneity, converts the incident wave into an infinite superposition of the spatial harmonics of circular polarization in the chiral medium. Thus in the presence of grating, the fulfilment of boundary conditions results in the appearance of the cross component in the reflected field even for normal incident wave. Initiation of new propagation circularly-polarized wave in the chiral medium at frequencies $\chi_n^{\pm}$ explains the increase in the module of reflection coefficients of the cross-polarized wave at these points, see Fig. 2(b). The value of the grating transparency $d/l = 0.5$ seems to be optimum for maximum excitation of cross-polarized component in the reflected field. Note that for the case of E- or H-linearly polarized wave incidence the reflection coefficients of the cross-polarized wave coincide.

The most pronounced peculiarities fall in the frequency interval $[0, \chi_1^w]$ to be near the points $\chi_{q^{\pm}}^{\pm}$, where integer $q^{\pm} \leq (1 \pm \eta)\sqrt{\varepsilon_2\mu_2/\varepsilon_1\mu_1}$. Because in the regime of existence of only the zeroth propagating harmonic in the upper half-space, the appearance of the additional propagating wave in the chiral medium leads to essential energy redistribution between the only wave in the first medium and the $q^+ + q^- + 1$ harmonics in the second one. The peculiarities are the weaker, the higher is the number of an additional propagating harmonic, which indicates the system tolerance to one more propagating wave. Therefore, despite the chiral medium maintains more propagating harmonics than the dielectric with the same permittivity and permeability, the χ-dependence of the reflection coefficient of co-polarization for the chiral and dielectric half spaces are practically the same for $\chi > \chi_3^w$. Note that the numerical results for a frequency dependence of reflection coefficients, obtained on the base of [2] for the structure with a very thin ($\sim 0.01\lambda$) dielectric layer separating a grating and chiral medium are rather close to the results for a grating placed directly on chiral medium.

Figure 3 shows the reflection coefficient of cross-polarized wave as a function of the relative chirality parameter $\eta = \gamma/\sqrt{\varepsilon_2\mu_2}$ for two chosen frequencies $\chi_1^{\pm}$, which give the first two maximums of the reflection coefficient of cross-polarized wave, see Fig. 2(b). These chirality dependent frequencies are given by $\chi_1^{\pm}(\eta) = [\sqrt{\varepsilon_2\mu_2}(1 \pm \eta)]^{-1}$. The growth of η gives an increase of χ_1^- and a decrease of χ_1^+ that means the transition to the short- and long-wavelength range, correspondingly. The appearance of new propagating waves at frequencies χ_n^w and $\chi_n^{\pm}$ explains the peculiarities of the reflection coefficient.

Now let us turn to the case of simultaneous incidence of E- and H-polarized plane waves. Introduce the phase difference of incident E- and H-waves $\Delta\varphi = \arg(\tilde{h}/\tilde{e})$. Depending on the value $\Delta\varphi$, the superposition of the incident waves would be generically elliptically-polarized plane wave. For $|\tilde{e}| = 1$, $|\tilde{h}| = 1/\rho_1$ and in-phase ($\Delta\varphi = 0$) incident waves, their superposition is linearly polarized wave (the angle between E-polarization plane and OX axis is $\pi/4$), whereas for values $\Delta\varphi = \pm\pi/2$ the resultant incident wave have the right- or left-hand circular polarization, respectively.

Diffraction for different orientations of the rotation of vector $\mathbf{E}$ in the incident circularly polarized wave occurs dissimilarly, as one can observe in Fig. 4. In the case of right-hand circular polarization of incident wave the peculiarity in the vicinity of frequency χ_1^- is more pronounced than at χ_1^+. The minus first propagating harmonic initiated at χ_1^- in chiral half-space has right-hand circular polarization that

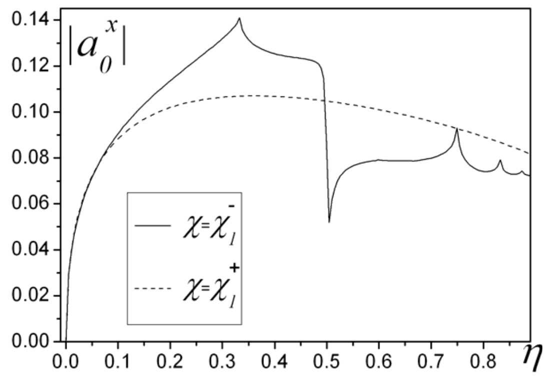

Figure 3. The reflection coefficient of the cross-polarized wave versus the relative chirality parameter η, ($\chi = \chi_1^{\pm}$, $d/l = 0.5$, $\varepsilon_1 = 1$, $\mu_1 = 1$, $\varepsilon_2 = 4$, $\mu_2 = 1$).

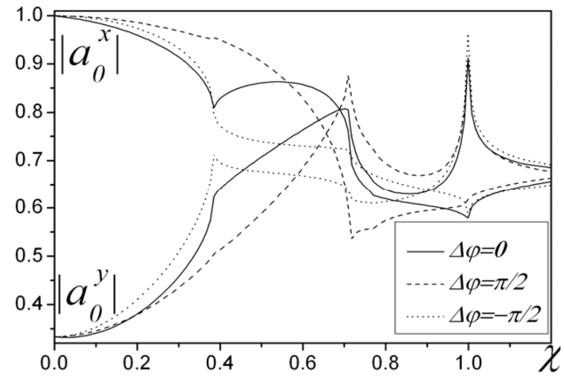

Figure 4. The frequency dependences of the reflection coefficients at simultaneous incidence of E- and H-waves ($d/l = 0.5$, $\varepsilon_1 = 1$, $\mu_1 = 1$, $\varepsilon_2 = 4$, $\mu_2 = 1$, $\gamma = 0.6$).

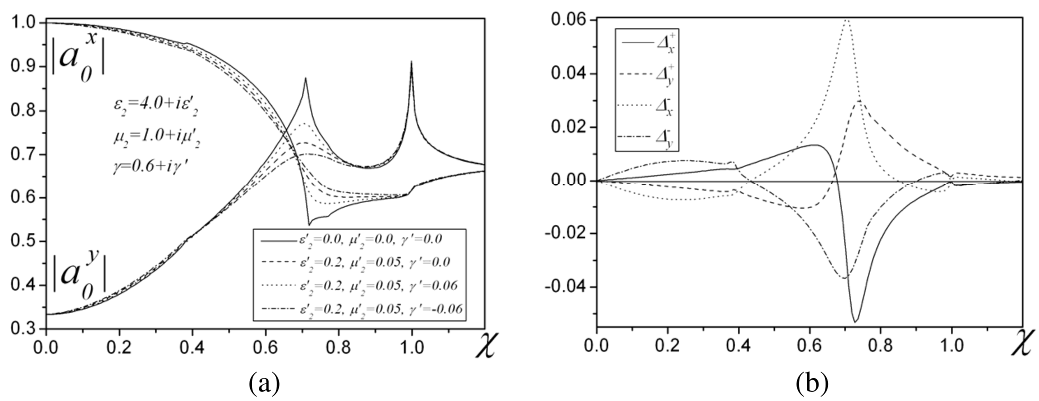

Figure 5. The frequency dependence of the (a) reflection coefficients and (b) $\Delta^{\pm}_{x,y}$ coefficients in the presence of losses, the case of the incidence of right-handed circularly polarized wave ($d/l = 0.5$, $\varepsilon_1 = 1$, $\mu_1 = 1$).

coincides with polarization of incident wave. Therefore, these waves effectively interact producing the distinct peculiarity at χ_1^-. On the other hand, at χ_1^+ these waves have different directions of the rotation of vector $\mathbf{E}$, and thus interact weakly. Analogically, for excitation with $\Delta\varphi = -\pi/2$ the peculiarity in the vicinity of frequency χ_1^+ is explained. In the case of the incidence of linearly polarized wave, the peculiarities in the vicinities of χ_1^+ and χ_1^- are exhibited almost equally.

In the presence of losses the frequency dependence of the reflection coefficients in the case of the incidence of right-hand ($\Delta\varphi = \pi/2$) circularly polarized wave is shown in Fig. 5(a). For this excitation the influence of losses is more evident in the vicinity of χ_1^-. The points $\chi_n^{\pm}$ associated with the diffraction peculiarities shift to the left, i.e., to the long wavelength region. For small losses we have $\chi_n^{\pm} = n[a'_{\pm} + (a''_{\pm})^2/a'_{\pm}]^{-1} \approx \bar{\chi}_n^{\pm}[1 - (a''_{\pm}/a'_{\pm})^2]$, where $a'_{\pm} + i\,a''_{\pm} = \sqrt{\varepsilon_2\mu_2} \pm \gamma$, $a'_{\pm}$, $a''_{\pm} \in R$ and $\bar{\chi}_n^{\pm} = n/a'_{\pm}$. As one would expect, the losses reduces and flattens the magnitude of the reflection coefficients.

The sign of imaginary part of complex-valued chirality parameter stipulates different attenuation for waves of the right- or left-hand circular polarization that is the essence of dichroism. Frequency behavior of the dichroism is illustrated in Fig. 5(b), where the coefficients, describing dichroism and introduced as

$$\Delta^{\pm}_{x,y} = \frac{|a_0^{x,y}|_{\gamma'=\pm 0.06} - |a_0^{x,y}|_{\gamma'=0}}{|a_0^{x,y}|_{\gamma'=0}},$$

where $\gamma' = \mathrm{Im}\gamma$, are plotted.

As seen from the z-coordinate dependence of the partial harmonics of the chiral medium, the right-handed waves are more absorbed when $\gamma' > 0$, and the left-handed waves are more absorbed when $\gamma' < 0$. Therefore, when the harmonics propagating in the chiral medium decay less, the diffraction behaviour is less affected by absorption.

6. CONCLUSION

Diffraction features of a periodic strip grating placed directly on a chiral medium are theoretically studied for excitation by linearly and left- and right-handed elliptically polarized waves. To get robust numerical results in the resonant domain, our solution approach employs the analytical regularization based on the solution to the vector Riemann-Hilbert boundary value problem.

The diffraction has peculiarities, caused by the energy redistribution between the propagating harmonics, not only a type of the classical Wood anomalies observed at the grazing points, but also anomalies in the vicinities of the points where new propagating harmonics of chiral media appear. The cross-polarized wave component is initiated in the reflected field even in the case of normal incidence of linearly polarized wave. The peculiarities are more pronounced in the resonant domain.

For elliptically polarized incident wave the diffraction character essentially depends on the polarization direction: when the frequency and direction of the circularly polarized incident wave coincide with the frequency and polarization direction of the harmonic propagating in chiral medium, the diffraction peculiarity is more pronounced. The influence of magnitude of chirality and dichroism caused by chiral medium losses are thoroughly studied in the resonant domain.

The carried out analysis shows that the combination of a chiral medium and a grating in the resonant domain can be efficiently used for a frequency and polarization selection and for a mode conversion.

APPENDIX A.

Analytical Regularization Method (ARM) should not be considered as a collection of various approaches only. ARM is rather a kind of philosophy of how to construct efficient and numerically stable methods that are relevant to the physical problems under consideration.

Taking into account of the well-known theorems of functional and numerical analysis, especially laws of propagation of the round off errors, we conclude that ARM must be targeted to equivalent reduction of the correspondent diffraction boundary value problem to an infinite algebraic system of the second kind $(\mathbf{I}+\mathbf{H})\mathbf{x} = \mathbf{b}$, $\mathbf{x}$, $\mathbf{b} \in l_2$ in space $l_2 = l_2(0)$ with compact $\mathbf{H}$ and identical $\mathbf{I}$ operators. Such a system is known having great numerical advantages [13, 14] over one of the first kind: $\mathbf{Ax} = \mathbf{b}$, $\mathbf{x}$, $\mathbf{b} \in l_2$ (i.e., non-representable as one of the second kind).

Historically, the first usage of ARM has been done in [20–22]. A brief explanation of the principal ideas of ARM can be found in [15], and their detailed description is the subject of a book [18]. The freshest explanation of the ARM methodology by Yu. A. Tuchkin is included in [16, 17]. As we mentioned, ARM is rather a philosophy than concrete techniques. Nevertheless, a few principal ARM steps can be outlined.

Let an equation of the first kind $\mathbf{Ax} = \mathbf{b}$ is given with operator $\mathbf{A}$: $H_1 \to H_2$ defined on a pair of functional spaces H_1 and H_2.

The steps are the following.

1) Proper choice of spaces H_1 and H_2 relevant to the physical problem and forming Tihonov's set of correctness of operator $\mathbf{A}$;
2) Reduction of the equation to canonical one, for which ARM construction is known or to such equation that known ARM can be easily adapted;

3) Additive splitting of $\mathbf{A}$ as $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}_1$, where $\mathbf{A}_0$ contains the principal part (singularity) of $\mathbf{A}$, $\mathbf{A}_1$ is operator subordinated (more smooth) to $\mathbf{A}_0$;
4) Multiplicative splitting, i.e., a representation of operator $\mathbf{A}_0$ as $\mathbf{A}_0 = \mathbf{L}^{-1}\mathbf{R}^{-1}$, where operators $\mathbf{L}$: $H_2 \to l_2$ and $\mathbf{R}$: $l_2 \to H_1$ are such that operator $\mathbf{H} = \mathbf{L}\mathbf{A}_1\mathbf{R}$: $l_2 \to l_2$ is compact;
5) Final regularization. Introducing new unknown $\mathbf{y}$ from equality $\mathbf{x} = \mathbf{R}\mathbf{y}$ and acting to $\mathbf{A}\mathbf{x} = \mathbf{b}$ by $\mathbf{L}$ from the left side, one obtains equation $(\mathbf{I} + \mathbf{L}\mathbf{A}_1\mathbf{R})\mathbf{y} = \mathbf{L}\mathbf{b}$, i.e., equation of the second kind $(I + H)x = b,\ x, b \in l_2$ with compact in space l_2 operator H.

Analytical implementation of all this steps constitutes ARM.

We see no choice, but have to forewarn our reader: do not mismatch ARM herein with Method of Analytical Regularization (MAR) described in survey [23], where explanation of MAR is replicated from the book in [24]. This approach is named in the book as Semi-inversion Method (SM) and under this name was and is well known and widely utilized in Eastern scientific hemisphere. We cannot see a *scientific* necessity to rename the well known for decades method SM by MAR, especially when the survey, introducing such renaming, is overcrowded by misleading and even wrong statements, such as that any bounded operator is compact, or that MAR itself removes spurious solutions. Formally SM is a special case of ARM, when $\mathbf{R} = \mathbf{I}$ and correspondingly $H_1 = l_2$. That is why MAR (i.e., SM) has small area of applicability in comparison with ARM. In particular, the problem considered here cannot be efficiently solved with MAR (i.e., SM).

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