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# Excitation of Azimuthal Surface Waves in Toroidal Waveguide by Rotating Electron Beam at the Range of Electron Cyclotron Resonance

**Volodymyr O. Girka***, **Igor O. Girka**, **Alexander V. Kostenko**, **and Ivan V. Pavlenko**

**Abstract**—Azimuthal Surface Waves (ASWs) are electromagnetic waves of the surface type, which propagate across an external steady magnetic field in plasma filled metal waveguides. The interaction between extraordinary ASWs and an electron beam that rotates along Larmor orbits in the gap between the plasma column and the metal wall is studied here. The initial stage of the ASW excitation is studied analytically and numerically. Growth rates of the ASW beam instability are analyzed as functions of the parameters of the plasma filled waveguide immersed in a steady magnetic field with toroidal nonuniformity. This nonuniformity leads also to the appearance of corrections to the ASW eigen frequencies. It is shown that the beam-wave interaction in a toroidally nonuniform steady magnetic field is not weaker than in the case of a uniform magnetic field. However, in the studied case, the efficiency of the power transfer from the beam into the excited waves becomes restricted due to the electron drift in the nonuniform magnetic field.

## 1. INTRODUCTION

Electronic devices with plasma filling are often used to generate and/or enhance electromagnetic radiation [1, 2]. The device geometry, parameters of the plasma filling and magnetic field configuration determine the eigen modes of the devices, which in its turn define the type and frequency of the generated radiation. The problems of construction of the plasma electronics devices can be divided into two types: studying the eigen modes of the devices and development of methods of the eigen mode excitation. This paper considers the second type problems, when the eigen modes of the electronic device which applies an external magnetic field are excited by an electron beam.

Doing that it is supposed to utilize ring electron beams, which rotate along large Larmor orbits in plasma filled waveguides. Ring beams of the charged particles [3] are widely used to excite electromagnetic waves for different purposes of plasma electronics, for example for elaboration free electron lasers [4] and for development of technologies for their application [5, 6]. Generally, electrons both gyrate across applied magnetic field lines and move along them. But if the longitudinal motion of electrons is not important for an excitation problem, such beams will be called as rotating beams.

The electronic devices based on a rotating beam are expected to have higher efficiency in comparison with devices based on the longitudinal ones, and are also expected to be more compact. One of the typical constructions of electronic devices based on rotating beams uses the electrons gyrating along the Larmor orbits in a gap between the chamber wall and the plasma column. The electrons transfer their energy to the electromagnetic waves as long as the particles approach the plasma surface as a result of their deceleration. In rotating beams, the particles can pass a way which is much larger than a size of the device, while the available efficiency of generators based on longitudinal beams is limited usually by the device length. For example, the instability growth rates and the efficiency of annular lasers based on rotating electron beams are larger (in $\gamma^{2/3}$ times) in comparison with the case of electron beams

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* Corresponding author: Volodymyr Oleksandrovych Girka (v.girka@gmail.com).
The authors are with the V.N. Karazin Kharkiv National University, Svobody Sq. 4, Kharkiv 61022, Ukraine.

propagating in the axial direction [4], (here, $\gamma$ is the relativistic factor). Also, electronic devices based on the rotating beams can generate electromagnetic radiation at different harmonics of the electron cyclotron frequency (up to the 20th harmonic) [6]. Their efficiency can reach values from one percent for small orbit gyrotrons to 20 percents for large orbit ones.

The excitation of the asymmetric traveling waves by the rotating electron beams has good prospective applications for radar systems and broadcast telecommunications as a high power efficient amplifier [7]. A modified relativistic (500 kV, 500 A) backward wave oscillator filled with a radially nonuniform preionized high density plasma is studied experimentally in [8]. The effects of high plasma density in the interaction region as well as the relativistic diode and the output horn regions are tested here. It is demonstrated [8] that filling the backward wave oscillator with high density radially nonuniform plasma leads to the following results: - suppression of high-order modes (so one can apply a single mode approach for theoretical description of the device) is proved; - the possibility of operating frequency tunability (from 8.5 to 11 GHz) by controlling the plasma density is demonstrated; - substantial increase in the measured microwave output power over the corresponding vacuum value is observed.

Interaction between eigen modes and rotating electron beams is applied as well for elaboration of new types of gyrotrons and cyclotron masers [9]. Authors of the paper [10] reported on successful single-mode operation of a prototype of large-diameter SW generator that utilized a moderate voltage electron beam for generation of Giga-Watt radiation. They have concluded that additional optimization of the extraction section, which was supplied with a coaxial waveguide and output window regions, was required in order to increase the radiated power.

The present paper is devoted to theoretical studying of specific surface waves' excitation by a rotating electron beam. Azimuthal surface waves (ASWs) propagating in a plasma filled cylindrical metal waveguide immersed into an axial magnetic field have been studied in [11] at first. A review of the ASW properties has been done in [12]. As far as ASWs are eigen modes of the waveguides filled with magnetized plasma, they can be effectively excited by an electron beam. But an applied magnetic field can be spatially nonuniform one. Influence of magnetic field nonuniformity on ASWs properties was studied in [12]. A magnetic field nonuniformity effects on features of charged particle beams propagating in a waveguide with such magnetic field and on its interaction with eigen modes of the waveguide as well. The effect of the external magnetic field nonuniformity on the wave excitation was studied experimentally in a combined axial and wiggler magnetic field [13]. That is why the toroidal nonuniformity of the external steady magnetic field has been chosen here to study its effect on the ASW excitation.

By the aid of theory of successive approximations ASWs propagation across the axis of a toroidal metal magnetized waveguide with plasma filling was studied in [14]. A space distribution of the wave field was obtained there analytically. When the external magnetic field has a toroidal nonuniformity, the ASWs propagate in the waveguide as a wave packet. The amplitudes of the satellite harmonics of the wave packet are much smaller than the amplitude of the main harmonic. An influence of the toroidal nonuniformity of the external magnetic field on the amplitude of the main harmonic is found to be of the second order of smallness with respect to the toroidicity parameter. The correction of the ASW eigen frequency caused by the toroidal nonuniformity of the external magnetic field was calculated in [14] also. It was proven that its value is of the second order of smallness.

In the case of strongly magnetized plasma a possibility of ASW excitation by rotating ion beam was proved theoretically in [15]. But influence of an applied magnetic field nonuniformity on interaction of charged particle beams with ASWs has not been analyzed yet. That is why this paper presents the results of studying the effect of the external magnetic field nonuniformity on the initial stage of the resonance beam-wave instability when the excited low frequency (LF) ASWs propagate in a toroidal metal waveguide filled partially by cold magneto-active plasmas.

The paper is organized as follows. The studied problem is formulated in Section 2. Section 3 presents the results of numerical analysis of the problem. Conclusions of the study are made in Section 4.

## 2. PROBLEM FORMULATION

Let consider a toroidal metal waveguide of a circular cross-section with the radius $b$. The waveguide is filled with circular plasma of the radius $a$ (see Figure 1). The waveguide is assumed to be symmetric along the axis, thus $\partial/\partial\zeta = 0$, in the right-handed quasi-toroidal coordinate system. The poloidal angle $\vartheta$ is measured from the direction to the symmetry center of the torus. Since the effect of the radial nonuniformity of the plasma density on the ASW dispersion properties has been studied in [12], then we do not consider this effect in the present study, and the plasma density is assumed to be uniform. This assumption is valid in particular for gas discharges sustained by surface waves. Such types of discharges are widely used in plasma technologies [16] where the plasma is uniform radially at distances which are of the order of the wave penetration depth.

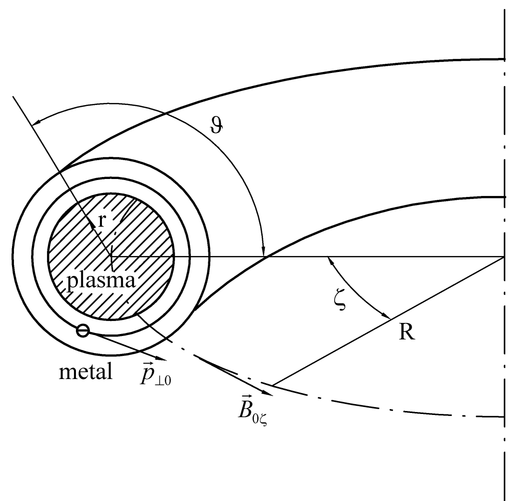


**Figure 1.** Scheme of the problem.

The analytical expression for the external confined magnetic field is as follows:

$$B_{0\zeta} = B_0/[1 - (r/R)\cos\vartheta], \tag{1}$$

where $R$ — large radius of the torus, $r$ — radial coordinate. An extraordinary wave with the electromagnetic field components $E_r$, $E_\vartheta$, $H_\zeta$ is under consideration here because the ordinary surface wave with the electromagnetic field components $E_\zeta$, $H_\vartheta$, $H_r$ does not propagate in the waveguides with a narrow gap between the plasma column and metal chamber [12]. In semiconductor physics, these types of surface waves are called magneto plasma polaritons. The presented direction of the external magnetic field to the plasma-metal interface is recognized as Voigt geometry. Generally application of surface waves in solid state plasmas has good prospects in photonics and plasmonics [17, 18].

Main achievements obtained in the development of surface waves theory are connected with application of fluid model of plasma; and theory of ASW propagating in non-uniform waveguides is created using the fluid model as well [12]. That is why here this model of plasma is applied once again. In this case, the relation between the electric induction and the electric field intensity is provided by the permittivity tensor of cold magnetized plasma with weak collisions. The following two components of the tensor will be used further:

$$\varepsilon_{11} = \varepsilon_0 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - \omega_{c\alpha}^2} \equiv \varepsilon_1, \tag{2}$$

$$\varepsilon_{12} = i \sum_\alpha \frac{\omega_{c\alpha}\omega_{p\alpha}^2}{\omega\left(\omega^2 - \omega_{c\alpha}^2\right)} \equiv i\varepsilon_2, \tag{3}$$

where $\omega_{p\alpha}$ and $\omega_{c\alpha}$ are plasma and cyclotron frequencies of plasma species $\alpha$, magnetic field, which determines value of the cyclotron frequencies, can be applied in relations (2) and (3) in the form of the expression (1) $B_{0\zeta}$. Plasma density is large enough to provide the inequality $\omega_{pe}^2 > \varepsilon_0\omega_{ce}^2$. For gas plasma $\varepsilon_0 = 1$; for $n$-semiconductor plasma, the dielectric constant of the lattice was defined as $\varepsilon_0 > 1$. Since the range of the electron cyclotron frequencies will be studied here, the ion terms can be neglected both in the expressions (2) and (3).

The form of relations (2) and (3) is inconvenient one because of that one can apply there the expression for the complete magnetic field (1). But in the case of weak toroidicity $\varepsilon_t = a/R \ll 1$, their form can be simplified by expanding these components of dielectric permittivity tensor into a series over the toroidicity parameter $\varepsilon_t$, that is of a small value:

$$\varepsilon_{1,2} = \varepsilon_{1,2}^{(0)} + \varepsilon_{1,2}^{(1)}\cos\vartheta + \varepsilon_{1,2}^{(2)}. \tag{4}$$

The main contributions to (4) don't depend on the coordinates, $\varepsilon_{1,2}^{(0)} = \varepsilon_{1,2|B_{0\zeta}=B_0}$. Therefore, they can be written explicitly:

$$\varepsilon_1^{(0)} = \varepsilon_0 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^{(0)2}}, \quad \varepsilon_2^{(0)} = \frac{\omega_{ce}^{(0)}\omega_{pe}^2}{\omega\left(\omega^2 - \omega_{ce}^{(0)2}\right)}. \tag{5}$$

The first order terms in (4) are linear in the small radius. Thus, they are proportional to the small parameter of toroidicity $\varepsilon_t$:

$$\varepsilon_1^{(1)} = -2\frac{r}{R}\frac{\omega_{pe}^2\omega_{ce}^{(0)2}}{\left(\omega^2 - \omega_{ce}^{(0)2}\right)^2}, \quad \varepsilon_2^{(1)} = -\frac{r}{R}\frac{\omega_{ce}^{(0)}\omega_{pe}^2\left(\omega^2 + \omega_{ce}^{(0)2}\right)}{\omega\left(\omega^2 - \omega_{ce}^{(0)2}\right)^2}. \tag{6}$$

The second order terms are quadratic in the small radius of torus,

$$\varepsilon_1^{(2)} = \frac{r^2}{2R^2}\frac{\omega_{pe}^2\omega_{ce}^{(0)2}\left(\omega_{ce}^{(0)2} + 3\omega^2\right)}{\left(\omega_{ce}^{(0)2} - \omega^2\right)^3}, \quad \varepsilon_2^{(2)} = \frac{r^2}{2R^2}\frac{\omega_{ce}^{(0)}\omega_{pe}^2\omega\left(\omega^2 + 3\omega_{ce}^{(0)2}\right)}{\left(\omega_{ce}^{(0)2} - \omega^2\right)^3}. \tag{7}$$

The expansion (4) does not contain the second order terms which are proportional to $\exp(2i\vartheta)$ because they issue a correction to the ASW eigen frequency which are of an order higher than the second. The electron cyclotron frequency in the expressions (5)–(7) is defined by the magnetic field value $B_0$ neglecting its nonuniformity (zero approximation).

The components of the ASW electric field are yielded by the Maxwell's equations from the magnetic field component [14], which is a solution of the nonhomogeneous Bessel's equation [19]. The components of the ASW field have to satisfy the following boundary conditions:

1) the wave field has to be a finite value inside the waveguide;
2) the tangential (poloidal in this consideration) component of the electric field is equal to zero on the inner surface of the metal chamber ($E_\vartheta(r = b) = 0$);
3) the tangential components of the electric and magnetic fields have to be continuous on the plasma-vacuum interface including the satellite harmonics.

The ASWs with different numbers of the azimuthal modes propagate independently in the zero approximation over the small parameter of toroidicity $\varepsilon_t$. The amplitude $H_\xi^{(0)}(r)$ of the main harmonic of the axial magnetic field in the zero approximation is proportional to the modified Bessel function $I_m(\xi)$ [19], where $\xi = rk_\perp$. The ASW field penetration depth in plasma $1/k_\perp$ is defined from the following relation,

$$k_\perp^2 = -(\omega/c)^2\,\varepsilon_\perp^{(0)}/\varepsilon_1^{(0)} > 0, \tag{8}$$

where $\varepsilon_\perp^{(0)} = (\varepsilon_2^{(0)})^2 - (\varepsilon_1^{(0)})^2$. The inequality (8) defines the frequency ranges where the ASWs can propagate: $\omega_{LH} < \omega < |\omega_{ce}|$, $|\omega_{ce}| < \omega < \omega_1 - |\omega_{ce}|$, $\omega_{UH} < \omega < \omega_1$. Here, $\omega_{LH}$ and $\omega_{UH}$ are the lower hybrid and upper hybrid frequencies, respectively, and $\omega_1 = 0.5|\omega_{ce}| + \sqrt{\omega_{pe}^2 + \omega_{ce}^2/4}$ is the cut-off frequency. This paper is devoted to studying the case of low LF ASWs, which propagate in the frequency ranges $\omega_{LH} < \omega < |\omega_{ce}|$ and $|\omega_{ce}| < \omega < \omega_1 - |\omega_{ce}|$.

Due to the periodic space nonuniformity of the external magnetic field (see expression (1)) the ASWs propagate as wave packets. Each of the packets contains the main harmonic (the wave field of the main harmonic is proportional to $\exp(im\vartheta - i\omega t)$) and an infinite number of satellite harmonics (the fields of the satellite harmonic change proportionally to $\exp[i(m \pm j)\vartheta - i\omega t]$, where $j = 1, 2, 3, \ldots$). It is known from the theory of wave propagation in media with periodic changes of the properties along the direction of the propagation [20] that the amplitudes of the satellite harmonics are small values (proportional to $(\varepsilon_t)^j$) in comparison with the amplitude of the main harmonic. The wave packet with two nearest satellite harmonics (their wave fields change proportionally to $\exp[i(m \pm 1)\vartheta - i\omega t]$) has been considered in [14] to study the effect of the toroidal magnetic field nonuniformity on the ASW dispersion properties. Considering the higher satellite harmonics yields corrections to the ASW eigen frequency, which are of the third and higher orders of smallness.

The first order terms in $\varepsilon_t$ do not change the amplitude of the main harmonic, but they are the reason for the appearance of the small satellite harmonics of the field which are proportional to $\exp[i(m \pm 1)\vartheta - i\omega t]$. Second order consideration in $\varepsilon_t$ shows that the amplitude of the main harmonic is corrected by the second order term, which together with the amplitude of the first satellite harmonic gives the second order contribution to the dispersion relation $D^{(2)}(\omega, m \ldots) \propto \varepsilon_t^2$. As a result, instead of the zero order dispersion relation $D^{(0)}(\omega, m \ldots) = 0$, one can apply the second order dispersion relation, which describes the present problem: $D^{(0)}(\omega, m \ldots) + D^{(2)}(\omega, m \ldots) = 0$. The explicit expression for $D^{(2)}(\omega, m \ldots)$ is too bulky and therefore it is not presented here because it is out of the scope of present paper, but it is described in detail in [14].

Let us suppose that the rotating electron beam is injected in the gap $b > r > a$ between the plasma column and the waveguide metal wall. The beam is described as a set of the oscillators with the same transverse momentum $p_{\perp 0}$ and zero axial momentum, $p_z = 0$. The plasma-beam system is assumed to be compensated in currents and charges. Such an electron beam is described by the equilibrium distribution function [21]:

$$f_0 = n_b \delta(p_\perp - p_{\perp 0})\delta(p_z)/(2\pi\, p_{\perp 0}), \tag{9}$$

where $p_{\perp 0} = m_e V_{\perp 0}\gamma$ is the electron transverse momentum (the direction of the beam rotation is defined by the direction of the external magnetic field), $\gamma = \sqrt{1 + p_{\perp 0}^2 m_e^{-2} c^{-2}}$ is the relativistic factor, $n_b$ is the beam electron density. The electrodynamics' properties of the waveguide part with the electron beam are described by the permittivity tensor $\varepsilon_{ik}^{(b)}$ [21]. The three components of the tensor $\varepsilon_{ik}^{(b)}$ that will be used below are:

$$\varepsilon_{11}^{(b)} = 1 + \frac{\Omega_b^2}{\omega\gamma} \sum_{s=-\infty}^{+\infty} s^2 \left[ \frac{\left(J_s^2(x)\right)'}{(s-y)\, k_\varphi V_{\perp 0}} + \frac{\omega J_s^2(x)}{(s-y)^2 c^2 k_\varphi^2} \right]; \tag{10}$$

$$\varepsilon_{12}^{(b)} = \frac{i\Omega_b^2}{\omega\,|\omega_e|} \sum_{s=-\infty}^{+\infty} s \left[ \frac{\left(J_s(x)\, J_s'(x)\right)'}{s-y} + \frac{J_s(x)\, J_s'(x)}{(s-y)\, x} + \frac{J_s(x)\, J_s'(x)\, \omega V_{\perp 0}}{(s-y)^2 c^2 k_\varphi} \right] = -\varepsilon_{21}^{(b)}; \tag{11}$$

$$\varepsilon_{22}^{(b)} = 1 + \frac{\Omega_b^2}{\omega\,|\omega_e|} \sum_{s=-\infty}^{+\infty} \left[ \frac{2\left(J_s'(x)\right)^2}{s-y} + \frac{2x J_s'(x)\, J_s''(x)}{s-y} + \frac{\left(J_s'(x)\right)^2 V_{\perp 0}^2 y}{(s-y)^2 c^2} \right]. \tag{12}$$

Here $\Omega_b^2 = 4\pi e^2 n_b m_e^{-1}$, $x = k_\varphi V_{\perp 0}\gamma/|\omega_e|$, $y = \omega\gamma/|\omega_e|$, $k_\varphi = |m| R_1^{-1}$, and $J_s(x)$ is the Bessel function of the first kind. A symbol 'prime' denotes the derivative of the Bessel function with respect to its argument [19].

Solving the Maxwell equations in the space occupied by the beam issues the expressions for the ASW field as a linear combination of the functions $J_m(\zeta)$ and the Bessel functions of the second kind $Y_m(\zeta)$ [19], and their derivatives with respect to their argument $\zeta = kr\sqrt{\psi_b}$, where $\psi_b = \varepsilon_{22}^{(b)} + (\varepsilon_{12}^{(b)})^2 (\varepsilon_{11}^{(b)})^{-1}$.

The application of the discussed above boundary conditions allows us to derive the following dispersion relation, which is the main object of our study in this paper:

$$\frac{I_m'(k_\perp a)k}{k_\perp I_m(k_\perp a)} + \frac{\mu m k a}{k_\perp^2 a^2} + D^{(2)}(\omega, m \ldots) = \frac{im\varepsilon_{12}^{(b)}}{\varepsilon_{11}^{(b)} \zeta_1 \sqrt{\psi_b}} - \frac{J_m'(\zeta_1) - \Phi Y_m'(\zeta_1)}{\sqrt{\psi_b}\left[J_m(\zeta_1) - \Phi Y_m(\zeta_1)\right]}, \tag{13}$$

where

$$\Phi = \left[\frac{im\varepsilon_{12}^{(b)}}{\varepsilon_{11}^{(b)}\zeta_2} J_m(\zeta_2) - J'_m(\zeta_2)\right]\left[\frac{im\varepsilon_{12}^{(b)}}{\varepsilon_{11}^{(b)}\zeta_2} Y_m(\zeta_2) - Y'_m(\zeta_2)\right]^{-1}, \quad \zeta_1 = \zeta(a), \quad \zeta_2 = \zeta(b).$$

Equation (13) is analyzed here, assuming that the LF ASW frequency is close to the $l$-th electron cyclotron harmonic, so the following resonant condition is satisfied:

$$\omega = \omega_0 + \Delta\omega_t = l\,|\omega_e|\,\gamma^{-1} + \Delta\omega_t, \tag{14}$$

where $l$ is a natural number, $\omega_0$ the eigen frequency of the studied extraordinary LF ASWs determined in the case when the electron beam is absent and when the external magnetic field is uniform, and $\Delta\omega_t$ the correction to the wave frequency caused by their interaction with the electron beam in the nonuniform magnetic field.

## 3. NUMERICAL ANALYSIS

The results of numerical analysis of Equation (13), which describes the excitation of the LF ASWs by the electron beam in the case of the toroidal nonuniformity of the external magnetic field, are presented in Figures 2–7. Each of the three couples of the figures: 2 and 3, 4 and 5, 6 and 7 has been built for the same parameters of the beam-plasma system. The ratio of the beam density to the plasma density is the same for all calculations, $n_b/n_p = 10^{-3}$. It was proven numerically that an increase of the beam density leads to the transfer from the excitation regime due to the resonant interaction of the wave with the separated cyclotron harmonics to the regime of resonance overlapping. Curves depicting dependence of LF ASW growth rates $\mathrm{Im}(\omega/|\omega_e|)$ on the effective wave number $k_{ef} = |m|c/(\omega_{pe}a)$ in Figure 8 illustrate exactly this peculiarity of the resonant beam instability of the studied waves.

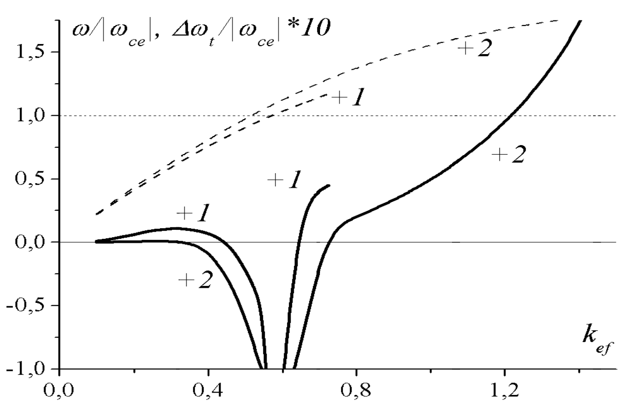


**Figure 2.** Correction to the ASW frequency caused by the toroidicity of the external magnetic field as a function of the effective wave number for different azimuthal mode numbers. The numbers correspond to the values of $m$; $Z = 3$; $\Delta = 0.3$; $\varepsilon_t = 0.1$.

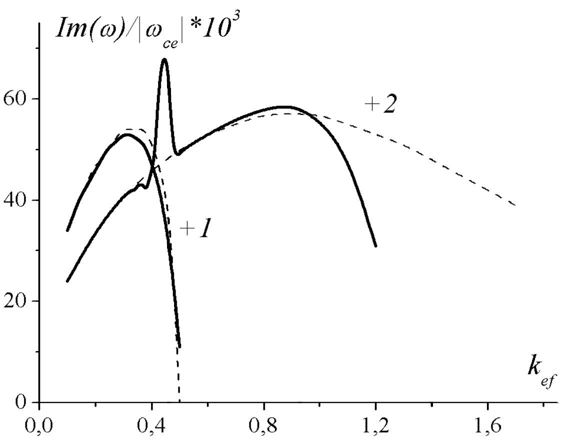


**Figure 3.** The growth rate of the ASW resonance beam instability as a function of the effective wave number for the different values of the azimuthal mode numbers. The parameters of the beam-plasma system are the same as in Figure 2.

Curves presented in Figure 8 are calculated for $Z = \omega_{pe}/|\omega_{ce}| = 10$, $\Delta = b/a - 1 = 0.2$, numbers indicate value of azimuthal mode numbers $m$, solid and dashed lines relate to the cases of a strong beam density $n_b/n_p = 10^{-2}$, and weak beam density $n_b/n_p = 10^{-4}$, correspondingly. Maximum meanings of $\mathrm{Im}(\omega/|\omega_e|)$ are obtained in the case of strong beam density, if $m = +1$, $k_{ef} = 0.075$ then $\mathrm{Im}(\omega/|\omega_e|) = 0.184$; if $m = +2$, $k_{ef} = 0.175$ then growth rate value becomes greater $\mathrm{Im}(\omega/|\omega_e|) = 0.207$. Doing that resonant condition (14) has been satisfied for the first beam's cyclotron harmonic $l = 1$ if $m = +1$ and for the second harmonic $l = 2$ if the mode number $m = +2$. Analyzing solid lines in

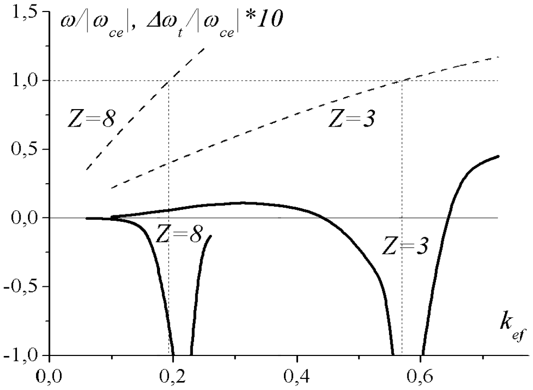


**Figure 4.** The correction to the ASW frequency caused by the toroidicity of the external magnetic field as a function of the effective wave number for different values of the external magnetic fields; $m = +1$; $\Delta = 0.3$; $\varepsilon_t = 0.1$.

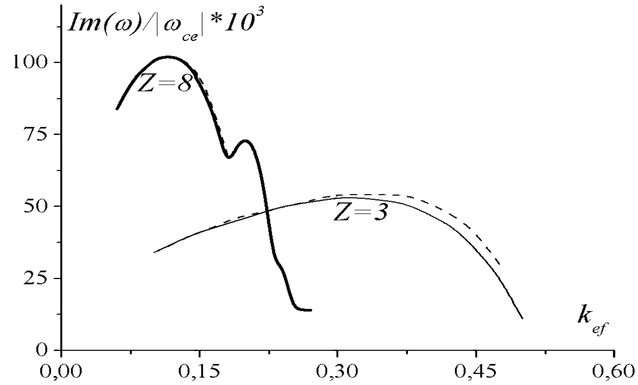


**Figure 5.** The growth rate of the ASW resonance beam instability as a function of the effective wave number for different values of the external magnetic field. The parameters of the beam-plasma system are the same as in Figure 4.

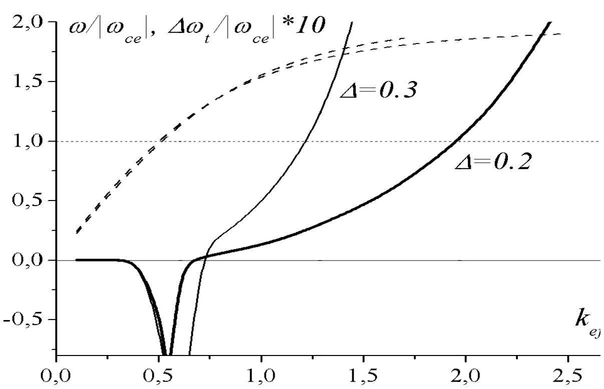


**Figure 6.** The correction to the ASW frequency caused by the toroidicity of the external magnetic field as a function of the effective wave number for different widths of the vacuum gap; $m = +2$; $Z = 3$. In case of $\Delta = 0.3$, the parameter of toroidicity $\varepsilon_t = 0.1$, in case of $\Delta = 0.2$, the parameter of toroidicity $\varepsilon_t = 0.05$.

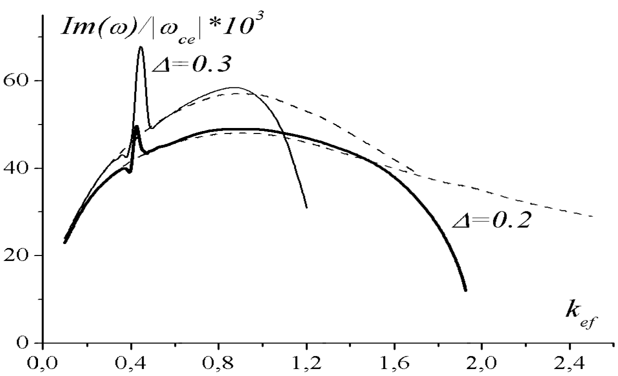


**Figure 7.** The growth rate of the ASW resonance beam instability as a function of the effective wave number for different widths of the vacuum gap. The parameters of the beam-plasma system are the same as in Figure 6.

Figure 8 one can see that there are two points of inflection ($k_{ef}$ is equal to 0.13 and 0.28 for the first and second azimuthal mode numbers, respectively). They indicate transfer from interaction between LF ASW with definite $m$ value and some beam's cyclotron harmonic $l$ value to the interaction between this eigen wave and the next number of the beam's cyclotron harmonic. In the present case it is a transfer from $l = 1$ to $l = 2$ if the LF ASW with $m = +1$ is excited and/or a transfer from $l = 2$ to $l = 3$ if the LF ASW with $m = +2$ is excited. Thus these solid lines show us "single bell" form of $\mathrm{Im}(\omega/|\omega_e|)$ curves, which describe the regime of resonance overlapping. But if we shall decrease the beam density then $\mathrm{Im}(\omega/|\omega_e|)$ curves will be depicted as "double bell" curves. In the present case transfer from the case $n_b/n_p = 10^{-2}$ to the case of weak beam density $n_b/n_p = 10^{-4}$ (see dashed lines in Figure 8) is accompanied not only by decreasing of the maximum $\mathrm{Im}(\omega/|\omega_e|)$ value, for instance from 0.187 to 0.054 for LF ASW with $m = +1$, but also by separation of the maximums of $\mathrm{Im}(\omega/|\omega_e|)$ related to the interaction with different numbers $l$ of the beam's cyclotron harmonics in the space of effective wave numbers $k_{ef}$. The indicated meanings of $\mathrm{Im}(\omega/|\omega_e|)$ are realized for $k_{ef}$ which distinguishes at four

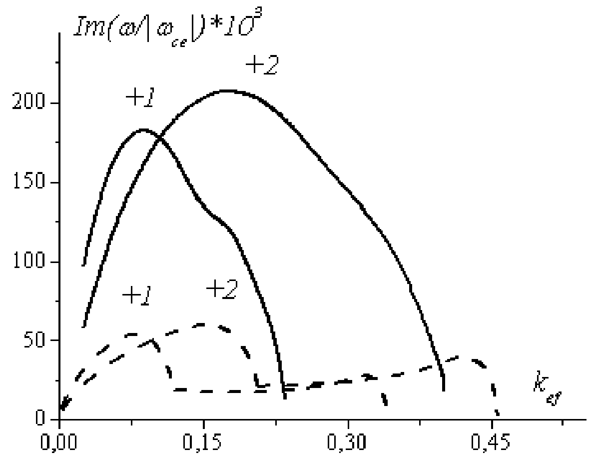


**Figure 8.** The growth rate of the ASW resonance beam instability as a function of the effective wave number for different beam densities. $Z = 10$, $\Delta = 0.2$. Solid lines correspond to strong beam, $\alpha = 10^{-2}$, dashed lines — weak beam, $\alpha = 10^{-4}$.

times, namely interaction with the first cyclotron harmonic takes place for $k_{ef} \approx 0.08$ and interaction of the same wave with the second cyclotron harmonic takes place for case $k_{ef} \approx 0.32$. Thus this regime of the LF ASW instability can be called as regime of interaction between this wave and separated beam's cyclotron harmonics.

The solid lines in Figures 2, 4 and 6 show the dependence of the correction to the LF ASWs frequency caused by the space nonuniformity of the external magnetic field, $\Delta\omega_t = -D^{(2)}(\partial D^{(0)}/\partial\omega)^{-1}_{|\omega=\omega_0}$, as functions of the $k_{ef}$. The correction $\Delta\omega_t$ is normalized there by the absolute value of the electron cyclotron frequency $|\omega_e|$. The dependence of the ASW frequency normalized by the absolute value of the electron cyclotron frequency $\omega/|\omega_e|$ is shown there by the dashed lines for comparison. To make the comparison convenient for readers, in the same graph the value of the correction was multiplied by 10.

The solid lines in the Figures 3, 5 and 7 show the dependence of the growth rate of the ASW resonant beam instability $\mathrm{Im}(\omega/|\omega_e|)$ which is normalized by the absolute value of the electron cyclotron frequency on the $k_{ef}$ value, taking into consideration the toroidal nonuniformity of the external magnetic field. The dependence of the growth rate in the case of a uniform magnetic field is shown there by the dashed lines for comparison.

The numbers +1, +2 in Figures 2 and 3 indicate the values of the azimuthal mode number. The curves for the ASWs with $m$=+1 have been built in Figure 2 for the smaller range of $k_{ef}$ which corresponds to the range in Figure 3 where the growth rate dependence was studied. The curves of the correction diverge near $k_{ef1}$, which corresponds to $\omega \approx |\omega_e|$. This divergence comes from the ratio $(\varepsilon_1^{(2)}/\varepsilon_1^{(0)})|_{\omega\to|\omega_e|} \propto (\omega - |\omega_e|)^{-2}$. The "hole" in the dependence of $\Delta\omega_t(k_{ef})$ is wider for larger values of $|m|$. This discontinuity in the graphs of the correction to the wave frequency is accompanied by a feature (but not a discontinuity) in the graphs of the instability growth rate near the same singular values of $k_{ef1}$. The physical reason for the frequency spectrum discontinuity in the vicinity of the electron cyclotron frequency is the application of the fluid model of plasma, which is not adequate at this frequency range [21].

The toroidicity has little effect on the maximal value of the growth rate, as can be seen from Figure 3, especially for small values of $k_{ef}$. For the ASWs with $m = +1$, the maximal value of the growth rate $\mathrm{Im}(\omega/|\omega_e|) = 0.054$ in the case of the uniform magnetic field is reached when $k_{ef} = 0.35$, and exceeds the maximal value of the growth rate in the case of the toroidal nonuniformity of the magnetic field by 3.7% only. For the ASWs with $m = +2$, the maximal value of the growth rate $\mathrm{Im}(\omega/|\omega_e|) = 0.057$ in the case of the uniform magnetic field is reached when $k_{ef} = 0.9$ and, oppositely, the maximal value of the growth rate in the case of the toroidal nonuniformity of the magnetic field is reduced by 3.4%. But it is found that the toroidal non-uniformity has an effect, essentially, on the width of the $k_{ef}$ range where the ASW excitation by the electron beam is effective. Increases of this range occur in the direction of the large values of $k_{ef}$, or in other words toward smaller of the plasma column radii and smaller plasma densities.

The effects of the magnetic field toroidicity on the ASW excitation are compared for different values of the external magnetic field in Figures 4 and 5. Decreasing of an external steady magnetic field $B_{0\zeta}$ leads to increasing of the studied ASW eigen frequency (see Figure 4), their growth rates (see Figure 5), shift of the frequency correction to the LF ASWs eigen frequency caused by the space non-uniformity of the $B_{0\zeta}$ into the region of lower values of $k_{ef}$, and a shift of the "bell" of the dependence of $\mathrm{Im}(\omega/|\omega_e|)$ upon $k_{ef}$ into the region of lower values of $k_{ef}$ as well.

In Figures 2, 4 and 6, one can see the singularity in the dependence of the correction $\Delta\omega_t$ on $k_{ef}$. It happens if the wave frequency is close to the $|\omega_e|$ value. Correlation between positions of the points of intersection between curves $\omega/|\omega_e|$ and asymptotic lines $\omega = |\omega_e|$ in these figures and "gaps" in dependences $\Delta\omega_t(k_{ef})$ can be easily seen from analysis of these curves presented in Figures 2, 4 and 6. Correction to the LF ASWs frequency caused by toroidicity of the $B_{0\zeta}$ is calculated here by the aid of theory of successive approximations. In the first approximation: $\Delta\omega_t = -D^{(2)}\left(\partial D^{(0)}/\partial\omega\right)^{-1}_{|\omega=\omega_0} \propto (\omega - |\omega_e|)^{-2}$, therefore one can take in mind that validity of this approximation is determined by satisfaction of the inequality $\omega \gg |\Delta\omega_t|$. Thus this approximation shows only a tendency of $|\Delta\omega_t|$ increasing if the $\omega \to |\omega_e|$. To obtain more correct expression for $|\Delta\omega_t|$ one can calculate next approximations. So to show tendency of $\Delta\omega_t$, to indicate position of such type resonance in the space of wave numbers it is enough to apply fluid model of plasma. But it should be remembered as well that at the vicinity of ion and electron cyclotron resonances just kinetic model is correct rather than fluid model, it is more difficult model as compared with the fluid model.

The difference in the dependences of the growth rates on $k_{ef}$ in the cases of the uniform and nonuniform magnetic field becomes smaller as the parameter $Z = \omega_{pe}/|\omega_{ce}|$ increases (see Figure 5). It is correlated with the result presented in Figure 3 since this difference becomes negligible just for the range of small values of $k_{ef}$. Unlike that near the right limit of the $k_{ef}$ range where the ASW excitation is effective, this difference becomes larger and is accompanied by broadening of this range of $k_{ef}$.

Influence of an external steady magnetic field toroidicity on the ASW growth rates value is mostly pronounced if eigen frequencies of the studied waves are close to electron cyclotron resonance, because $\Delta\omega_t \propto (\omega - |\omega_e|)^{-2}$ in the first approximation. That is why one can see sharp increasing of the growth rate value for ASW with $m = +2$ in the case $Z = 3$ in Figures 3 and 7 nearby $k_{ef} = 0.5$. In the Figure 5 results related to the case of $m = +1$ are presented; one can see that since curve $\omega/|\omega_e|$ crosses asymptotical line of electron cyclotron resonance for $Z = 3$ at $k_{ef} = 0.6$, where resonant condition (14) is not satisfied, then related growth rate curve has no any peaks. But if $Z = 8$, then (see Figure 4) the indicated above crossing point is located nearby effective wave number $k_{ef} = 0.2$. Then the curve $\mathrm{Im}(\omega/|\omega_e|)$ has additional peak just near this value of effective wave number (see Figure 5).

The effect of the toroidicity on the ASW excitation was studied for different widths of the gap between the plasma column and the metal wall, and these results are presented in Figures 6 and 7. Since the gap's width weakly affects the ASW frequency for small values of the parameter $Z$, the singularity in the dependence of the frequency correction $\Delta\omega_t$ caused by the toroidicity is seen in Figure 6 almost in the same place where $k_{ef} = 0.55 \div 0.6$. In the case of a large gap, the related "hole" in the figure becomes wider. On the left side from the "hole", the dependences are similar, but on the right side the correction to the frequency is smaller than in the case of the wider gap. This decrease of the correction is only partly due to the difference in $\Delta$, but also due to the difference in $\varepsilon_t$ ($\varepsilon_t = 0.1$ for the graph in the case $\Delta = b/a - 1 = 0.3$ and $\varepsilon_t = 0.05$ for the case $\Delta = 0.2$). The point is that the strong inequality $\varepsilon_t \ll \Delta$ should be satisfied when solving the problem. The dependencies of the ASW growth rate $\mathrm{Im}(\omega/|\omega_e|)$ on the effective wave number for different gap widths are presented in Figure 7. The differences in the graphs for the cases of uniform and non-uniform magnetic fields are qualitatively the same for both gap widths. But increasing of the gap leads to increasing of the $\mathrm{Im}(\omega/|\omega_e|)$.

The inequality $\varepsilon_t \ll 1$ should be satisfied in our consideration to utilize the expansion (4) and dispersion Equation (13). But the toroidal nonuniformity of the external magnetic field has an effect not only on the electrodynamics' plasma properties, but also on the dynamics of the beam motion. The beam electrons rotating along Larmor radii in the toroidal nonuniform external magnetic field move with a drift velocity

$$\vec{v}_D = \frac{v_\perp \rho_{L\perp}}{2B_0^2}\left[\vec{B}_0, \nabla B_0\right], \tag{15}$$

here $\rho_{L\perp} = cp_\perp/(eB_0)$ is the Larmor radius. Due to the drift motion, the electron beam will annihilate on the metal wall or the plasma surface during the time $\tau = (b-a)/v_D$. The excitation of the ASWs is possible only when the time $\tau$ is much larger than the ASW period. This request leads to more strict inequality $\varepsilon_t \ll \Delta$, which has been satisfied in all the calculations.

## 4. DISCUSSIONS AND CONCLUSIONS

The initial stage of the resonant beam excitation of the extraordinary azimuthal surface waves by an electron beam rotating in the gap between the metal wall and the plasma column of toroidal waveguides with small values of the toroidicity parameter is studied. Usually, the toroidicity of the external magnetic field $\vec{B}_0$ is accompanied by a shift of the plasma column axis from the waveguide axis and a change of the plasma column cross-section. But these two effects on the excitation of the azimuthal surface modes have been studied in [22], and therefore they were out of the scope of this paper.

It is shown that the toroidal non-uniformity of $\vec{B}_0$ affects most essentially the dispersion properties of the LF ASWs when their eigen frequency is close to the absolute value of the electron cyclotron frequency. The toroidal non-uniformity of $\vec{B}_0$ weakly affects the maximal (for the complete range of the effective wave numbers where the instability can be realized) value of the ASW beam instability growth rate. However, under the considered conditions, the range of $k_{ef}$ where the instability can be realized becomes larger by shifting the right limit of the range toward the side of larger values of $k_{ef}$. This shift is larger for larger azimuthal mode numbers $m$. The effect of the $\vec{B}_0$ toroidicity on the ASW growth rates is weaker for a plasma filling with larger plasma density.

The toroidal non-uniformity of $\vec{B}_0$ leads to the drift of the electron beam, because of that these electrons will disappear on the metal wall or the plasma surface faster. But if the toroidicity parameter is weak so that $a/R \ll (b-a)/a < 1$, then beam will have enough time during the drift to excite the LF ASWs. Since the correction to the ASW eigen frequency caused by the toroidal non-uniformity of $\vec{B}_0$ is quadratic in the small parameter of the toroidicity, $\Delta\omega_t \propto \varepsilon_t^2$, the effect of the toroidicity on the ASW excitation is larger for larger gaps between the plasma column and the toroidal chamber.

Thus, the studied features of the LF ASWs beam instability, which develops in toroidal waveguides, allow one to control an excitation of these modes by the way of changing plasma density and other parameters of the waveguide.

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