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Influence Analysis of Stochastic Translation of Transmission Lines over Ground

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Abstract—This paper proposes a method for the quick estimation of the average voltages at terminal loads when the transmission line translate randomly and analyzes the sensitivities of the loads' voltages to the translation. Because nonuniform transmission lines can be approximated as **n**-cascaded uniform lines, the study of uniform lines is the basis. Based on the transmission-line equations, the equations are derived to estimate the average voltages, the voltage variations, and the sensitivity of the voltage to the random translation when transmission lines have random translation in their cross sections. With these equations, the average voltages at the loads, the probability distributions of the voltage variations, and the sensitivity of the voltage to the random translation can be obtained quickly. A two-wire line over the ground is studied by using the proposed method. The average voltages and the voltage variations' probability distributions agree well with those via the Monte Carlo (MC) method and the proposed method is more efficient. The results show that the sensitivities of the voltages at the loads to the random height increase with the terminal sources but decrease with the height.

1. INTRODUCTION

Due to the randomness of wire positions, the crosstalk between wires and the coupling of external fields to wires is stochastic. The effect induced by the randomness has attracted lots of attention and many methods and models have been developed to analyze it.

At the beginning, many measurements were carried out to study the effects of the random wire positions and the mean and the variance were calculated from the measured data [1, 2]. The experimental method is straightforward, but takes much time and costs a lot. Afterward, the random lines or cables were always modeled first by using the Monte Carlo (MC) method, where the nonuniform transmission lines were generally modeled as n-cascaded segments of a uniform multiconductor transmission line, then the transmission line theory was adopted to compute the terminal voltages of each sample, and statistical values were obtained from the numerical results [3–7]. This method needs large repeated computations and the results are only suitable for the case studied. Many researches proposed the worst-case methods for transmission lines in some specific cases, such as weak coupling and strong coupling [8–11]. The worst-case methods can provide quick estimations for designs; however, the results may differ greatly from the actual values. For three-conductor transmission lines under specific conditions, analytical expressions for the crosstalk could be derived and the effects of the random parameters on the crosstalk could be obtained directly from the expressions [12–14]. This method is efficient but has lots of limits.

The above study mainly focuses on the computation of the statistical quantities for a random line which obeys a given distribution. However, the randomness of a transmission line may appear diversely. For example, the height of the line over the ground may obey Gauss distribution or other distributions. The results of one specific case are hard to be applied for others. In addition, only the statistical quantities, such as the mean, the variance, and the probability distribution et al., are the main concern

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in these researches. The sensitivity of the crosstalk or coupling to the random position is an important issue but has not been studied.

The random transmission lines may be nonuniform ones. Because nonuniform transmission lines can be approximated as cascaded series of many short sections of uniform lines [15, 16], random uniform transmission lines are the basis. The random movement of a random line in its cross section can be divided into random translation and random rotation. This paper mainly studies the influence induced by the stochastic translation of uniform lines, proposes a method for the quick estimation of the average voltages at terminal loads, and analyzes the sensitivities of the loads' voltages to the translation.

2. THEORETICAL ANALYSIS

2.1. Estimation of the Average Voltages

The telegraph equation for a uniform transmission line can be written as

$$\begin{aligned} &\frac{d}{dz}\mathbf{V}(x,y,z) + j\omega\mathbf{L}(x,y)\,\mathbf{I}(x,y,z) = \mathbf{0} \\ &\frac{d}{dz}\mathbf{I}(x,y,z) + j\omega\mathbf{C}(x,y)\,\mathbf{I}(x,y,z) = \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{V}$ and $\mathbf{I}$ are the voltage and current vectors of the line, and $\mathbf{L}$ and $\mathbf{C}$ are the per-unit-length inductance and capacitance matrices. Since the translation of a transmission line without the ground in its cross sections does not change the terminal voltages, only the line with the ground is considered. Due to the random translation in the cross section, the matrices $\mathbf{L}$ and $\mathbf{C}$ are random, and this results in the stochastic voltage and current vectors, which are the functions of the random wire positions.

When the translation is a small quantity, then the voltage can be expressed through Taylor expansion as

$$V(x,y) \approx V(\bar{x},\bar{y}) + (x-\bar{x})\frac{\partial}{\partial x}V(\bar{x},\bar{y}) + (y-\bar{y})\frac{\partial}{\partial y}V(\bar{x},\bar{y}) \tag{2}$$

As a result, the mean of the voltage can be approximated by

$$\bar{V}(x,y) \approx V(\bar{x},\bar{y}), \tag{3}$$

where $\bar{x}$ and $\bar{y}$ are the average coordinates. Similarly, the average current can be approximated by

$$\bar{I}(x,y) \approx I(\bar{x},\bar{y}), \tag{4}$$

Then the average voltage can be computed approximately by using the Baum-Liu-Tesche (BLT) equation [17, 18], which is network equation and can be used to obtain the terminal voltage and current of transmission line, and is given by

$$\begin{bmatrix}\bar{\mathbf{V}}_{\mathbf{ZS}}\\ \bar{\mathbf{V}}_{\mathbf{ZL}}\end{bmatrix} = \begin{bmatrix}\mathbf{1}+\boldsymbol{\rho}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{1}+\boldsymbol{\rho}_2\end{bmatrix}\begin{bmatrix}-\boldsymbol{\rho}_1 & e^{\gamma l}\\ e^{\gamma l} & -\boldsymbol{\rho}_2\end{bmatrix}^{-1}\begin{bmatrix}\left(\mathbf{V}_S - e^{\gamma l}\mathbf{V}_L\right)/2\\ \left(-e^{\gamma l}\mathbf{V}_S + \mathbf{V}_L\right)/2\end{bmatrix}\Bigg|_{x=\bar{x},y=\bar{y}}. \tag{5}$$

Here $\boldsymbol{\rho}_i(i = 1,\, 2)$ and $\boldsymbol{\gamma}$ are the reflection coefficients at the ends and propagation constant when the transmission line is situated at the average position, respectively. l is the length of the line, and $\mathbf{1}$ is a unit matrix. This equation can be used to estimate the average voltages in the case of small position variations.

2.2. Analysis of Voltage Variations

The voltage variation $\Delta V(x,y) \doteq V(x,y) - \bar{V}(x,y)$ can be approximated as

$$\Delta V(x,y) \approx (x-\bar{x})\frac{\partial}{\partial x}V(\bar{x},\bar{y}) + (y-\bar{y})\frac{\partial}{\partial y}V(\bar{x},\bar{y}). \tag{6}$$

And it is related to the position variations and the derivative of the voltage at the average position. However, the direct solution of the derivative is nontrivial. We give the derivation in another way. In the case of transmission line over an infinite ground, the movement of the transmission line along the y axis does not change the voltage, so only $\partial V(x,y)/\partial x$ is derived.

The telegraph equation at the average position can be written as

$$\begin{aligned} \frac{d}{dz}\mathbf{V}(\bar{x},\bar{y},z)+j\omega\mathbf{L}(\bar{x},\bar{y})\mathbf{I}(\bar{x},\bar{y},z)=\mathbf{0} \\ \frac{d}{dz}\mathbf{I}(\bar{x},\bar{y},z)+j\omega\mathbf{C}(\bar{x},\bar{y})\mathbf{I}(\bar{x},\bar{y},z)=\mathbf{0} \end{aligned} \tag{7}$$

Subtracting (7) from (1) and neglecting the second-order small quantity yield

$$\begin{aligned} \frac{d}{dz}\Delta\mathbf{V}(z)+j\omega\mathbf{L}(\bar{x},\bar{y})\Delta\mathbf{I}(z)=-j\omega\Delta\mathbf{L}\bar{\mathbf{I}}(z) \\ \frac{d}{dz}\Delta\mathbf{I}(z)+j\omega\mathbf{C}(\bar{x},\bar{y})\Delta\mathbf{V}(z)=-j\omega\Delta\mathbf{C}\bar{\mathbf{V}}(z) \end{aligned}, \tag{8}$$

where

$$\begin{aligned} \Delta\mathbf{I}(z)=\mathbf{I}(z)-\bar{\mathbf{I}}(z)\approx\mathbf{I}-\mathbf{I}(\bar{x},\bar{y}) \\ \Delta\mathbf{L}=\mathbf{L}-\mathbf{L}(\bar{x},\bar{y}), \quad \Delta\mathbf{C}=\mathbf{C}-\mathbf{C}(\bar{x},\bar{y}) \end{aligned} \tag{9}$$

When the medium is homogenous and uniform,

$$\begin{aligned} \Delta\mathbf{C}=\frac{1}{c^2}\left(\mathbf{L}^{-1}-\mathbf{L}^{-1}(\bar{x},\bar{y})\right)\approx-\frac{1}{c^2}\mathbf{L}^{-1}(\bar{x},\bar{y})\Delta\mathbf{L}\mathbf{L}^{-1}(\bar{x},\bar{y}) \\ \mathbf{Z}_C(\bar{x},\bar{y})\Delta\mathbf{C}\approx-\Delta\mathbf{L}\mathbf{Z}_C^{-1}(\bar{x},\bar{y}) \end{aligned}, \tag{10}$$

where c is the velocity of the light in the medium. Then the voltage variations $\Delta\mathbf{V_{ZS}}$ and $\Delta\mathbf{V_{ZL}}$ can be solved by using the BLT equation as

$$\begin{bmatrix}\Delta\mathbf{V_{ZS}}\\ \Delta\mathbf{V_{ZL}}\end{bmatrix}=\begin{bmatrix}\mathbf{1}+\mathbf{\Gamma}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{1}+\mathbf{\Gamma}_2\end{bmatrix}\begin{bmatrix}-\mathbf{\Gamma}_1 & e^{\gamma l}\\ e^{\gamma l} & -\mathbf{\Gamma}_2\end{bmatrix}^{-1}\Bigg|_{x=\bar{x},y=\bar{y}}\cdot\begin{bmatrix}\Delta\mathbf{L} & \mathbf{0}\\ \mathbf{0} & \Delta\mathbf{L}\end{bmatrix}\begin{bmatrix}\mathbf{M}_1\\ \mathbf{M}_2\end{bmatrix}, \tag{11}$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the values at the average positions and are given by

$$\begin{aligned} \mathbf{M}_1=\frac{j\omega}{2}\int_0^l e^{\gamma z}\left[\mathbf{Z}_C^{-1}\mathbf{V}(z)-\mathbf{I}(z)\right]dz\Big|_{x=\bar{x},y=\bar{y}} \\ \mathbf{M}_2=\frac{j\omega}{2}\int_0^l e^{\gamma(l-z)}\left[\mathbf{Z}_C^{-1}\mathbf{V}(z)+\mathbf{I}(z)\right]dz\Big|_{x=\bar{x},y=\bar{y}} \end{aligned}. \tag{12}$$

Equation (11) can not only be used to estimate the values of $\Delta\mathbf{V_{ZS}}$ and $\Delta\mathbf{V_{ZL}}$, but also give the probability distribution of the $\Delta\mathbf{V}$ $(=[\Delta\mathbf{V_{ZS}}\,\Delta\mathbf{V_{ZL}}])^T$ once the probability distribution of $\Delta\mathbf{L}$ is given. In addition, the upper limit of $\Delta\mathbf{V}$ can be estimated quickly if a norm is used.

From (1), the forward and backward waves at the position z can be expressed as

$$\begin{aligned} \mathbf{Z}_C^{-1}\mathbf{V}(z)-\mathbf{I}(z)=\left[\mathbf{Z}_C^{-1}\mathbf{V}(0)-\mathbf{I}(0)\right]e^{\gamma z} \\ \mathbf{Z}_C^{-1}\mathbf{V}(z)+\mathbf{I}(z)=\left[\mathbf{Z}_C^{-1}\mathbf{V}(l)+\mathbf{I}(l)\right]e^{\bar{\gamma}(l-z)} \end{aligned} \tag{13}$$

Substituting (13) into (12) and using the boundary conditions, (12) can be written as

$$\begin{aligned} \mathbf{M}_1=\frac{j\omega}{4}\gamma^{-1}\left(e^{2\gamma l}-\mathbf{1}\right)\left[\mathbf{Z}_C^{-1}\mathbf{V_{ZS}}+\mathbf{Z}_C^{-1}\mathbf{V}_S+\mathbf{Z}_S^{-1}\mathbf{V_{ZS}}\right]\Big|_{x=\bar{x},y=\bar{y}} \\ \mathbf{M}_2=\frac{j\omega}{4}\gamma^{-1}\left(e^{2\gamma l}-\mathbf{1}\right)\left[\mathbf{Z}_C^{-1}\mathbf{V_{ZL}}+\mathbf{Z}_C^{-1}\mathbf{V}_L+\mathbf{Z}_L^{-1}\mathbf{V_{ZL}}\right]\Big|_{x=\bar{x},y=\bar{y}} \end{aligned}, \tag{14}$$

It can be inferred from (11) and (14) that the variations $\Delta\mathbf{V_{ZS}}$ and $\Delta\mathbf{V_{ZL}}$ not only depend on the inductance variation $\Delta\mathbf{L}$ but also the parameters of the line, such as the characteristic impedance, terminal loads, and terminal voltage sources. For a multiconductor transmission line with a given distribution, the probability distribution of the $\Delta\mathbf{V}$ mainly depends on the probability distribution of $\Delta\mathbf{L}$.

For a multiwire transmission line over a perfectly conducting and infinite ground, as shown in Figure 1, $\mathbf{L}$ and $\mathbf{C}$ can be expressed as

$$\begin{aligned}\mathbf{L} &= \mu\mathbf{F}\\ \mathbf{C} &= \varepsilon\mathbf{F}^{-1},\end{aligned} \tag{15}$$

where μ and ε are the permeability and permittivity of the medium, respectively. Denoting $\delta x_i = x_i - \bar{x}_i$ and $\delta y_i = y_i - \bar{y}_i$, the matrix $\mathbf{F}$ can be written as [17]

$$\mathbf{F} = \frac{1}{2\pi}\begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n}\\ f_{21} & f_{22} & \cdots & f_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ f_{1n} & f_{2n} & \cdots & f_{nn}\end{bmatrix} \tag{16}$$

$$f_{ii} = \ln\left(\frac{2\left(\bar{x}_i + \delta x_i\right)}{a_i}\right) \tag{17}$$

$$f_{ij} = \ln\left(\frac{d'_{ij}}{d_{ij}}\right) = \frac{1}{2}\ln\frac{\left(\bar{x}_i + \bar{x}_j + 2\delta x\right)^2 + \left(\bar{y}_i - \bar{y}_j\right)^2}{d_{ij}^2} \quad (i \neq j). \tag{18}$$

Here a_i is the radius of the ith wire. The parameter d_{ij} is the distance between ith wire and the jth wire, while d'_{ij} is the distance between the ith wire and the image of the jth wire, as shown in Figure 1. (17) and (18) also indicate that only the translation along the x axis affect the parameters of the transmission line.

When δx is small compared with x_i, then f_{ii} and f_{ij} can be approximated by

$$f_{ii} \approx \ln\left(\frac{2\bar{x}_i}{a_i}\right) + \frac{\delta x}{\bar{x}_i} \tag{19}$$

$$f_{ij} \approx \frac{1}{2}\ln\left(1 + \frac{4\bar{x}_i\bar{x}_j}{d_{ij}^2}\right) + \frac{2\delta x\left(\bar{x}_i + \bar{x}_j\right)}{d_{ij}^2 + 4\bar{x}_i\bar{x}_j}. \tag{20}$$

From (9), (15), (19) and (20), $\Delta\mathbf{L}$ can be written as

$$\Delta\mathbf{L} = \left.\mathbf{A}\right|_{x_i=\bar{x}_i, y_i=\bar{y}_i}\delta x, \tag{21}$$

where

$$\mathbf{A} = \frac{\mu}{2\pi}\begin{bmatrix} \frac{1}{x_1} & \cdots & \frac{2(x_1+x_n)}{d_{1n}^2+4x_1x_n}\\ \vdots & \ddots & \vdots\\ \frac{2(x_1+x_n)}{d_{1n}^2+4x_1x_n} & \cdots & \frac{1}{x_n}\end{bmatrix}. \tag{22}$$

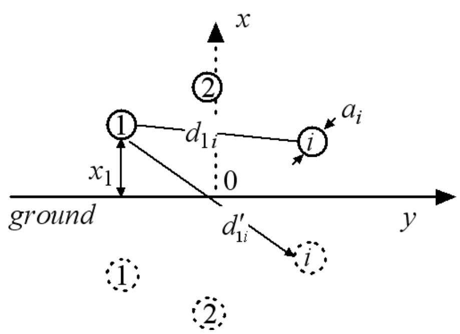


Figure 1. A multiconductor transmission line over the ground.

Then voltage variations $\Delta\mathbf{V_{ZS}}$ and $\Delta\mathbf{V_{ZL}}$ can be written as

$$\begin{bmatrix}\Delta\mathbf{V_{ZS}}\\ \Delta\mathbf{V_{ZL}}\end{bmatrix} = \begin{bmatrix}\mathbf{1}+\mathbf{\Gamma}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{1}+\mathbf{\Gamma}_2\end{bmatrix}\begin{bmatrix}-\mathbf{\Gamma}_1 & e^{\gamma l}\\ e^{\gamma l} & -\mathbf{\Gamma}_2\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{A}\end{bmatrix}\begin{bmatrix}\mathbf{M}_1\\ \mathbf{M}_2\end{bmatrix}\Bigg|_{x_i=\bar{x}_i, y_i=\bar{y}_i} \delta x \tag{23}$$

From (23), the values and the probability distributions of the voltage variations depend on the translation component δx, which is perpendicular to the ground, when it is small.

2.3. Sensitivity of Voltage Variation to Random Height

Because only the perpendicular translation component δx, corresponding to the variation of the height, has an influence on the voltages, the sensitivities of the voltages to the random height should be studied. The sensitivity S_h can be defined as

$$S_h \doteq \lim_{\delta x\to 0}\frac{\Delta V}{\delta x}. \tag{24}$$

Then

$$\begin{bmatrix}\mathbf{S}_{h\mathbf{ZS}}\\ \mathbf{S}_{h\mathbf{ZL}}\end{bmatrix} = \begin{bmatrix}\mathbf{1}+\bar{\mathbf{\Gamma}}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{1}+\bar{\mathbf{\Gamma}}_2\end{bmatrix}\begin{bmatrix}-\bar{\mathbf{\Gamma}}_1 & e^{\bar{\gamma} l}\\ e^{\bar{\gamma} l} & -\bar{\mathbf{\Gamma}}_2\end{bmatrix}^{-1}\begin{bmatrix}\bar{\mathbf{A}} & \mathbf{0}\\ \mathbf{0} & \bar{\mathbf{A}}\end{bmatrix}\begin{bmatrix}\bar{\mathbf{M}}_1\\ \bar{\mathbf{M}}_2\end{bmatrix} \tag{25}$$

It can be concluded from (11), (14), and (25) that the sensitivity vector increases with the terminal sources V_S and V_L, but decreases with the height of the transmission line over the ground. With the definition of the sensitivity, the relative sensitivity can be defined as $S_{hr} = S_h/V$. Compared with the sensitivity S_h, the relative sensitivity S_{hr} is influenced less by the terminal voltage sources and is a more suitable variable representing the influence of the random height.

When the translation component δx is a small quantity, then voltage variations at the loads can be deduced from (24) as

$$\Delta\mathbf{V} \approx \mathbf{S}_h \cdot \delta x \tag{26}$$

3. EXAMPLE

Figure 2 shows a two-wire line over the ground, where the separation d between the wires is 2 mm and the radius a is 0.5 mm. The height of the line is denoted by x and obeys Gaussian distribution with a mean of 15 mm and a variance σ_x of 1 mm. The terminal loads Z_{S1}, Z_{S2}, Z_{L1}, and Z_{L2} are all 50 Ω and the voltage source V_{S1} is 1 V with a frequency of 100 MHz. The proposed method is adopted to analyze this model and the results are compared with those obtained by the MC method where the number of samples is 5000 and the BLT equation is used to obtain the voltages for each sample.

The approximate means obtained by the proposed method are given in Table 1 and agree well with those from the MC method. When the height x of the line is 12 mm, 3 mm smaller than the average height, the estimated $\Delta\mathbf{V}$ by using (11) are given in Table 2, which are compared with those obtained by using the BLT equation twice.

The probability distribution of $\Delta\mathbf{V}$ will be analyzed by using the proposed method. Due to the small height variation compared with the average height, the voltage variations can be approximated by $\Delta\mathbf{V} \approx \mathbf{S}_h \cdot \delta x$. Consequently, the real part $\mathrm{Re}(\Delta V_i)$ and the imaginary part $\mathrm{Im}(\Delta V_i)$ of the voltage

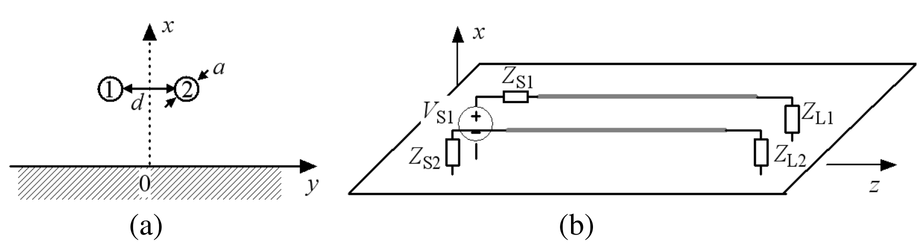


Figure 2. The model to be analyzed. (a) Cross section. (b) Configuration.

Table 1. Means obtained by the proposed method and the MC method.

	Proposed method	Monte Carlo
V_{ZS1} (mV)	$-169.5 - 80.7i$	$-169.5 - 80.7i$
V_{ZS2} (mV)	$145.4 + 14.1i$	$145.4 + 14.1i$
V_{ZL1} (mV)	$-113.2 - 270.8i$	$-113.2 - 270.8i$
V_{ZL2} (mV)	$94.1 + 133.9i$	$94.0 + 133.9i$

Table 2. $\Delta\mathbf{V}$ obtained by the proposed method and the BLT equation.

	ΔV_{ZS1} (mV)	ΔV_{ZS2} (mV)	ΔV_{ZL1} (mV)	ΔV_{ZL2} (mV)
Proposed method	$-1.7 - 1.8i$	$-1.4 - 2.0i$	$-1.3 - 4.2i$	$-1.1 - 4.1i$
BLT equation	$-1.8 - 1.9i$	$-1.5 - 2.1i$	$-1.4 - 4.5i$	$-1.2 - 4.4i$

Table 3. Sensitivities obtained by the proposed method and the difference method.

	S_{VZS1} (V/m)	S_{VZS2} (V/m)	S_{VZL1} (V/m)	S_{VZL2} (V/m)
Proposed method	$0.49 + 0.55i$	$0.42 + 0.58i$	$0.37 + 1.26i$	$0.34 + 1.24i$
Difference Method	$0.49 + 0.55i$	$0.41 + 0.58i$	$0.37 + 1.25i$	$0.34 + 1.23i$

variation at the ith load conform to the Gaussian distributions with the mean of 0 and variances of $|\mathrm{Re}(\mathbf{S}_{hi})|\sigma_x$ and $|\mathrm{Im}(\mathbf{S}_{hi})|\sigma_x$, respectively. The sensitivities of the load voltages obtained by (24) when the height is 15 mm are given in Table 3 and the results agree well with those obtained by the difference method, where the voltage variation ΔV when $\delta x = 0.01$ mm is calculated first and then the sensitivity is estimated by dividing ΔV by δx. Figures 3 and 4 show the probability distributions of the real and imaginary parts of ΔV_{ZS1} and ΔV_{ZL2} obtained by the proposed method, respectively, and they agree well with those obtained via the MC method.

Figures 5 and 6 show the sensitivities and the relative sensitivities of the load voltages to the height

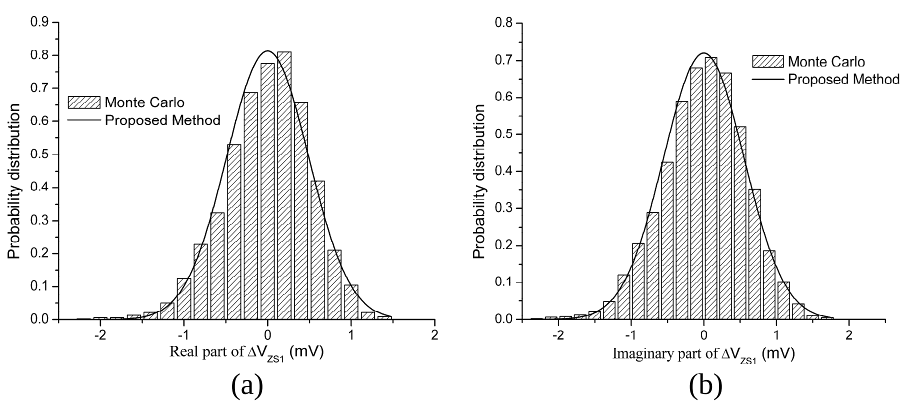
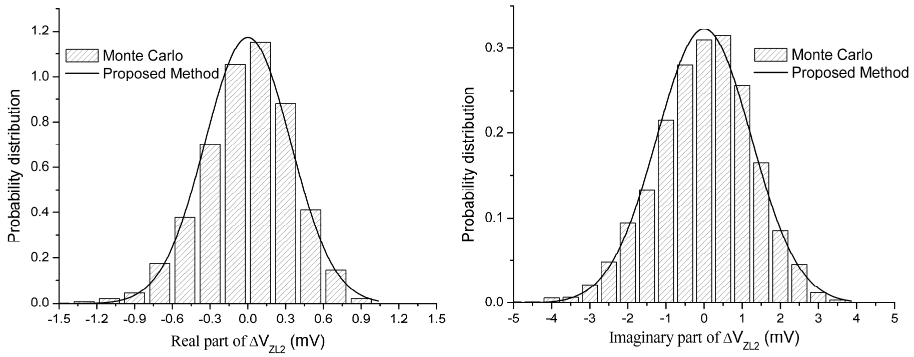
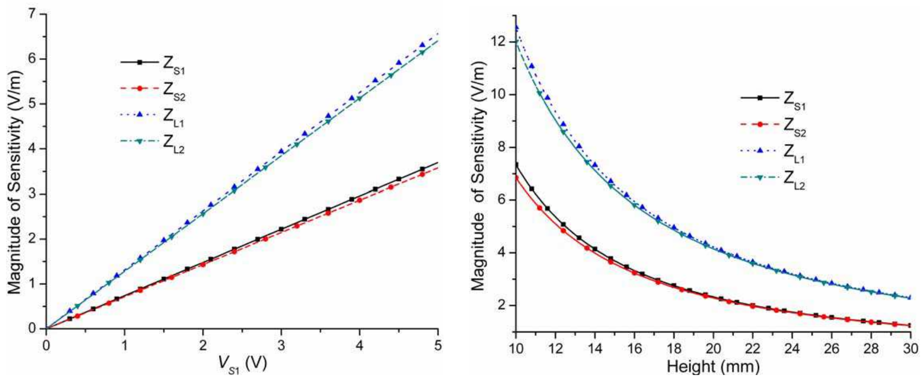


Figure 3. The probability distribution of ΔV_{ZS1}. (a) The real part. (b) The imaginary part.



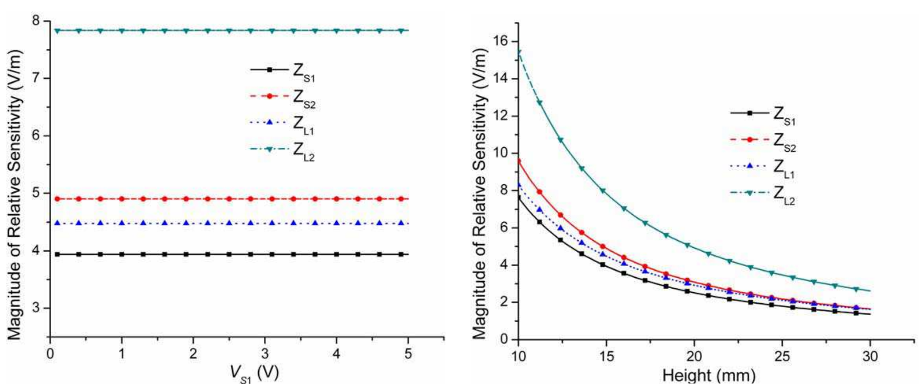
(a) (b)

Figure 4. The probability distribution of ΔV_{ZL2}. (a) The real part. (b) The imaginary part.



(a) (b)

Figure 5. The sensitivities of the load voltages change with the terminal voltage V_{S1} and the height of the transmission line. (a) Changing with the terminal voltage. (b) Changing with the height.



(a) (b)

Figure 6. The relative sensitivities of the load voltages change with the terminal voltage V_{S1} and the height of the transmission line. (a) Changing with the terminal voltage. (b) Changing with the height.

changing with the terminal voltage source V_{S1} and the height x obtained by the proposed method, respectively. The results show that the sensitivities increase with the voltage source V_{S1} but decrease with the height x. However, the relative sensitivities are independent of the voltage source V_{S1}. This is because that this configuration is linear and the voltages and the voltage variations increase with the voltage source V_{S1} together. Consequently, the relative sensitivity is a more suitable quantity to describe the influence of the random height than the sensitivity.

4. CONCLUSIONS

The influence induced by the random translation on the voltages of the uniform transmission line over the ground has been studied. The quick estimations of the average voltages at the loads and the probability distributions of the voltage variations have been proposed first and then the sensitivities of the load voltages to the random height have been studied. An example of a two-wire line over the ground is researched by using the proposed method. The average voltages and probability distributions of the voltage variations estimated by the proposed method agree well with those obtained by MC method. The proposed method is much more efficient than the MC method which needs a large number of repeated computations. The results show that: 1) only the translation component which is perpendicular to the ground affect the voltages at the loads; 2) when the translation component perpendicular to the ground is small quantity, the means of the voltages equal approximately to the voltages when the line is located at its average position; 3) the voltage variations not only depend on the inductance variation induced by the translation, but also relate to the characteristic impedance, terminal loads, and terminal sources of the line; 4) the sensitivities of the load voltages to the random height increase with the terminal sources but decrease with the height.

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