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About the Physical Reality of "Maxwell's Displacement Current" in Classical Electrodynamics

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Abstract—This work aims to provide a physical interpretation of the "Maxwell's displacement current" and generalize the use of the derivative function of the electrical flux "$d\Psi/dt$" in the magnetic field calculation. The innovative contribution of this study is a mathematical model to describe the origin of magnetic field as a variation of electric flux. By this approach, it follows that only the function "$d\Psi/dt$" can generate a "magnetic tension": this leads to an interesting unification of electrical phenomena. Displacement current and conduction current are interpreted as complementary aspects of the same phenomena. It is shown how the use of the "$d\Psi/dt$" allows the recovery of a formal symmetry in Maxwell's relations of electromagnetic induction law and circuit magnetic law. Included also is a generalized expression of the function "$d\Psi/dt$".

1. INTRODUCTION

As pointed out by Heras in [1]: "Maxwell's displacement current has been the subject of controversy for more than a century. Questions on whether the displacement current represents a true current like the conduction current and whether it produces a magnetic field have recently been discussed in the literature." On this problem many publications have recently been written, and some of them are cited in [1–18].

In the Faraday-Ampere-Coulomb's formulation of electro-magnetism the interaction in distance between source and field points plays a crucial role. The Maxwell's synthesis represents a modern approach because it interprets the electromagnetic phenomena replacing the concept of "distance effect" to "field": the field becomes the mediator of the interaction between source and field point. Another important Maxwell's result is the explicit inter-dependence between electrical and magnetic component of the field. It may be interesting to express the magnetic field through a function of electric field (electrical flux which we call "Ψ") and not by means of the conduction current that identifies a "cause far" and not local.

Maxwell, as known, introduced the "displacement current" because he believed that ether exists. Although mathematically the equation in which the displacement current appears to retain legitimacy, the same cannot be said for the physical interpretation which is now widely recognized as outdated. In the traditional interpretation, the magnetic field can be generated by the conduction current and/or by the displacement current, a phenomenon that appears as "*the son of two mothers*". In the study follows the intention is to introduce a single physical quantity that describes, for the generation of the magnetic field "H", the effect of displacement current and the effect of the conduction current. The analysis presented in this work also aims to provide a definitive answer to these authors, such as [3], denying the existence of the magnetic field generated by the displacement current. The contribution of this study is, according to [11], to associate a variation of the electric flux (attributable to a motional phenomenon) with the electric current and also provide a mathematical model to understand how the magnetic field arises.

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2. ORIGIN OF THE MAGNETIC FIELD

Now we aim to investigate the origin of the magnetic field generated by individual charges in motion, electric currents and the so-called "displacement current".

2.1. Magnetic Field Generated by Moving Single Charges

According to the classical treatment of electromagnetism a moving charge remains associated with the origin of the magnetic field.

The magnetic field origin is the direct result of the charge motion and is calculated using the Biot-Savart's equation. For a single moving charge the following relation:

$$\vec{B} = \frac{1}{c^2}\vec{v} \times \vec{E} = \mu\varepsilon\vec{v} \times \vec{E} \tag{1a}$$

In it, the magnetic field is expressed as the product of the velocity vector of the charge and the electric field generated in the same point. A representation of what is expressed in (1a) is shown schematically in Fig. 1.

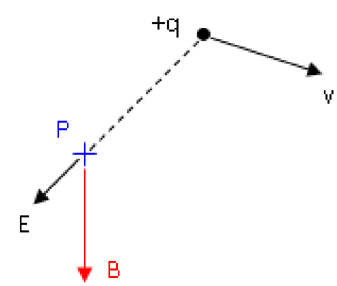

Figure 1. Single moving charge "$+q$"; electric and magnetic fields associated with the point "P".

I can calculate the total magnetic field for "aggregate charges in motion", or electric currents, with a simple integration from (1a):

$$\vec{B}_{TOT} = \sum \mu\varepsilon\vec{v}_i \times \vec{E}_i \quad \rightarrow \quad \vec{B}_{TOT} = \int \mu\varepsilon\vec{v} \times d\vec{E} \tag{1b}$$

If I have discrete electric charges the summation must be employed while if I have a continuous system of charges the use of the integral is appropriate.

In this paper we will study those particular configurations where the speed of each charge is the same and the relation (1b) can be simplified as follows:

$$\vec{B}_{TOT} = \vec{v} \times \sum \mu\varepsilon\vec{E}_i \quad \rightarrow \quad \vec{B}_{TOT} = \vec{v} \times \int \mu\varepsilon d\vec{E} \tag{1c}$$

The magnetic field origin can also be interpreted according to the relativistic approach. In such a case two reference systems are considered one of which is integral with the point field of investigation and the other integral with the moving charge. The magnetic field is again interpreted as a direct result of the relative motion between two reference systems. As reported in literature the motion of the charge is the "first cause" in the generation of magnetic field.

In the classical treatment, the following local and integral equations:

$$rot\,\vec{H} = \vec{j} + \frac{\partial\vec{D}}{\partial t} \tag{2a}$$

It is important remember the observation of Prof. Jackson [6]: under Coulomb's gauge the term called "displacement current" at the right-hand side of (2a) appears a field source comparable to the conduction current.

$$\oint \vec{H} \cdot d\vec{l} = i + \iint \frac{\partial\vec{D}}{\partial t} \cdot d\vec{s} \tag{2b}$$

The term "j" in (2a) identifies the current density of conduction: it is the direct expression of the motion of charge. The second term "dD/dt", also known as the "*displacement current contribution*", is not direct expression of the motion of any charge, but "mathematically assimilable" to a moving charge: this term is only the rate of change of an electric field over time. The traditional interpretation of (2a) and (2b) leads to the observation that two different physical phenomena such as moving charge and electric field varies in time produce the same effect: generation of magnetic field.

Also it is important to remember the other Maxwell's Equations (3a) and (3b). As known, the interdependence of Equations (2a) and (3a) due to the term "$\partial D/\partial t$" called "displacement current" generates the propagation of electromagnetic field.

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3a}$$

$$\oint \vec{E} \cdot d\vec{l} = \iint \left(-\frac{\partial \vec{B}}{\partial t}\right) \cdot d\vec{s} \tag{3b}$$

Several spontaneous questions arise:

- What is the common intrinsic phenomena between motion of charge and the variation of the electric field?
- Why "dD/dt" can generates "H" even if I haven't motion of charge?
- There is only one physical quantity whose variation generates the magnetic field and contains as limiting cases the two phenomena described above (variation of the electric field and motion of charge)?

A first level of investigation might suggest that simply "dD/dt", or equivalently "dE/dt", is the primary cause attributable to the generation of the "H". But it isn't right as shown by some interesting examples. For example, let's consider the motion of two charges with two parallel paths, but with opposite directions of motion (as in Fig. 2) and an internal point to the trajectories: the electric field "E" experienced by the point "P" is identically zero because the two charges generate electric field vectors equal and opposites, also the "H" field point in question is nonzero.

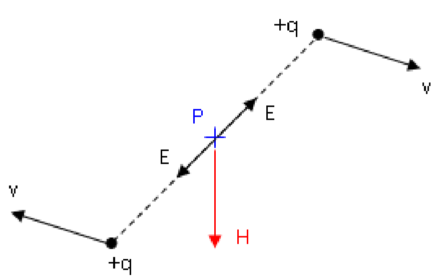

Figure 2. A pair of moving charges "$+q$"; electric and magnetic fields associated with the point "P".

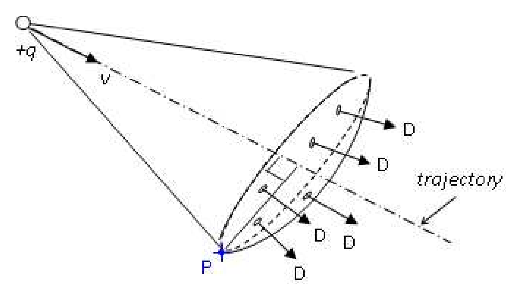

Figure 3. A moving charge "$+q$" and electrical flux associated on spherical cap for the calculation of "H" at point "P".

Consider now a single charge in motion as that of Fig. 3 and calculate the magnetic field at a point following the Cullwick's method [19]. The geometric model for the charge in motion is illustrated in Fig. 4.

The point "P" of Fig. 3 experiences the origin of the field "H". We can consider the volume in front of the charge, in the direction of motion, represented by a cone such as Fig. 3. In it the base circle is the line closed curve around which it is possible to calculate the circuitry of the field "H". On the circumference of the cone base lies the spherical cap whose points are affected by vectors "D" which, in a certain instant, are constant in magnitude on the cap. Now I can introduce "Ψ" as a general flux of the vector "D" through a generic area by means Equation (4):

$$\Psi = \iint \vec{D} \cdot d\vec{s} \tag{4}$$

Now applying the Gauss's theorem to a closed spherical surface, since the field on the spherical surface is uniform, for the spherical cap I have:

$$q = \oiint_S \vec{D} \cdot d\vec{s} \to \Psi_{CAPS} = DS_{CAPS} = q\frac{\gamma_{CAPS}}{4\pi} \tag{5}$$

γ_{CAPS} represents the solid angle subtended by the spherical cap.

$$\gamma_{CAPS} = \frac{2\pi Rh}{R^2} = \frac{2\pi(R-b)}{R} = \frac{2\pi(R - R\cos\alpha)}{R} = 2\pi(1-\cos\alpha) \tag{6}$$

$$\Psi_{CAPS} = DS_{CAPS} = q\frac{\gamma_{CAPS}}{4\pi} = q\frac{2\pi(1-\cos\alpha)}{4\pi} = q\frac{(1-\cos\alpha)}{2} \tag{7}$$

It is important remember that:

$$\frac{vdtsen\alpha}{R} = d\alpha \tag{8}$$

So we can write:

$$\frac{d\Psi_{CAPS}}{dt} = q\frac{sen\alpha}{2}\frac{d\alpha}{dt} = q\frac{sen\alpha}{2}\frac{vsen\alpha}{R} = q\frac{vsen^2\alpha}{2R} \tag{9}$$

as the following relationship holds:

$$\oint \vec{H} \cdot d\vec{l} = \frac{d\Psi}{dt} \tag{10}$$

$$H \cdot 2\pi Rsen\alpha = q\frac{vsen^2\alpha}{2R} \quad \to \quad \vec{H} = \frac{q}{4\pi R^3}\left(\vec{v} \times \vec{R}\right) = \vec{v} \times \vec{D} \tag{11}$$

As shown in the last Equation (11), obtained in literature from [19], the use of the "Ψ" has made it possible to achieve the same result expressed by Equation (1a). As shown if I have a single charge in motus I can calculate the magnetic field "H" by means "$d\Psi/dt$".

The function "$d\Psi/dt$" is defined around the moving charges but also within the armor of capacitors where I have no movement of charge. The origin of "$d\Psi/dt$" is therefore a phenomenon common to both the case of moving charge, both to the case of variation of the electric field. The "$d\Psi/dt$" contains the limiting cases of conduction current and displacement current.

It is possible to show as the case of the point charge in motion (11) is equivalent to the formulation of the law of Biot-Savart differential making sure to replace the product "qv" the infinitesimal element of the current "idl":

$$\vec{H} = \frac{q}{4\pi R^3}\left(\vec{v} \times \vec{R}\right) = \vec{v} \times \vec{D} \quad \to \quad \vec{H} = \frac{1}{4\pi R^3}\left(id\vec{l} \times \vec{R}\right) \tag{12}$$

Equation (11) allows to derive the magnetic field in the case where the source is a single moving charge. I would like to emphasize that the case of moving charge represents a not stationary configuration. In fact, the motion of the charge causes variation of the vector "D", and according to what described by Cullwick [19], a variation of the area is also associated to the flux of "D". In other words, the "*flux of D*" changes because with the motion of the charge, there is variation of "D" and the area associated with the flux relevant to them.

2.2. Some Considerations about Maxwell's Equations

At this point, we believe that it is important to comment on the passage algebraic (14) described in [20] obtained starting from (13):

$$rot\left(\vec{A} \times \vec{B}\right) = \left(\vec{B}\nabla\right)\vec{A} - \left(\vec{A}\nabla\right)\vec{B} + \vec{A}div\vec{B} - \vec{B}div\vec{A} \tag{13}$$

$$rot\,\vec{H} = rot\left(\vec{v} \times \vec{D}\right) = rot\left(-\vec{u} \times \vec{D}\right) = -\left(-\vec{u}\nabla\right)\vec{D} - \vec{u}div\vec{D} = \frac{\partial\vec{D}}{\partial t} + \vec{v}div\vec{D} = \frac{\partial\vec{D}}{\partial t} + \vec{j} \tag{14}$$

The shift (14) suggests that the known Maxwell Equation (2), deduced by means of an algebraic method from a relationship with a clear physical meaning, is not a simple phenomenological schematization but springs from deep reasons and has intrinsic meaning. It should be noted that the product "$v \times D$" represents the field "H" because it quantifies the change in electrical flux caused either by the variation of vector "D" (due to the motion) or by the variation of the area of the flux (due to the motion of charge). It does not appear that it represents, however, the field generated by reasons not directly attributable to the motion of charge, but we cannot exclude it.

2.3. Magnetic Field Due to an Electric Current

Based on the previous observations, I can calculate the variation of the electrical flux for "aggregate charges in motion", or electric currents, with a simple integration by (1b).

As already mentioned, in this paper we will investigate those particular configurations in which relationship (1b) can be simplified in (1c):

$$\vec{B}_{TOT} = \vec{v} \times \sum \mu\varepsilon \vec{E}_i \quad \rightarrow \quad \vec{v} \times \int \mu\varepsilon d\vec{E} \tag{15}$$

Let's see now, using the "Ψ", to analyze the magnetic field due to an electric current represented by a charge in motion and linearly distributed as in Fig. 5. For a charge with linear density "λ" it is possible to calculate the electric field "E":

$$E = \frac{\lambda}{2\pi\varepsilon R} \tag{16}$$

Suppose that we want to calculate the field "H" lying in the plane containing the circular cross-section "A" which is fixed. Consider a second circular section, "B", integral with the moving charges, and then in motion together with the charges themselves. After a time "dt", the surface "B" will have covered the distance "dx". The cylindrical surface defined by the sections "A" and "B" will progressively grow with the flux of the vector "D" intercepted by her (see Fig. 5).

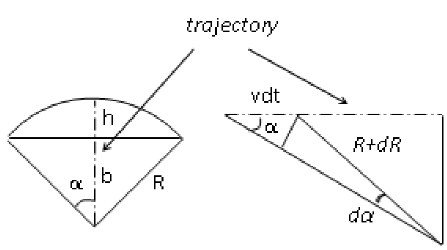

Figure 4. Geometric model for system of Fig. 3.

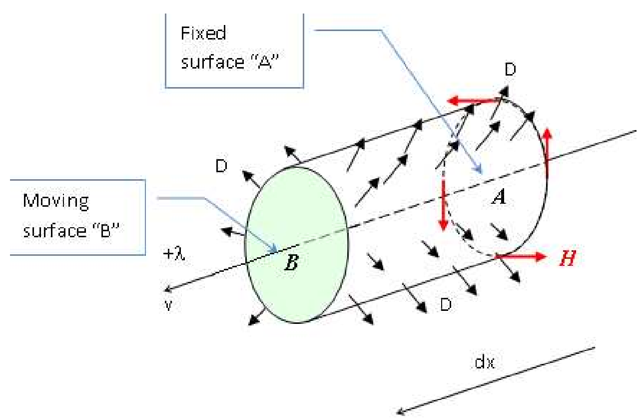

Figure 5. Model of electric current and electric flux intercepted by the cylindrical surface in expansion.

The variable section constitutes an infinitesimal variation of the electrical flux "$d\Psi$" calculated on the cylindrical side surface:

$$d\Psi = DdS = D2\pi Rdx = \varepsilon \frac{\lambda}{2\pi\varepsilon R} 2\pi Rdx = \lambda dx \tag{17}$$

$$\frac{d\Psi}{dt} = \frac{\lambda dx}{dt} = \lambda v = i \tag{18}$$

Now, considering the expression (18), we propose, in accord to [15], the following relationship:

$$\oint \vec{H} \cdot d\vec{l} = \frac{d\Psi}{dt} \tag{19}$$

In reality, the conductors are generally neutral, thus the external field "D" becomes zero. The configuration of the conductors can be modeled as two linear distributions "$+\lambda$" and "$-\lambda$" of which one is fixed and the other in motion. The motion of a single charge distribution generates magnetic field because it generates a variation of the electrical flux "*within its exclusive competence*". This happens even though the overall flux is zero because the distributions have opposite signs, according to (1a).

2.4. Magnetic Field around an Ohmmic-capacitive Circuit

Now we can consider a capacitive circuit such as Fig. 6. Based on previous observations, magnetic field "H" can be generated by the derivative of the function "Ψ", both from the conduction current, and both from the so-called "displacement currents".

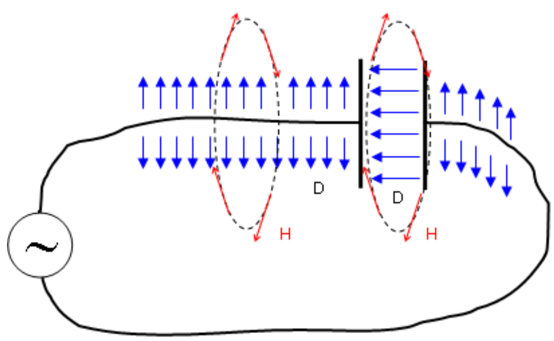

Figure 6. Example with creation of a magnetic field for conduction current and displacement current.

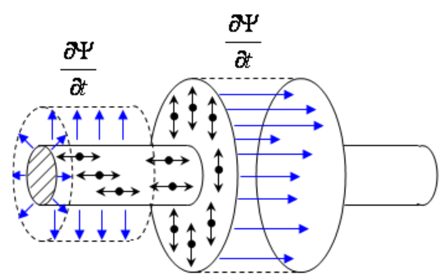

Figure 7. Schematization of electrical flux in the vicinity of the conductor and inside of the dielectric of the capacitor.

The role of the capacitor appears as a component "channeling" the flux of electric field within their armor. In wires the flux variation is due to the motion of the charges because there is a variation of the cylindrical surface intercepted by the flux, while inside the condenser the same flux variation is due to a variation of the vector "D". In this way, the capacitor is able to transform the flux variation due to change of the intercepted surface (conductive phenomenon) in the flux variation due to temporal variation of "D" (capacitive phenomenon), so the flux variation for conductive phenomenon is equal to the flux variation for capacitive phenomenon.

In the conductor the variation of the electrical flux is canceled by the presence of fixed charges with opposite sign. The condenser, separating the charge with opposite sign in the armor, makes clear that electric flux changes while in the conductor it is hidden.

With this interpretation, rather than assimilate the displacement current to the conduction current, we have followed the reverse method: also the conducting current phenomenon has seen like a change of electric flux. With what we believe we have answered the three questions posed at the beginning:

1. What is the common intrinsic phenomena between motion of charge and the variation of the electric field?

R. The phenomenon of motion of the charge produces a change in the electrical flux "Ψ"; the variation of the electric field inside a capacitor's plates also produces a variation of the electrical flux.

2. Why can "dD/dt" generate "H" even if there is no motion of charge?

R. The variation of flux "$d\Psi/dt$" generates the magnetic field in both the cases when it is due to variations of "D" and when there is a change in surface intercepted (moving charges).

3. There is only one physical quantity whose variation generates the magnetic field and contains as limiting cases the two phenomena described above (the motion of charge and variation of electric field)?

R. We suggest, in accord to [11], that "$d\Psi/dt$" is the real cause that generates the magnetic field.

The displacement current introduced by Maxwell represents a correct choice from a mathematical point of view but wrong in its physical interpretation: the displacement current was associated with an oscillation of an elastic medium called "ether" whose existence was ruled out by physicists Michelson and Moreley and not considered necessary according to the relativistic theory of A. Einstein.

The phenomenon of electromagnetic induction (Faraday's law) is described by the magnetic flux variation that is symmetrically corresponding to the electrical flux variation. This symmetry is described later in Section 4.

3. GENERAL EXPRESSION FOR THE VARIATION OF THE ELECTRICAL FLUX

At this point, in order to build an appropriate mathematical model for our examples it appears necessary to introduce a mathematical expression of the generalized flux variation.

3.1. Mathematical Expression for the Variation of a Generical Flux

Now I want to provide a general treatment for the expression of the electrical flux variation "$d\Psi/dt$". This variation can be caused either by a change of vector "D", by a change of area of the flux, or by both.

The general expression (20) to calculate the flux derivative of a vector field "A" in the case of moving surfaces is known in the literature

$$\frac{d\Phi}{dt} = \iint_S \frac{\partial \vec{A}}{\partial t} \cdot d\vec{S} + \iint_S \left(\vec{v}\, div \vec{A}\right) \cdot d\vec{S} - \oint_l \left(\vec{u} \times \vec{A}\right) \cdot d\vec{l} \tag{20}$$

Recalling the analysis of [21] it is possible to interpret the terms of function (20). The first term represents the change of flux through the surface caused by the variation in time of the vector field "A". The second term is derived from the passage of the moving surface through an inhomogeneous field: I have a change in flux for the acquisition or the expulsion of charges by the moving surface. The third term identifies the loss of flux through the outline of the moving surface, i.e., the lateral surface generated by the trace of the contour of the surface in motion. In literature, the use of (20) is frequently employed for the calculation of the derivative of the magnetic flux; in this case, moreover, (19) is simplified thanks to the fact that "$divB = 0$".

With reference to Fig. 8, the demonstration of (20) can be obtained, following the method of [21], developing the following considerations.

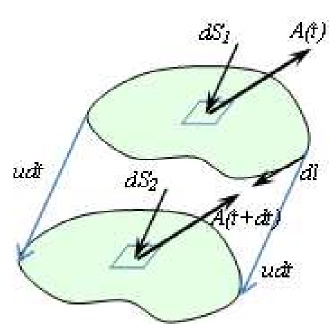

Figure 8. Generic moving surface in a region of space affected by the field "$A(t)$".

3.1.1. Demonstration of the Relationship (20)

We can consider the variation of flux relative to a surface in motion with velocity "u" because of the passage of the surface from position "1" to position "2" (Fig. 8):

$$\frac{\Delta}{\Delta t}\iint\limits_S \vec{A}\cdot d\vec{S} = \frac{1}{\Delta t}\iint\limits_S \vec{A}(t+\Delta t)\cdot d\vec{S}_2 - \vec{A}(t)\cdot d\vec{S}_1 \tag{21}$$

Now apply the Gauss theorem to the total volume represented in Fig. 8 (note that the theorem refers to the same instants "non-deferred", then, for the two surfaces "S_1" and "S_2"):

$$\iiint\limits_V div\vec{A}d\vec{\tau} = \iint\limits_S \left[\vec{A}(t)\cdot d\vec{S}_2 - \vec{A}(t)\cdot d\vec{S}_1\right] - \oint\limits_l \vec{A}(t)\cdot\left(\vec{u}dt\times d\vec{l}\right) \tag{22}$$

For Taylor's theorem:

$$\vec{A}(t+\Delta t) = \vec{A}(t) + \frac{\partial\vec{A}}{\partial t}dt + \ldots \tag{23}$$

Now replacing (23) in (21) to give:

$$\frac{\Delta}{\Delta t}\iint\limits_S \vec{A}\cdot d\vec{S} = \frac{1}{\Delta t}\iint\limits_S\left[\vec{A}(t)+\frac{\partial\vec{A}}{\partial t}dt\right]\cdot d\vec{S}_2 - \vec{A}(t)\cdot d\vec{S}_1 = \frac{1}{\Delta t}\iint\limits_S\left[\vec{A}(t)\cdot d\vec{S}_2 - \vec{A}(t)\cdot d\vec{S}_1 + \frac{\partial\vec{A}}{\partial t}dt\cdot d\vec{S}_2\right]$$

$$= \frac{1}{\Delta t}\iint\limits_S\left[\vec{A}(t)\cdot d\vec{S}_2 - \vec{A}(t)\cdot d\vec{S}_1\right] + \frac{1}{\Delta t}\iint\limits_S \frac{\partial\vec{A}}{\partial t}dt\cdot d\vec{S}_2 \tag{24}$$

Replacing (22) in (24) and engaging in the passage to the limit yields:

$$\frac{d}{dt}\iint\limits_S \vec{A}\cdot d\vec{S} = \frac{1}{dt}\left[\iiint\limits_V div\vec{A}d\tau + \oint\limits_l \left(\vec{A}\times\vec{u}\right)\cdot d\vec{l}\right] + \frac{1}{dt}\iint\limits_S \frac{\partial\vec{A}}{\partial t}dt\cdot d\vec{S}_2 \tag{25}$$

being:

$$d\tau = d\vec{S}\cdot\vec{u}dt \tag{26}$$

(25) is transformed into the following Equation (27) equivalent to (20):

$$\frac{d}{dt}\iint\limits_S \vec{A}\cdot d\vec{S} = \iint\limits_S \left(\vec{u}div\vec{A}\right)\cdot d\vec{S} + \oint\limits_l \left(\vec{A}\times\vec{u}\right)\cdot d\vec{l} + \iint\limits_S \frac{\partial\vec{A}}{\partial t}d\vec{S} \tag{27}$$

3.2. Expression for the Variation of an Electrical Flux

By introducing the electric displacement field "D" in (27) yields:

$$\frac{d}{dt}\iint\limits_S \vec{D}\cdot d\vec{S} = \iint\limits_S \left(\vec{u}div\vec{D}\right)\cdot d\vec{S} + \oint\limits_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} + \iint\limits_S \frac{\partial\vec{D}}{\partial t}d\vec{S} \tag{28}$$

As already described, in the right-hand side of Equation (28), the first term is derived from the passage of the moving surface through an inhomogeneous field: I have a change in flux for the acquisition or the expulsion of charges by the moving surface. The second term identifies the loss of flux through the outline of the moving surface, i.e., the lateral surface generated by the trace of the contour of the surface in motion. The third term represents the change of flux through the surface caused by the variation in time of the vector field "D".

3.3. Expression for the Electrical Flux Variation with Straight Electric Current

For the evaluation of the magnetic field around a straight wire it is possible to study a fixed surface "S" and a linear charge "λ" in motus with velocity "v" or an equivalent model in which the relative motion is got by the motion of the surface "S" (with velocity "u"), and the charge linear "λ" is fixed as shown in Fig. 9. The relation between "u" and "v" is therefore:

$$\vec{u} = -\vec{v} \tag{29}$$

Now we are going to study (28) in order to extract those terms useful for the calculation of the field "H" for an electric charge straight.

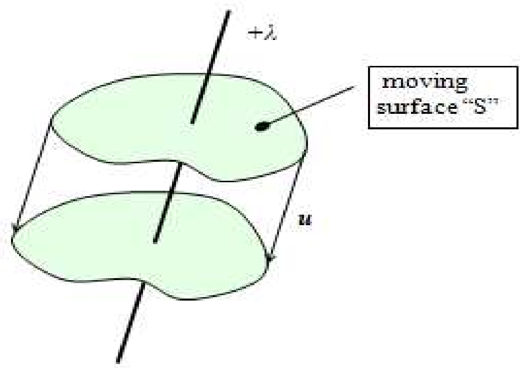

Figure 9. Moving surface and fixed linear charge.

Being:

$$\vec{u} div \vec{D} = \vec{u}\rho = -\vec{J} \tag{30}$$

Replacing (30) in (28) we got Equation (30):

$$\frac{d}{dt}\iint_S \vec{D}\cdot d\vec{S} = \iint_S -\vec{j}\cdot d\vec{S} + \oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} + \iint_S \frac{\partial\vec{D}}{\partial t} d\vec{S} \tag{31}$$

In the right-hand side of Equation (31), the first addend represents the contribution to the change in flux due to the current. The third summand, in stationary hypothesis, is equal to zero. Furthermore, the variation of flux (see left-hand side of Equation (31)) for this case is identically zero because the surface in motion meets, moment by moment, always the same field configuration.

For the case of Fig. 9, we can assert that:

$$\frac{d}{dt}\iint_S \vec{D}\cdot d\vec{S} = \iint_S -\vec{j}\cdot d\vec{S} + \oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} = 0 \tag{32}$$

$$\iint_S \vec{j}\cdot d\vec{S} = \oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} \tag{33}$$

$$i = \iint_S \vec{j}\cdot d\vec{S} \tag{34}$$

$$\oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} = \oint_l \left(\vec{v}\times\vec{D}\right)\cdot d\vec{l} \tag{35}$$

$$i = \oint_l \left(\vec{v}\times\vec{D}\right)\cdot d\vec{l} \tag{36}$$

Resorting to the meaning of (28), (36) can be interpreted as follows: the current flowing in the conductors produces a contribution to the variation of electric flux (relative to the surface "S" of Fig. 9). In other words, the current flowing is exactly the measure of the rate of change of electric flux.

This contribution is equal and opposite to the variation of flux which affects the surface generated by the contour of the surface "S" in motion (Fig. 9); the derivative of this contribution is equal to the current itself. Thanks to (33) I can replace the flux of "j" with a function of "D".

3.4. Magnetic Field near a Capacitor

Similar to the classical treatment, with reference to the circuit of Figs. 6, 7 where there is a capacitor in the hypothesis of "almost stationary", the previous considerations invite to introduce the following analysis. We have conservation of charge and solenoidal total current; so one has:

$$div\left(\vec{J}+\frac{\partial\vec{D}}{\partial t}\right)=0 \quad \rightarrow \quad \iint_S \frac{\partial\vec{D}}{\partial t}\cdot d\vec{S}=\iint_S -\vec{j}\cdot d\vec{S} \tag{37}$$

Then (31), (32), (33), (34) and (35) suggest that, in a circuit like that of Fig. 6, the flux variation that affects the capacitor is transformed into a variation of flux which affects the area generated by the contour of the surface in motion. Equations (31), (32), (33), (34) and (35) can be summarized as follows:

$$\oint \vec{H}\cdot d\vec{l}=\frac{\partial\Psi_{TOT}}{\partial t}=\frac{\partial\Psi_{DISP}}{\partial t}+\frac{\partial\Psi_{COND}}{\partial t} \tag{38}$$

where"Ψ_{TOT}" is the total electrical flux, "Ψ_{DISP}" the electrical flux of the capacitor, and "Ψ_{COND}" the electrical flux of the conductor.

$$\frac{\partial\Psi_{DISP}}{\partial t}=\iint_{Sdisp}\frac{\partial\vec{D}}{\partial t}\cdot d\vec{S} \tag{39}$$

$$\frac{\partial\Psi_{COND}}{\partial t}=\iint_{Scond}\left(div\vec{D}\vec{v}\right)\cdot d\vec{S}=\oint_l\left(\vec{v}\times\vec{D}\right)\cdot d\vec{l} \tag{40}$$

Expressions (38)–(40) are interesting for the following reasons:

- the link between magnetic field "H" and electric displacement field "D" appears explicit.
- The explicit dependency of "H" by the current has been removed.
- Appear interesting from a relativistic point of view since it is explicit dependence of the field "H" on the relative velocity "v" between two reference systems.

3.5. Ampere's Law in the Case of Straight Wires

For the evaluation of the magnetic field around a straight wire it is possible to study a model in which there is a fixed surface and a linear charge "λ" in motus on a wire or equivalently a linear fixed charge and the mobile surface as shown in Fig. 10. Vectors "u" and "v" are defined by Equation (29) in Section 3.2.

Remember Equation (41) to estimate the variation of flux that affects the surface in motion:

$$\frac{d}{dt}\iint_S \vec{D}\cdot d\vec{S}=\iint_S div\vec{D}\vec{u}\cdot d\vec{S}+\oint_l\left(\vec{D}\times\vec{u}\right)\cdot d\vec{l}+\iint_S\frac{\partial\vec{D}}{\partial t}d\vec{S} \tag{41}$$

In the right-hand side of Equation (41), the first addend represents the contribution to the change in flux due to the current. The third summand, in stationary hypothesis, is equal to zero. Furthermore, the variation of flux for this case is identically zero because the surface in motion meets, moment by

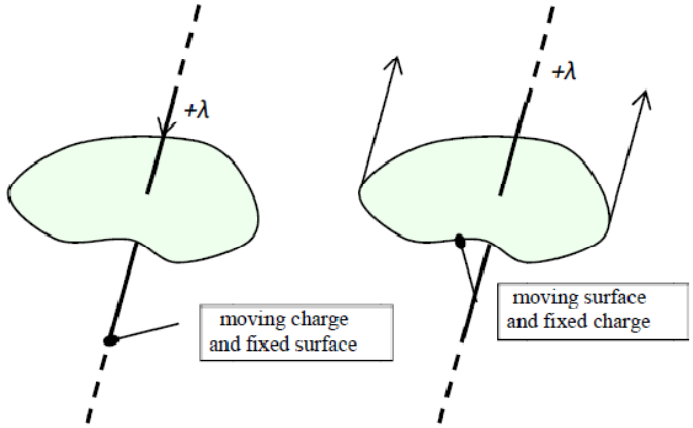

Figure 10. Model of electric current to study Ampere law.

moment, always the same field configuration. For the case of Fig. 10, we can assert that, as already written:

$$\frac{d}{dt}\iint_S \vec{D}\cdot d\vec{S} = \iint_S -\vec{j}\cdot d\vec{S} + \oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} = 0 \tag{42}$$

$$\iint_S \vec{j}\cdot d\vec{S} = \oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} \tag{43}$$

It follows that the passage of charges through the fixed surface produces a cut of the field lines "D" by the contour of the fixed surface. Equation (43) says that this variation of flux is exactly equal to the current concatenated with the loop along which I can calculate the circuitry of "H". So it is possible to rewrite Ampere's Law in function of vector "D".

$$\oint \vec{H}\cdot d\vec{l} = i = \oint_l \left(\vec{D}\times\vec{u}\right)\cdot d\vec{l} \tag{44}$$

These analytical steps can be interpreted as an immediate demonstration of Ampere's law if it is assumed that the magnetic tension arises from the change in flux caused by the motional phenomenon of cutting the vector "D".

It is a remarkable fact that Ampere's law, as known, is valid for any geometry of the conductors. The link between Biot-Savart's law and Ampere's law is not trivial: the demonstrations that I want to remember are those written in [23, 24].

4. CONSIDERATIONS ON THE FORMAL ANALOGY BETWEEN ELECTROMAGNETIC INDUCTION LAW VS MAGNETIC CIRCUIT LAW

Now we want to consider a formal aspect of Maxwell's equations. Let's remember Electromagnetic Induction Law:

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial \Phi}{\partial t} \tag{45}$$

The local formulation of (45) becomes:

$$rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{46}$$

Let's remember Magnetic Circuit Law:

$$\oint \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{s} + \int_S \frac{\partial \vec{D}}{\partial t}\cdot d\vec{s} \tag{47}$$

The local formulation of (47) becomes:

$$rot\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{48}$$

Equations (45) and (46) give meaning to phenomenon of electromagnetic induction: a change in magnetic flux is associated with a induced voltage which is opposed to the cause that generates it. Equations (47) and (48) describe the magnetic circuit law. Equation (45) vs (47) and (46) vs (48) are not exactly symmetrical relations. Resorting to (49) it is possible to restore symmetry:

$$\oint \vec{H} \cdot d\vec{l} = \frac{\partial \Psi_{TOT}}{\partial t} \tag{49}$$

Now if we interpret the Magnetic Circuit Law as the origin of "H" because of the electric flux change, an interesting phenomenological analogy, which we are going to show, emerges. In Fig. 11 shows a fixed linear charge and a surface with velocity "u". The speed "u" corresponds to an opposite speed "v" with which an surface integral observer apparently translates the linear charge "λ". Vectors "u" and "v" are defined by Equation (29) in Section 3.2. The running surface has a closed line "s" as a side dish also moving with a speed "u". This outline, cutting the lines of the "D", generates a magnetic tension "h" expressed by the following relation directly from Equation (44):

$$h = \oint \vec{H} \cdot d\vec{l} = \oint_l \left(\vec{D} \times \vec{u}\right) \cdot d\vec{l} = \oint_l \left(\vec{v} \times \vec{D}\right) \cdot d\vec{l} \tag{50}$$

Equation (50) then describes the origin of a "magnetic tension induced h" similar to what happens to the induced voltages in electromagnetic induction. In Fig. 12, the conductor cuts the field lines "B" with a speed "v" generating an induced voltage "e"; this electric voltage can be expressed by the following relationship (51) known in the literature and in any case derivable from (45):

$$e = \int_A^B \vec{E} \cdot d\vec{l} = \int_A^B \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l} \tag{51}$$

As you can see, expressions (50) and (51) are presented formally similar to the exclusion of the path of integration: in a case represented by a closed line while the other consists of an open line. This difference is due to the solenoidal nature of "H field" and the conservative nature of "E field".

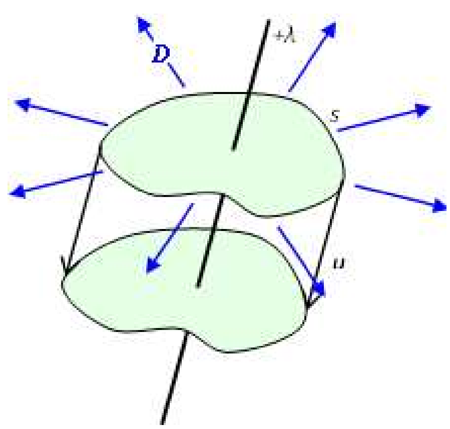

Figure 11. Model for studying Magnetic Circuit Law.

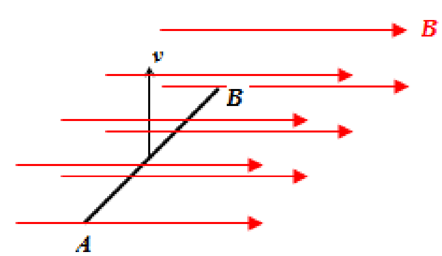

Figure 12. Model for studying Electromagnetic Induction Law.

5. BRIEF MENTION TO RELATIVISTIC ELECTRODYNAMICS AND QUANTUM ELECTRODYNAMICS

As known in the literature [25], the following Equation (52) represents the "not decoupled electrodynamic equations":

$$\begin{cases} \nabla^2 V + \dfrac{\partial}{\partial t}\left(\nabla \cdot \vec{A}\right) = -\dfrac{\rho}{\varepsilon} \\ \nabla^2 \vec{A} - \varepsilon\mu \dfrac{\partial^2 \vec{A}}{\partial t^2} - \nabla\left(\nabla \cdot \vec{A} + \varepsilon\mu \dfrac{\partial V}{\partial t}\right) = -\mu j \end{cases} \tag{52}$$

A suitable gauge transformation allows us to get Equations (52) decoupled. Using the following Lorentz condition (53), Equation (52) is transformed into "decoupled electrodynamic equations" (54a) or equivalent (54b)

$$\nabla \cdot \vec{A} + \varepsilon\mu \frac{\partial V}{\partial t} = 0 \tag{53}$$

$$\begin{cases} \nabla^2 V - \varepsilon\mu \dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho}{\varepsilon} \\ \nabla^2 \vec{A} - \varepsilon\mu \dfrac{\partial^2 \vec{A}}{\partial t^2} = -\mu j \end{cases} \tag{54a}$$

$$\begin{cases} \Box^2 V = -\dfrac{\rho}{\varepsilon} \\ \Box^2 \vec{A} = -\mu j \end{cases} \tag{54b}$$

From a relativistic point of view, charge density and current density are both sources of field and different aspects of the same reality which may be considered as components of the same vector (55).

$$j^\alpha = (\rho c, j) \tag{55}$$

Also the potentials "V" and "A" may be considered as a 4-vector potential (56):

$$A^\alpha = \left(\frac{V}{c}, \vec{A}\right) \tag{56}$$

So Equation (54b) can be written in the covariant form of (57):

$$\Box^2 A^\alpha = -\mu j^\alpha \tag{57}$$

Or in the following covariant form equivalent to (57):

$$\rho^\alpha = \left(\rho, \frac{j}{c}\right) \tag{58}$$

$$\Box^2 A^\alpha = -\frac{1}{\varepsilon c}\rho^\alpha \tag{59}$$

In Equations (57) and (59) I can see the sources of field, which are the components of the vector at the right-hand side. When I choose a system of reference with charges not in motion and in motion, the first generates electric field and the second magnetic field, but all contributions are different aspects of the same reality. The sources called "j" are associated with the state of motion and therefore are a measure of the rate of change of something. This interpretation does not seem to contradict what I said earlier: I can see the electric current (charges in motion) such as the measure of the rate of change of electric flux generate by fixed charges.

In Quantum Electrodynamics (QED) Equations (54a), (54b), (57) and (59) are modified by replacing the right-hand members with functions that take into account the quantization of the sources. It would be extremely interesting to analyze the meaning of the EM fields in the context of QED, but this is beyond the scope of this paper.

6. CONCLUSION

The need to give a physical interpretation to the "Maxwell's displacement current" has led to a revision of the concept of conduction current which causes generating the magnetic field.

According to some considerations the derivative electric flux "$d\Psi/dt$" has been recognized as the main cause in generation of "magnetic tension". On the basis of this position, for the purpose of magnetic field generation, displacement current and conduction current are interpreted as complementary aspects of the same phenomena in the strict context of Maxwell's field equations using an electric flux change in appropriate computing areas.

With the interpretation presented in this study, rather than assimilating the displacement current to the conduction current, the conduction current phenomenon is also seen as an electric flux variation. Thus I can have variations in electrical flux by conduction and/or capacity. A generalized expression that satisfactorily describes the functional links with the quantities of interest was finally presented.

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