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Energy Characteristics of a Slot Cut in an Impedance End-Wall of a Rectangular Waveguide and Radiating into the Space over a Perfectly Conducting Sphere

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Abstract—A mathematical model of a spherical antenna excited by a slot cut in an impedance end-wall of a semi-infinite rectangular waveguide was built using a rigorous solution of the problem. Control of energy characteristics is accomplished by changing impedance distributed on the end-wall of the waveguide section. If the waveguide is excited by the wave H_{10}, the wavelength tuning reaches (30–35)%, i.e., about a half of the wavelength range of single mode waveguide regime.

1. INTRODUCTION

Low-power microwave devices controlled by lumped components, e.g., by PIN diodes, are extensively used in modern electronics [1, 2]. Further progress in microwave technology is characterized by using film hybrid circuits. Application of hybrid circuits can significantly improve the technical and operational performance of microwave devices; it determines the device miniaturization and increases efficiency of device functioning in the automatic control mode. However, development of controlled microwave devices with hybrid circuits is hampered by lack of adequate mathematical models. This is especially true for waveguide radiators, in which the controlled film element can be a part of the surface, affecting formation of radiation field. The role of mathematical modeling in developing new microwave devices is very important, since experimental design and optimization of the functional elements are time consuming and expensive processes.

The integration of various elements in a single device demands more complicated mathematical models for devices design and analysis. As a rule, for each element used in combined devices a rigorous numerical-analytical method of analysis can be selected that takes into account geometrical and physical features of a local electrodynamic problem. A direct reconciliation of such methods for finding the total electromagnetic field in most cases is impossible, and a direct numerical simulation is usually required. Conversely, the use of direct numerical simulations for the open surface antenna devices are also become limited by their electrical dimensions. In some cases it is possible to use a general formulation of the problem using approximate one-sided boundary conditions, for example, impedance boundary conditions allowing to reduce the number of coupled electrodynamic volumes that must be taken into account. The impedance boundary conditions are known for ability to exclude a need to define fields inside metal and dielectric structural elements at the stage of problem formulation [3–5].

The paper is aimed at creation of new radiating microwave devices with controllable energy characteristics, namely, a spherical antenna, excited by a narrow slot cut in a semi-infinite impedance end-wall of a rectangular waveguide. The paper is based on a solution of a problem of electromagnetic waves radiation into the space above a sphere through a slot in the end-wall of a semi-infinite rectangular waveguide under conditions that all surface elements are perfectly conducting [6]. Design of the external controller for changing the surface impedance will not be considered in the paper.

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2. PROBLEM FORMULATION AND INTEGRAL EQUATION SOLUTION

Let a fundamental wave H_{10} propagates in a hollow semi-infinite rectangular waveguide with perfectly conducting walls (index V^i) from $z = \infty$ (Fig. 1). The waveguide cross-section is $\{a \times b\}$. A Cartesian coordinate system related to the waveguide is shown in Fig. 1(a). A narrow transverse slot is cut in the waveguide end-wall symmetrically relative to the waveguide's longitudinal axis ($x_0 = a/2$). The width of slot aperture S_i is d and its length is $2L_i$ are such that inequalities $[d/(2L_i)] \ll 1$, $[d/\lambda] \ll 1$ hold (λ is free space wavelength). Constant surface impedance, $\bar{Z}_S = Z_S/Z_0$, normalized to the impedance of free space, $Z_0 = 120\pi$ Ohms is distributed continuously over the internal side of the waveguide end-wall. The slot radiates into the free space outside a perfectly conducting sphere (index V^e). The radius of the sphere is R. A spherical coordinate system, associated with the spherical scatterer (flange) is shown in Fig. 1(c). The geometric center of the slotted element in the Cartesian coordinate system is defined by coordinates ($a/2$, y_0, 0); the coordinates of external aperture S_e center in the spherical coordinate system are (R, $\pi/2$, 0). The length of slot aperture S_e measured along the arc on the sphere is $2L_e$.

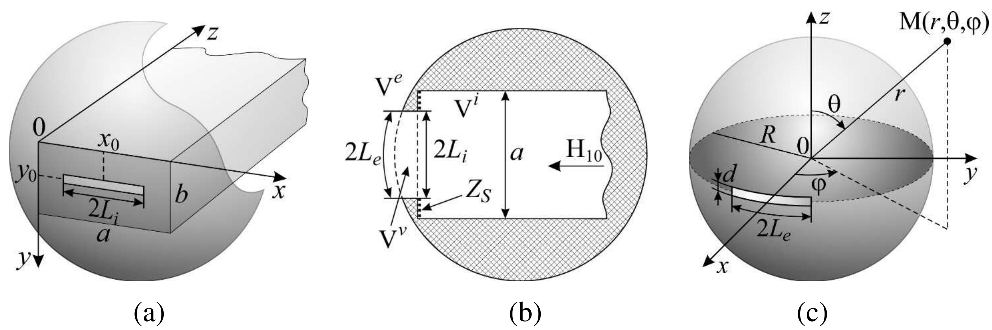


Figure 1. The geometry of the spherical antenna.

The tunneling slot cavity is an area (index V^v), bounded between apertures S_i and S_e, represents an irregular shape resonator, whose boundaries could not be described in either coordinate system (Fig. 1(c)). This defines the principal difficulty for analytical problem analysis. The cavity volume changes as the sphere radius and waveguide cross-sectional dimensions $\{a \times b\}$ are varied, since these geometrical parameters determine the mutual positions of the slot apertures S_i and S_e (Fig. 1(b)).

The initial system of integral equations for the spherical antenna can be formulated using continuity conditions for tangential components of the magnetic fields on the inner and outer slot apertures as

$$\begin{cases} \text{for } S_i: & \vec{H}^i_\tau\left(\vec{e}_{si}\right) + \vec{H}^i_{0\tau} = \vec{H}^v_\tau\left(\vec{e}_{si}\right) + \vec{H}^v_\tau\left(\vec{e}_{se}\right), \\ \text{for } S_e: & \vec{H}^v_\tau\left(\vec{e}_{si}\right) + \vec{H}^v_\tau\left(\vec{e}_{se}\right) = \vec{H}^v_\tau\left(\vec{e}_{se}\right), \end{cases} \tag{1}$$

where $\vec{e}_{si}$, $\vec{e}_{se}$ are the electric fields on the surfaces S_i and S_e; $\vec{H}^i_\tau(\vec{e}_{si})$, $\vec{H}^e_\tau(\vec{e}_{se})$, $\vec{H}^v_\tau(\vec{e}_{si})$, $\vec{H}^v_\tau(\vec{e}_{se})$ are tangential components of magnetic fields with respect to the slot aperture, excited by fields $\vec{e}_{si}$, $\vec{e}_{se}$, in the corresponding electrodynamic volumes, and $\vec{H}^i_{0\tau}$ is the component of the extraneous magnetic field in the waveguide.

A rigorous mathematical justification concerning reduction of simultaneous Equation (1) to a single equation

$$\vec{H}^i_\tau\left(\vec{e}_{si}\right) + \vec{H}^i_{0\tau} = \vec{H}^e_\tau\left(\vec{e}_{se}\right) \tag{2}$$

can be found in [6]. The equation does not contain fields, defined in the slot cavity V^v and it is written for a fictitious slot aperture whose equivalent width is d_e. The magnetic fields in Equations (1) and (2) can be expressed in local coordinate system for each of the coupled volumes.

We will solve Equation (2), following the procedure described in [6], by the generalized method of induced magnetomotive force (MMF) justified in [7]. Then, under the conditions that the fields $\vec{e}_{si(e)}$ are constant in the direction transverse to the slot, and the parameters $\theta_0 = \pi/2$ and $x_0 = a/2$ are constant, we obtain using the local coordinate systems (Fig. 1)

$$\int\limits_{-L_e/R}^{L_e/R}\int\limits_{\theta_0-\frac{d_e}{2R}}^{\theta_0+\frac{d_e}{2R}}\vec{H}^e_\tau\Big(f(\varphi')\vec{\theta}^0\Big)f(\varphi)\,d\theta d\varphi-\int\limits_{x_0-L_i}^{x_0+L_i}\int\limits_{y_0-d_e/2}^{y_0+d_e/2}\vec{H}^i_\tau\left(f(x')\vec{y}^0\right)f(x)dydx=\frac{1}{I_0}\int\limits_{x_0-L_i}^{x_0+L_i}\int\limits_{y_0-d_e/2}^{y_0+d_e/2}\vec{H}^i_{0\tau}f(x)dydx. \tag{3}$$

In deriving the equality (3) we have made use of the following relations:

$$\vec{e}_{se}=\vec{\theta}^0\frac{I_0}{d_e}\delta(r'-R)f(\varphi')=\vec{\theta}^0\frac{I_0}{d_e}\delta(r'-R)\left[\cos(kR\varphi')\cos\frac{\pi}{a}L_e-\cos kL_e\cos\frac{\pi R\varphi'}{a}\right]$$

in the spherical coordinate system, and

$$\vec{e}_{si}=\vec{y}_0\frac{I_0}{d_e}\delta\left(z'\right)f(x')=\vec{y}_0\frac{I_0}{d_e}\delta\left(z'\right)\left[\cos k\left(x'-\frac{a}{2}\right)\cos\frac{\pi}{a}L_i-\cos kL_i\cos\frac{\pi}{a}\left(x'-\frac{a}{2}\right)\right]$$

in the rectangular coordinate system. Here $k=2\pi/\lambda$ is wavenumber of the free space; $\vec{\theta}_0$, $\vec{y}_0$ are the unit vectors in the corresponding coordinate system, $\delta(x')$ is the Dirac delta function, ω is circular frequency. We took into account that the electromagnetic fields depend on time t as $e^{i\omega t}$.

The complex current amplitude

$$I_0=\frac{1}{Y^i+Y^e}\int\limits_{a/2-L_i}^{a/2+L_i}\int\limits_{y_0-d_e/2}^{y_0+d_e/2}H^i_{0x}f\left(x\right)dydx \tag{4}$$

can be found as solution of Equation (3). The slot conductivities in corresponding electrodynamic volumes are

$$Y^e=\int\limits_{-L_e/R}^{L_e/R}\int\limits_{\frac{\pi}{2}-\frac{d_e}{2R}}^{\frac{\pi}{2}+\frac{d_e}{2R}}H^e_\varphi\left(f(\varphi')\vec{\theta}^0\right)f\left(\varphi\right)d\theta d\varphi,\quad Y^i=-\int\limits_{a/2-L_i}^{a/2+L_i}\int\limits_{y_0-d_e/2}^{y_0+d_e/2}H^i_x\left(f(x')\vec{y}^0\right)f\left(x\right)dydx. \tag{5}$$

The external conductivity on the sphere is equal

$$\begin{aligned}Y^e&=Y^e(kd_e,kL_e,kR)=-\frac{4kR}{(kd_e)^2}\sum_{n=1}^{\infty}\frac{1}{n(n+1)}\times\frac{1}{(n+1)-kRh^{(2)}_{n+1}(kR)/h^{(2)}_n(kR)}\\&\quad\times\left\{k^2R^2C_0^2\left(A_n^0\right)^2-2\sum_{m=1}^{n}C_m^2\left[m^2\left(n(n+1)-k^2R^2\right)\left(B_n^m\right)^2-k^2R^2\left(A_n^m\right)^2\right]\right\},\end{aligned} \tag{6}$$

where

$$A_n^m\approx\sin\theta\left[\bar{P}_n^m\left(\cos\left(\theta_0+\frac{d_e}{2R}\right)\right)-\bar{P}_n^m\left(\cos\left(\theta_0-\frac{d_e}{2R}\right)\right)\right],\quad B_n^m=\int\limits_{\theta_0-\frac{d_e}{2R}}^{\theta_0+\frac{d_e}{2R}}\bar{P}_n^m(\cos\theta)\,d\theta,$$

$$\begin{aligned}C_m&=\frac{\cos(\pi L_e/a)}{m^2-(kR)^2}\left[m\sin\frac{mL_e}{R}\cos kL_e-kR\cos\frac{mL_e}{R}\sin kL_e\right]\\&\quad-\frac{\cos kL_e}{m^2-(\pi R/a)^2}\left[m\sin\frac{mL_e}{R}\cos\frac{\pi L_e}{a}-\frac{\pi R}{a}\cos\frac{mL_e}{R}\sin\frac{\pi L_e}{a}\right]=C_m^I-C_m^{II},\\C_m^I\big|_{m\to kR}&=\left(\frac{L_e}{2R}+\frac{\sin(2kL_e)}{4kR}\right)\cos\frac{\pi L_e}{a},\quad C_m^{II}\big|_{m\to\frac{\pi R}{a}}=\left(\frac{L_e}{2R}+\frac{\sin(2\pi L_e/a)}{4\pi R/a}\right)\cos kL,\end{aligned}$$

$\bar{P}_n^m(\cos\theta)=\frac{(2n+1)(n-m)!}{2\pi(n+m)!}P_n^m(\cos\theta)$ are associated Legendre functions of the first kind; $h_n^{(2)}(kr)=\sqrt{\frac{\pi}{2kr}}H^{(2)}_{n+1/2}(kr)$ are the spherical Hankel function of the second kind; $H^{(2)}_{n+1/2}(kr)$ are Hankel function of the second kind [9,10].

The internal slot conductivity in the waveguide section is equal

$$Y^i = Y^i(kd_e, kL_i, \bar{Z}_S) = \frac{4\pi}{ab} \sum_{m=1,3,5\ldots}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_n \left(k^2 - k_x^2\right)}{kk_z} \left(\cos k_y y_0\right)^2 \left(\frac{\sin(k_y d_e/2)}{k_y d_e/2}\right)^2 F_Z(k_z, \bar{Z}_S) g^2(kL_i), \quad (7)$$

where

$$F_Z(k_z, \bar{Z}_S) = \frac{kk_z\left(1+\bar{Z}_S^2\right)}{\left(ik + k_z\bar{Z}_S\right)\left(k\bar{Z}_S - ik_z\right)} \left(1 - i\frac{kk_z\bar{Z}_S}{k^2 - k_x^2}\right), \quad g(kL_i) = I(kL_i)\cos\frac{\pi L_i}{a} - I\left(\frac{\pi L_i}{a}\right)\cos kL_i,$$

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}, \quad k_z = \sqrt{k_x^2 + k_y^2 - k^2}, \quad \varepsilon_n = \begin{cases} 1, & n = 0 \\ 2, & n \neq 0 \end{cases},$$

$$I(kL_i) = \frac{2}{k^2 - k_x^2}\left(k \sin kL_i \cos k_x L_i - k_x \cos kL_i \sin k_x L_i\right),$$

$$I\left(\frac{\pi L_i}{a}\right) = \frac{2}{(\pi/a)^2 - k_x^2}\left(\frac{\pi}{a}\sin\frac{\pi L_i}{a}\cos k_x L_i - k_x\cos\frac{\pi L_i}{a}\sin k_x L_i\right)\Bigg|_{k_x = \frac{\pi}{a}} = \frac{\sin\frac{2\pi L_i}{a} + \frac{2\pi L_i}{a}}{(2\pi/a)}.$$

The expressions (6) and (7) were obtained by replacing the electric field in the slot with equivalent currents and by using Green's functions of Helmholtz equations for the Hertz potentials presented in Appendix A. Details of obtaining the expression (6) can be found in [6]. The expression (7) was derived using methodology presented in works [7, 11], where the expression for external conductivity of the slot cut in the impedance end of the rectangular waveguide and radiating in the half-space above a perfectly conducting plane was given.

If the waveguide section is excited by a fundamental wave

$$H_{10}(x, z) = H_0 \sin\frac{\pi x}{a} e^{-i\gamma z},$$

where H_0 is amplitude and $\gamma = \sqrt{k^2 - (\pi/a)^2}$ is propagation constant, the formula for the magnetic current in the slot aperture may be written as

$$J(s) = -\frac{i\omega}{k^2} H_0 F(kL_i) \frac{\cos ks \cos(\pi L_{i(e)}/a) - \cos kL_{i(e)}\cos(\pi s/a)}{Y^i(kd_e, kL_i, \bar{Z}_S) + Y^e(kd_e, kL_e, kR)}, \quad (8)$$

where

$$F(kL_i) = \frac{2}{\pi} \times \frac{\pi \sin kL_i \cos(\pi L_i/a) - ka\cos kL_i \sin(\pi L_i/a)}{1 - [\pi/(ka)]^2},$$

Here the coordinates $s = R\varphi'$ and $s = x' - a/2$ if the fields are defined outside the sphere and inside the waveguide section, respectively.

Thus, both energy and spatial characteristics can be defined by using the asymptotic solution (8) of the Equation (2) for the magnetic field in the slot. For example, the reflection coefficient in the waveguide can be written as

$$S_{11} = \left\{\frac{1 - (\gamma/k)\,\bar{Z}_S}{1 + (\gamma/k)\,\bar{Z}_S} - \frac{8\pi\gamma\, g^2(kL_i)}{iabk\,(Y^i + Y^e)} \cdot \frac{1 + \bar{Z}_S^2}{1 + (\gamma/k)\,\bar{Z}_S}\right\} e^{-2i\gamma z} \quad (9)$$

and the radiation coefficient as

$$|S_\Sigma|^2 = \frac{P_\Sigma}{P_{10}} = \frac{|I_0|^2}{2}\mathrm{Im}Y^e(kd_e, kL_e, kR), \quad (10)$$

where P_Σ is the mean power radiated through the slot aperture, i.e., flux of a Umov-Poynting vector through the slot, P_{10} the power of H_{10} wave, and $\mathrm{Im}Y^e(kd_e,\, kL_e,\, kR)$ the imaginary part of the external slot conductivity.

If the slot parameter h satisfy the inequality $\frac{h}{d} \geq 1$, the equivalent slot width can be evaluated using the formula [12] $d_e \approx \frac{8d}{\pi}\exp(-\frac{\pi V}{2dS_i} + 1)$ where V is the slot cavity volume, S_i is the square of the internal slot aperture and h is maximal dimension of the tunnel slot cavity in the radial direction adjusted for the thickness of impedance coating of the waveguide end-wall. The loss power P_σ in the impedance coating can be found using the energy balance condition, $|S_{11}|^2 + |S_\Sigma|^2 + P_\sigma = 1$. The same condition will be used for verification of computational procedures during energy parameter estimation for imaginary values of surface impedance $\bar{Z}_S$, when the losses in the impedance element are absent and $P_\sigma = 0$.

3. NUMERICAL RESULTS

Truncation of infinite series in the formula (6) was carried out according to the method proposed in [6]. The maximum value of the summation index was selected so that the value of the imaginary part of the conductivity was calculated with accuracy of 1%. The number of members in double series in the expression for Y^i (7) was selected, as in [7, 8], to provide calculation $|Y^i|$ with accuracy up to 0.1%. To establish correspondence between our mathematical model and the real physical process we have conducted: (1) analytical comparison and (2) test calculations. In the first case, the formulas obtained in this study were compared with corresponding expressions presented in [6], where surfaces of all antenna elements were supposed to be perfectly conducting. It is easy to see that formulas (6), (7), (8) (9) and (10) are transformed to corresponding formulas in [6] if $\bar{Z}_S = 0$. Test calculations were carried out to compare our results with results given in [11]. Comparison of the energy parameters of the two devices with different geometry of the waveguide radiator flange were done in the range $10 \leq kR \leq 50$. Comparisons were made for the radiation coefficient (10) and the absolute value of reflection coefficient (9) in the operating wavelength of standard rectangular waveguide $\{a \times b\} = 23 \times 10\,\mathrm{mm}^2$ for the varying value of complex impedance $\bar{Z}_S$ of a magneto-dielectric film TDK IR-E110 on a metal layer

$$\bar{Z}_S = i\sqrt{\frac{\mu_1}{\varepsilon_1}}tg\left(\sqrt{\varepsilon_1\mu_1}kh_d\right), \tag{11}$$

with material parameters are $\varepsilon_1 = 8.84 - i0.084$, $\mu_1 = 2.42 - 24.75/\lambda - i0.994$ [13]. The maximum difference between the calculated values of the energy parameters for both cases did not exceed 2.5%. The loss power in the impedance surface of the waveguide end-wall increases with the increase of $\bar{Z}_S$ both for a flat screen [11] and for the spherical antenna with various values of sphere radiuses. Thus, the overall slot radiation coefficient, $|S_\Sigma|^2$, is decreased. We have also concluded that variation of the real part of the surface impedance $\mathrm{Re}(\bar{Z}_S)$ does not change the antenna resonant frequency at which a maximum level of radiated power is observed. Therefore, to determine the possibility of frequency antenna tuning we will considered only pure imaginary impedance ($\mathrm{Re}(\bar{Z}_S) = 0$).

It was shown [6] that maximal radiation level at any given frequency in the range of waveguide single mode (excluding the area adjacent to the critical frequency) can be achieved by variation of the slot length. For a small sphere radii, for example if $\pi R/(2L_e) = 3$, the resonant slot length is close to a half-wave, $2L_e \approx 0.5\lambda$. If the sphere radius is increased, the effect of the sphere shortening becomes apparent. Maximal reduction of the resonant slot length, $2L_e \approx 0.48\lambda$, is observed for an infinite screen. Therefore, for practical applications, there arises an important question: what is the extent of frequency antenna tuning attainable by changing the impedance of the film element? Fig. 2 shows, the wavelength dependence of energy characteristics for the four variants of the spherical antenna which radii are $R = 10L_e/\pi$ and $R = 20L_e/\pi$ and two fixed slot lengths are $2L_e = 14\,\mathrm{mm}$ and $2L_e = 16\,\mathrm{mm}$. In all cases, the calculations were made in the single-mode range of the waveguide section with cross section $\{a \times b\} = 23 \times 10\,\mathrm{mm}^2$. It was assumed that the value of the surface impedance do not change its sign and always is the inductive-type impedance for the lossless magneto-dielectric layer.

Figure 2 shows that the maximum of radiated power is shifted to the long-wavelength limit if the value of impedance $\bar{Z}_S$ is increasing. The variation interval of $\bar{Z}_S \in [0; i0.2]$ ensures resonant wavelength tuning within (30–35)% as compared with the case when $\bar{Z}_S = 0$. Although the tuning range covers almost half the range of single-mode regime of the waveguide section, the antenna reflection coefficient is not increased and the level of the maximum radiation coefficient is not reduced. Thus, we can affirm the antenna and waveguide transmission line are matched in the entire tuning range. If the sphere radius is increased, a minor decrease of the tuning range is observed. For any value of the impedance $\bar{Z}_S$, the bandwidth of the antenna radiation at the half-power level is maximal for large sphere radii and significantly reduces when the sphere is decreased. Fig. 3 illustrates an example of this trend for the fixed impedance value $\bar{Z}_S = i0.05$.

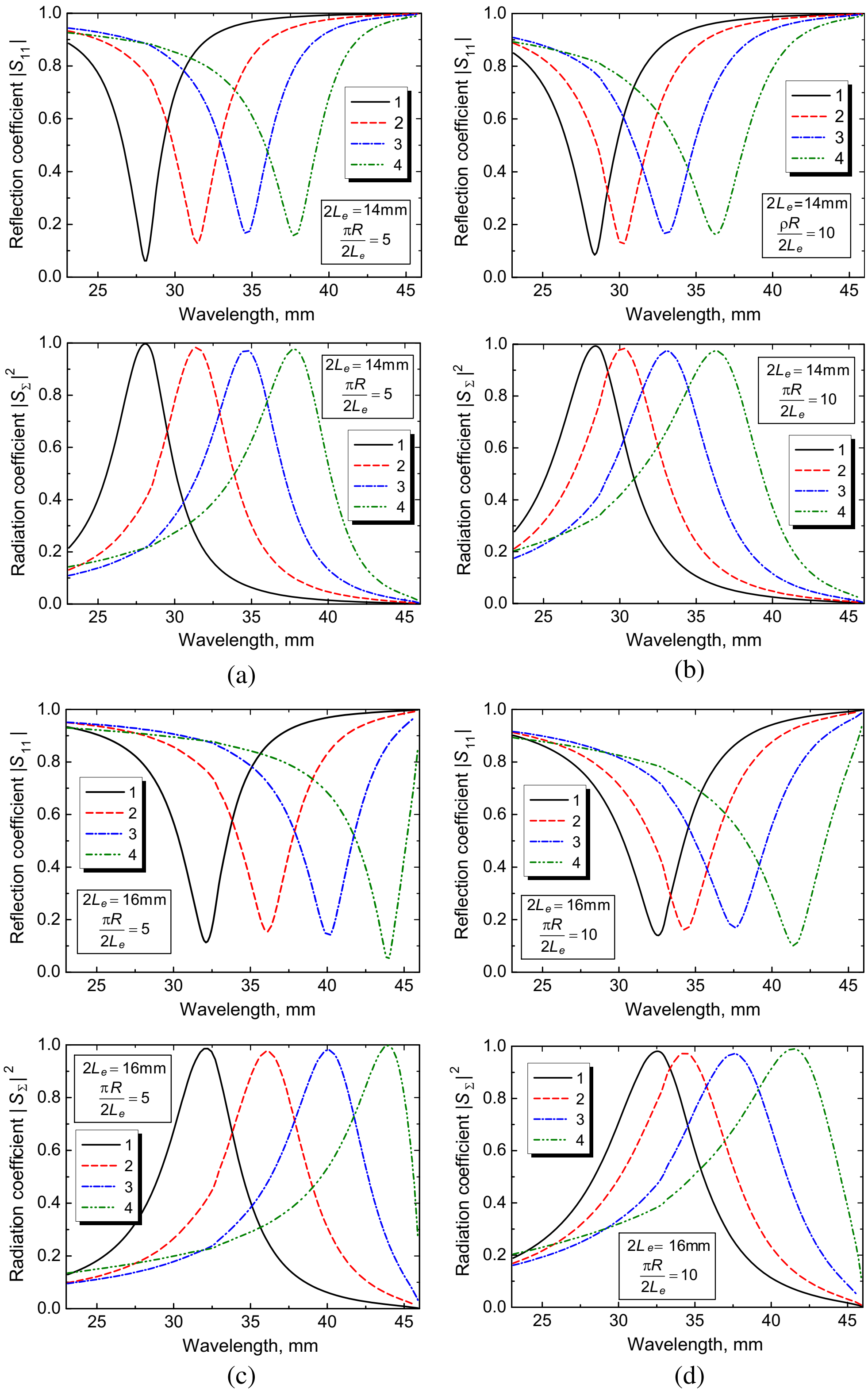
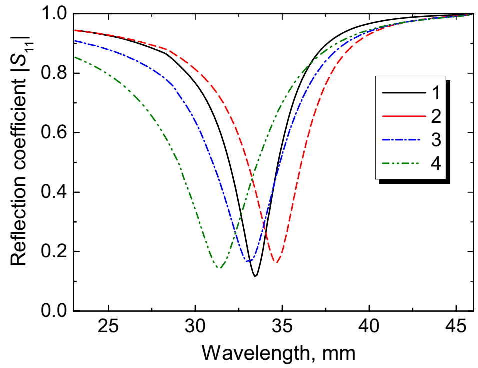


Figure 2. Wavelength dependence of energy characteristics for the spherical slot antenna for variable imaginary impedance: $1 - \bar{Z}_S = 0$, $2 - \bar{Z}_S = i0.01$, $3 - \bar{Z}_S = i0.05$, $4 - \bar{Z}_S = i0.2$.



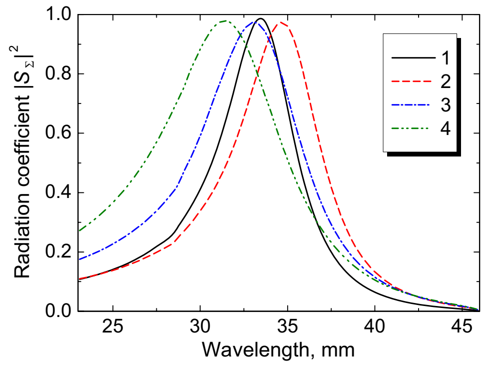


Figure 3. The wavelength dependence of the energy characteristics of the spherical slot antenna for various sphere radii, $2L_e = 14\,\text{mm}$, $\bar{Z}_s = i0.05$: $1 - \pi R/2L_e = 3$, $2 - \pi R/2L_e = 5$, $3 - \pi R/2L_e = 10$, $4 - \pi R/2L_e = 20$.

4. CONCLUSION

The electrodynamic problem of electromagnetic wave radiation into the space over the perfectly conducting sphere through the narrow slot aperture cut in the semi-infinite impedance end-wall of the rectangular waveguide excited by the H_{10} wave was solved in strict formulation by the generalized method of induced MMF. This method is a variety of the well-known method of moments, but it can be classify as a separate method due to specific selection of basis functions. A single basis function is defined as the analytical solution of the integral equation, obtained by the asymptotic method of averaging for the problem of wave diffraction by a transverse slot cut in the end-wall of a semi-infinite rectangular waveguide radiating into the half-space over the perfectly conducting screen. The advantage of the method is in using of two different local coordinate systems for the coupled electrodynamic volumes. Thus, the fields in the space outside the spherical scatterer and inside the waveguide section can be found using the corresponding Green's functions.

Conceptually, the impedance of the waveguide end-wall may be controlled by film coating, distributed over it. The parameters of the magneto-dielectric coating and, hence, the value of the surface impedance can be changed by hypothetical external action. The article is aimed mainly at the investigation of possibility for a spherical antenna energy characteristics control by changing the values of the impedance of the end-wall cover. Analysis of test calculations obtained using the mathematical model and comparison test results for special cases with that known previously confirmed the model validity. Thus, we may state that the energy characteristics of the spherical antenna can be controlled in a fairly wide range by varying the impedance of the end-wall covering of the waveguide section. It was found the wavelength tuning of the resonant antenna radiation is possible within (30–35)%. This result proves the efficiency of the method for energy characteristics control of the spherical antenna. The proposed mathematical model can be directly used in the antenna development and design.

APPENDIX A. MAGNETIC DYADIC GREEN'S FUNCTIONS FOR VARIOUS ELECTRODYNAMIC VOLUMES

1. For a semi-infinite rectangular waveguide with perfectly conducting walls

$$\begin{aligned}\hat{G}^m(\vec{r},\vec{r}') = \frac{2\pi}{ab}\sum_{m,n}\frac{\varepsilon_m\varepsilon_n}{k_z}\Big\{&(\vec{e}_x\otimes\vec{e}_{x'})\,\Phi_x^m\left(x,y;x',y'\right)\left[e^{-k_z|z-z'|}+e^{-k_z(z+z')}\right]\\ &+\left(\vec{e}_y\otimes\vec{e}_{y'}\right)\Phi_y^m\left(x,y;x',y'\right)\left[e^{-k_z|z-z'|}+e^{-k_z(z+z')}\right]\\ &+\left(\vec{e}_z\otimes\vec{e}_{z'}\right)\Phi_z^m\left(x,y;x',y'\right)\left[e^{-k_z|z-z'|}-e^{-k_z(z+z')}\right]\Big\}.\end{aligned} \tag{A1}$$

2. For a semi-infinite rectangular waveguide with impedance end in the case where impressed sources are located on the end-wall surface

$$\begin{aligned}\hat{G}^m\left(\vec{r},\vec{r}'\right) &= \frac{2\pi}{ab}\sum_{m,n}\frac{\varepsilon_m\varepsilon_n}{k_z}\left\{(\vec{e}_x\otimes\vec{e}_{x'})\,\Phi_x^m\left(x,y;x',y'\right)f_{\mathrm{II}}\left(k_z,\bar{Z}_S\right)2e^{-k_z z}\right.\\ &\left.+\left(\vec{e}_y\otimes\vec{e}_{y'}\right)\Phi_y^m\left(x,y;x',y'\right)f_{\mathrm{II}}\left(k_z,\bar{Z}_S\right)2e^{-k_z z}\right\},\end{aligned}\tag{A2}$$

where

$$f_{\mathrm{II}}\left(k_z,\bar{Z}_S\right)=\frac{kk_z\left(1+\bar{Z}_S^2\right)}{\left(ik+k_z\bar{Z}_S\right)\left(k\bar{Z}_S-ik\right)}.$$

The following notations are adopted in expressions (A1), (A2):

$$\begin{aligned}&\Phi_x^m\left(x,y;x',y'\right)=\sin k_x x\sin k_x x'\cos k_y y\cos k_y y',\\ &\Phi_y^m\left(x,y;x',y'\right)=\cos k_x x\cos k_x x'\sin k_y y\sin k_y y',\\ &\Phi_z^m\left(x,y;x',y'\right)=\cos k_x x\cos k_x x'\cos k_y y\cos k_y y',\\ &\varepsilon_{m,n}=\begin{cases}1, & m,n=0\\ 2, & m,n\neq 0\end{cases},\quad k_x=\frac{m\pi}{a},\quad k_y=\frac{n\pi}{b},\quad k_z=\sqrt{k_x^2+k_y^2-k^2},\end{aligned}$$

m and n are integers; $\bar{Z}_S$ is the normalized surface impedance; $\vec{e}_x$, $\vec{e}_y$, and $\vec{e}_z$ are the unit vectors of the Cartesian coordinate system fixed to the waveguide; "$\otimes$" stands for dyadic product.

3. For the space outside a perfectly conducting sphere of radius R [3, 9]

$$\begin{aligned}\hat{G}^m\left(\vec{r},\vec{r}'\right) &= \left(\vec{e}_\theta\otimes\vec{e}_{\varphi'}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{mu_n\left(r,r'\right)\sin m\left(\varphi-\varphi'\right)}{n\left(n+1\right)C_{nm}}\\ &\times\left[\frac{dP_n^m\left(\cos\theta\right)}{d\theta}\times\frac{P_n^m\left(\cos\theta'\right)}{\sin\theta'}+\frac{P_n^m\left(\cos\theta\right)}{\sin\theta}\times\frac{dP_n^m\left(\cos\theta'\right)}{d\theta'}\right]\\ &-\left(\vec{e}_\varphi\otimes\vec{e}_{\varphi'}\right)\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{\left(2-\delta_{om}\right)u_n\left(r,r'\right)\cos m\left(\varphi-\varphi'\right)}{2n\left(n+1\right)C_{nm}\sin\theta\sin\theta'}\\ &\times\left[m^2P_n^m\left(\cos\theta\right)P_n^m\left(\cos\theta'\right)+\sin\theta\sin\theta'\frac{dP_n^m\left(\cos\theta\right)}{d\theta}\times\frac{dP_n^m\left(\cos\theta\right)}{d\theta}\right],\end{aligned}\tag{A3}$$

where $(\vec{e}_r,\ \vec{e}_\theta,\ \vec{e}_\varphi)$ are the unit vectors of the spherical coordinate system, $C_{nm}=\frac{2\pi(n+m)!}{(2n+1)(n-m)!}$, $u_n(r,r')=\begin{cases}kh_n^{(2)}(kr')[h_n^{(2)}(kr)\bar{Q}_n(y_n(kR))-y_n(kr)], & R\leq r<r';\\ kh_n^{(2)}(kr)[h_n^{(2)}(kr')\bar{Q}_n(y_n(kR))-y_n(kr')], & r>r',\end{cases}$ $\bar{Q}_n(y(kR))=\frac{kRy_{n-1}(kR)-ny_n(kR)}{kRh_{n-1}^{(2)}(kR)-nh_n^{(2)}(kR)}$; $P_n^m(\cos\theta)$ is the associated Legendre function; $h_n^{(2)}(kr)=j_n(kr)-iy_n(kr)=\sqrt{\frac{\pi}{2kr}}H_{n+1/2}^{(2)}(kr)$ is the spherical Bessel function of the second kind; $j_n(kr)=\sqrt{\frac{\pi}{2kr}}J_{n+1/2}(kr)$ is the spherical Bessel function; $y_n(kr)=\sqrt{\frac{\pi}{2kr}}N_{n+1/2}(kr)$ is the spherical Neumann function; $J_{n+1/2}(kr)$, $N_{n+1/2}(kr)$ and $H_{n+1/2}(kr)$ are the Bessel, Neumann and Hankel function of half-integer indexes.

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