Progress In Electromagnetics Research M, Vol. 34, 99–105, 2014

Design of Miniature Coil to Generate Uniform Magnetic Field

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Abstract—In various technological and scientific applications, different types of coil systems are being used to produce uniform alternating magnetic field. The dimensions of these coil systems are considerably larger than the volume of interest. There is a necessity to reduce the dimension of the coil system without sacrificing the extent of uniformity of the magnetic field. This problem has a wide audience and still remains as a topic of contemporary research in the development of miniaturized devices especially for calorimetric measurements of nano-particles, cancer therapy, and detection of minute surface defects by eddy current probes, etc. In this paper we present how we can modify the shape of a miniature solenoid to produce uniform magnetic field. A Genetic algorithm has been implemented to get the optimum dimension of the miniature solenoid. Our distinct shape design has achieved 97% uniformity for a 60% volume of interest.

1. INTRODUCTION

Solenoids are usually cylindrical inductive coils to produce magnetic field in various technological and scientific applications [1–4]. Many applications demand uniformity of the magnetic field within a volume of interest. Infinite solenoids produce uniform magnetic fields but in reality, magnetic field uniformity is a challenging task especially for the solenoids used in the bio-electromagnetic experiments [4]. Helmholtz coil system [4] is used in laboratory for the generation of uniform magnetic field in smaller volume of interest. Merritt and Ruben coil systems are implemented where large volume of uniform magnetic field is required [2, 3]. However, a Ruben coil system is more difficult to build [6]. Specific applications like calorimetric measurements of nano-particles, cancer therapy, eddy current probes, etc. demand uniformity of magnetic field generated by solenoids of much smaller dimensions than those used in Magnetic Resonance Imaging (MRI) [5] systems. Some researchers have attempted to build minimum volume coil configurations using a linear-programming technique [9] or finite-element method (FEM) [6] to produce uniform magnetic field. The magnetic field strength is maximum at the center of a finite solenoid and it reduces towards the ends. Improvement of uniformity of magnetic field inside a solenoid is realized with the help of structural modifications along with the magnetic flux concentrator rings at the coil ends [4]. Two-dimensional finite element analysis simulation software, e.g., FEMM [11] is usually used to validate the design. In this paper, we describe the principle, design methodology and performance of a kind of miniature solenoid which can be used for some specific purposes. We arrive at a typical shape of the winding surface to achieve the required uniformity of magnetic field of the solenoid. In engineering shape design there are various methods. We have adopted here an optimization technique using non-dominated sorting genetic algorithm (GA), which is a multi-objective optimization method in real engineering problems [7, 8]. In our case we have identified the required region and extent of uniformity of the magnetic field within the solenoid and optimized its design by using GA as explained in the later sections.

Received 26 November 2013, Accepted 18 December 2013, Scheduled 20 December 2013
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2. PRINCIPLE OF FIELD UNIFORMITY

The uniformity of the magnetic field within a specified region of interest is defined as the measure of the maximum deviation of the magnitude of the magnetic field in relation to the average value within the specified domain [12]. Thus, the magnetic field uniformity, η, is expressed in Eq. (1) where $B_{\max}$, $B_{\min}$ and B_{avg} are the maximum, minimum and average values of the magnetic field within the working volume respectively.

$$\eta = 1 - \frac{B_{\max} - B_{\min}}{B_{avg}} \tag{1}$$

The magnetic field components (Figure 1) produced at any arbitrary location $(r,\, z)$ by an infinitely thin circular current loop of radius R_i carrying current I_i located at position Z_i, is given by (2) [10] where $\alpha = r/R_i$, $\beta = (z - Z_i)/R_i$, $\gamma = (z - Z_i)/r$, $Q = [(1+\alpha)^2 + \beta^2]$, $k = (4\alpha/Q)^{1/2}$ and $K(k)$ and $E(k)$ are the complete elliptical integral functions of first and second kind respectively. The field, B_0 at the center of the coil is $\mu_0 I_i/2R_i$. $B_z^i(r,z)$ and $B_r^i(r,z)$ are the axial and radial components of the magnetic field respectively.

$$\begin{aligned} B_z^i(r,z) &= \frac{B_0}{\pi\sqrt{Q}}\left[E(k)\,\frac{1-\alpha^2-\beta^2}{Q-4a} + K(k)\right] \\ B_r^i(r,z) &= \frac{B_0\gamma}{\pi\sqrt{Q}}\left[E(k)\,\frac{1+\alpha^2+\beta^2}{Q-4a} - K(k)\right] \end{aligned} \tag{2}$$

The total magnetic field of a solenoid of N_{turns} at a location $(r,\, z)$ is given in (3). For uniformity of the magnetic field, $B_z(r,\, z)$ and $B_r(r,\, z)$ should be better than a specified η value for certain volume of interest.

$$\begin{aligned} B_z(r,z) &= \sum_{i=1}^{N_{turns}} B_z^i(r,z) \\ B_r(r,z) &= \sum_{i=1}^{N_{turns}} B_r^i(r,z) \end{aligned} \tag{3}$$

One can observe in Equations (2) and (3) that the radial and axial magnetic field components inside the solenoid depend on the diameter of the current loops. Therefore, it is possible to change the strength of the magnetic field inside a solenoid by varying R along the axis of the coil. This is the motivation behind the shape modification of the coil to achieve field uniformity. Based on this understanding many geometrical shapes are possible where discrete changes of R with z are suggested by Kasuya et al. in large coil systems [13]. In the case of a small coil, these discrete changes of radius

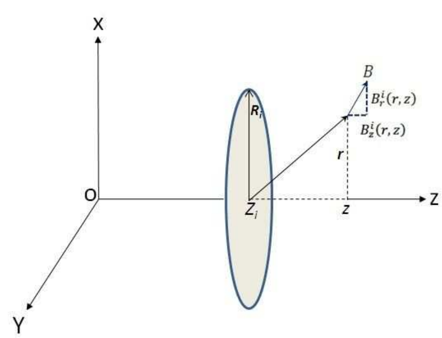

Figure 1. Axial and radial components of magnetic field due to a single circular current loop of radius R_i and carrying current I_i.

of the bobbin in the form of collars impose discontinuities on the mechanical winding as well as on the magnetic field distribution. This constraint has been eliminated in our design by a gradual variation of R with z. The basic design criteria to make a uniform magnetic field inside a solenoid is to concentrate the magnetic field strength at the ends in the form of a field concentrator by reducing the radius of the bobbin compared to that at the centre. Thus sigmoid curve can be assumed to be a better replacement of discrete steps for the profile of the bobbin (see Figure 2). The windings made on the curved surface of the bobbin will have a tendency to slip off the surface. Therefore, the frictional force between the bobbin and the windings should be sufficient to hold the windings in position. Thus, the slope of the sigmoid curve at any location should not be more than the co-efficient of static friction (μ) between the windings of the coil and the bobbin surface. It is better to use glue after winding the coil for rugged usage.

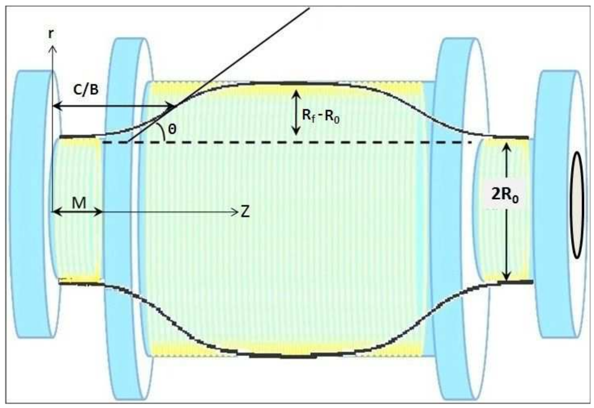

Figure 2. Combined representation of discrete step bobbin and shape modified bobbin. M is the field concentrated region for the discrete coil design. The profile of the shape modified bobbin, for the left half of its length, is defined by the generalized logistic function. The profile of the rest half is the mirror image of the former. $\theta = \tan^{-1} B\frac{R_f-R_0}{4}$ for the shape modified bobbin.

The silhouette of the bobbin is determined by the generalized logistic function [14] as given in Equation (4).

$$R(z) = R_0 + \frac{R_f - R_0}{1 + \exp(C - Bz)} \tag{4}$$

$$\frac{dR}{dz} = \frac{B(R_f - R_0)\exp(C - Bz))}{[1 + \exp(C - Bz)]^2} \tag{5}$$

In the above equation, the lower asymptote, R_0, is the minimum radius at the ends of the bobbin. The upper asymptote, R_f determines the maximum allowable radius of the bobbin at the center. R_0 is one of the design parameters of the coil. Here C and B determine the location of the maximum growth rate of $R(z)$. The growth rate in Equation (5) of the logistic curve is slow initially and it increases to the maximum at the point where $z = \frac{C}{B}$, and $R = \frac{R_f+R_0}{2}$. Hence, $\tan\theta = (\frac{dR}{dz})_{\max} = B\frac{(R_f-R_0)}{4}$ should not exceed μ for the mechanical stability of the windings. Therefore, B is restricted by the coefficient of static friction μ as given in (6). The end field concentrator region of this solenoid is from $\frac{C}{B}$ to 0 as $\frac{dR}{dz}$ reduces at that region.

$$B = 4\frac{\mu}{R_f - R_0} \tag{6}$$

3. OPTIMIZATION OF η

The basic parameters for any engineering design evolve from the requirement specifications. In our case R_0, μ, η (desired) and L (length of the solenoid) are the fundamental required parameters for the design of the miniature solenoid. Based on this input parameters our aim is to find suitable values of R_f and C to achieve the desired η. Using the logistic function from Equation (3), Pareto optimal situation can be achieved. Keeping in mind the feasibility of windings over the bobbin and the dimensions of miniature coil, the optimization of R_f and C is obtained from the objective function. GA algorithm is inspired by the mechanisms of the natural evolution, are usually effective in rapidly searching for the global optimum when a number of design variables need to be adjusted. This multiobjective optimization problem consists of a number of objectives is linked with a number of inequality and equality constraints. If optimization function $\phi_i(x)$ has to be optimized as per the fitness limit, the problem can be symbolically expressed as follows.

Maximize or Minimize

$$\phi_i(x) \quad i = 1 \text{ to } N$$

Subjected to

$$\theta_j(x) \quad j \leq 1 \text{ to } J$$
$$\zeta_k(x) \quad k = 1 \text{ to } K$$

where x is a multi-dimensional vector having specific number of design variables.

Here R_f and C are taken from random pool of variables and their ranges are judiciously chosen. B is calculated from (6) which in turn provides the profile of the bobbin and the effective length L_e of the curved profile is obtained from Equation (7).

$$L_e = 2\int_0^{\frac{L}{2}} \sqrt{1 + \left(\frac{dR}{dz}\right)^2} dz \tag{7}$$

The number of turns N_{turns} is equal to L_e/d of the coil where d is the diameter of the wire used to wound the coil. A simplified flow chart for GA computation is given in the Figure 3. From the generated profile the η is calculated from Equation (1). This process is iterated to arrive at the desired η. Figure 4 shows the axial field distribution and the optimum profile of the bobbin obtained by GA using MATLAB.

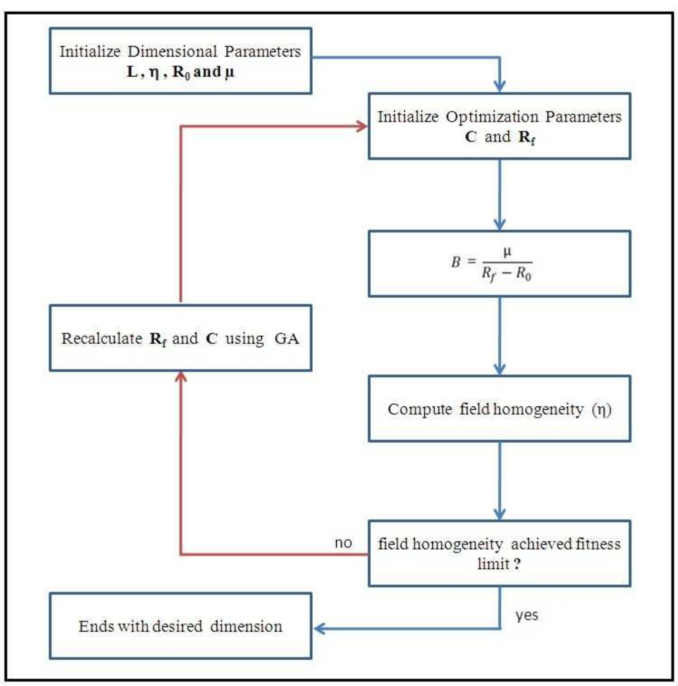

Figure 3. Flow chart for the optimization of the parameters R_f and C.

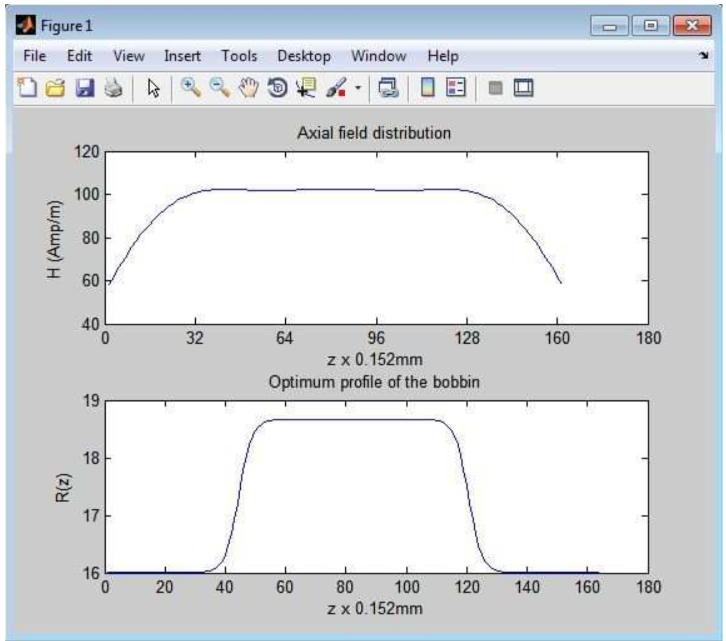

Figure 4. Axial field distribution and the optimum profile of the bobbin obtained by GA using MATLAB. The abcisa is in terms of the width (0.152 mm) of the wire (38 SWG). The values $R_f = 8.28$ and $C = 15.4$ as obtained by GA optimization.

4. DESIGN VALIDATION

We have constructed a miniature solenoid coil of length of 25 mm and $R_0 = 5$ mm. If the ratio of the diameter (d) of the wire to the coil-diameter (D) is of the order of 10^{-3} then it can safely be considered that d is infinitesimal small. The coil has (38 SWG) 164 numbers of turns, the dc resistance of the coil is 34.5 Ohms and $\frac{d}{D} = 15.2 \times 10^{-3}$. The GA optimization technique has estimated the parameters $R_f = 8.28$ mm and $C = 15.4$ respectively. FEMM simulation of the coil results $\eta = 88\%$ when volume of interest is 80% of total volume. η improves reasonably (around 97%) if we consider the volume of interest to be 60% of the total volume. The designed parameters have been validated before fabrication of the bobbin by finite element method [11]. Axis symmetric modeling is done considering the estimated dimension of the bobbin. The uniformity of the magnetic field from finite element simulation is shown in pseudo color in Figure 5. In Figure 6 we have demonstrated the axial uniformity inside the coil derived from the simulation. The figure indicates the uniformity of the field in the central part of the coil.

5. DESIGN VERIFICATION

The uniformity of the magnetic field for the constructed coil has also been verified by measuring axial magnetic field by a magnetic field sniffer-set (Model No. SPECTRAN NF5035 manufactured by Aaronia AG, Germany). The coil has been excited by alternating current of 10 mA and frequency 600 kHz. The measurements made by the magnetic field probe is compared with the computed results. To measure the axial field, the probe was positioned along the z-axis of the coil using a solid non-magnetic fixture outside the coil. The probe was inserted from one end into its central hole to the other end along the axis of the solenoid with a help of a slide caliper. The field was measured throughout the length of the of the coil axis by shifting the fixture along the axis in 1 mm increments. Figure 7 shows the complete experimental setup used and the fabricated bobbin for the construction of the miniature solenoid. Figure 8 shows the experimental results obtained from this setup.

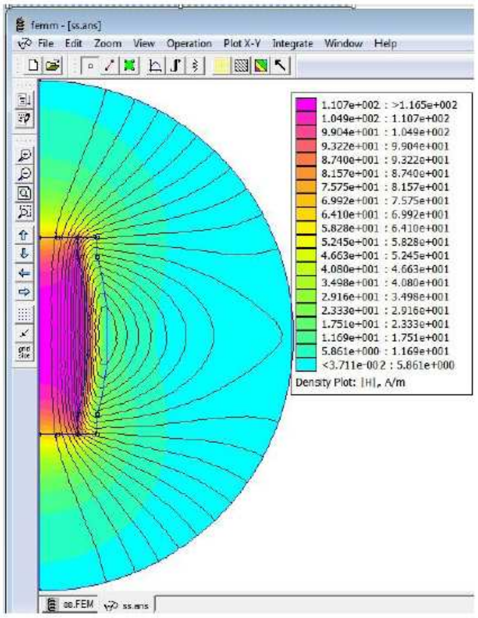

Figure 5. FEMM plot demonstrates uniformity of magnetic field at 600 kHz.

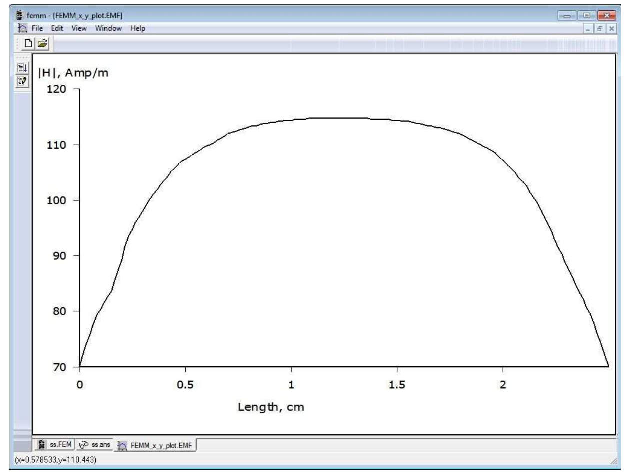

Figure 6. Axial homogeneity of the magnetic field obtained from FEMM.

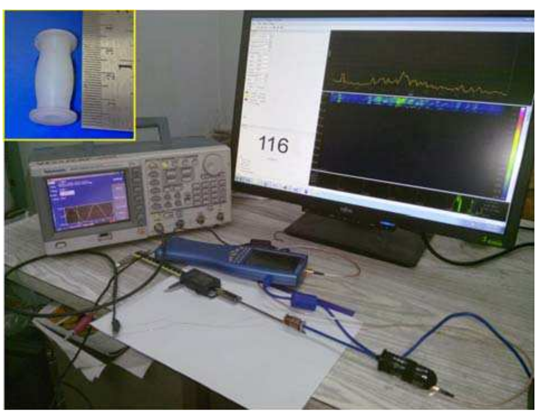

Figure 7. Experimental setup to measure the axial field distribution by SPECTRAN Analyzer. Inset of the figure shows the bobbin (at the left top corner).

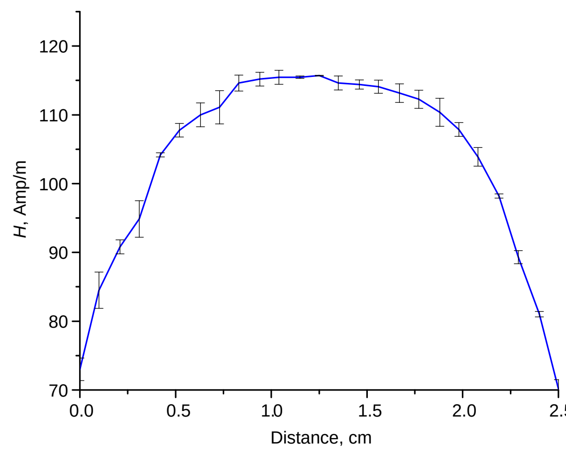

Figure 8. The axial distribution of the magnetic field measured in the experimental setup shown in the Figure 7.

6. CONCLUSION

We have presented a generalized design procedure for the generation of uniform magnetic field inside a miniature cylindrical coil. Optimizing technique of the design process is addressed in detail. Central field uniformity and low stray field is achieved inside the coil. η improves to 97% if we consider the volume of interest to be 60% of the total volume. The presented procedure is very useful in the development of any small magnetic gadgets and it supplements the concept of concentrator coils. In tiny eddy current probe design, ferrite cores are used to focus the field lines. In that case the probe becomes non-linear over frequency due the presence of ferrite core. The focusing achieved by our geometry modification technique on air-core based probe has linearity response over a wide range of frequency. Our shape modified coil has been used in a table top susceptometer for the measurement of susceptibility of nano-magnetic composites.

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