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Experimental Research of the Electric Field Potential of a Rotating Magnetized Sphere

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Abstract—We performed an experiment for the verification of existing theoretical formulas for the electric field potential of a rotating magnetized sphere. Measurements of the electric field potential differences across cylindrical capacitors were carried out. Experimental results are essentially in accord with the potential obtained using the special relativity transformations and contradict formula for the quadrupole field potential.

1. INTRODUCTION

Below, in a brief review, the theoretical formulas for the electric potential and electric field intensity of a rotating magnetized sphere are given, and some experimental results are discussed.

It is known that a homogeneously magnetized sphere rotating around the axis parallel to the magnetic moment of the sphere is the source of the potential electric field [1]. There exist different calculation methods for the electric field potential φ of the rotating magnetized sphere.

The first, the simplest one, supposes that the Lorentz force induces a displacement of free or bond electric charges rotating with a magnet. The Lorentz force is produced by the rotating magnetized body's self-magnetic field [2]. This method gives the correct value of the unipolar induction e.m.f. acting between the pole and the equator of the sphere and the quadrupole electric field. However, in the experiment of Wilson and Wilson [3], the Einstein's formula has been confirmed. Based on this result we can conclude that the charge polarization of the rotating magnetic insulator is observed only in the external magnetic field of a coil that is at rest in the lab frame, and the self-magnetic field of a rotating magnetic dielectric does not produce any polarization. In the experiment in [4], we attempted to detect free charges displacement in an aluminum conductor which was mounted on a magnet rotating together with the conductor. The experiment did not detect any displacement of the free electric charges in the conductor under the Lorentz force action.

The second method is based on several elements of the special relativity followed by the boundary-value electrostatic problem solving. In this method, the material Minkowski equations [1], the magnetic field transformation [2, 6] inside the magnetic sphere and the magnetic moment transformation [5, 6] are used. In this case, it is implicitly assumed that the special relativity transformations are applicable in the rotating frame of reference. Solutions obtained by this method also give the quadrupole electric field potential of the rotating magnetized sphere.

In particular, the expression for the scalar potential of the sphere — conductor ($\varepsilon = \infty$, ε — permittivity of the sphere), which was obtained using the material Minkowski equations, is [1]

$$\varphi = -\frac{1}{3c}\frac{m\omega R^2}{r^3}\left(3\cos^2\theta - 1\right), \tag{1}$$

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where m is the magnetic moment, R the radius, ω the angular velocity of the magnetized sphere, c the light speed, and r, θ are spherical coordinates of the observation point, $r \geq R$. The expression for the electric field obtained in [1] depends on the sphere material (dielectric, conductor) and radius of the sphere. The radial component of the electric field deduced from the potential (1) has the form [6]

$$E_r = -\frac{m\omega R^2}{cr^4}\left(3\cos^2\theta - 1\right). \tag{2}$$

The quadrupole electric field intensity falls off as $1/r^4$. The results of the experiment [4] exhibit that the discrepancies between the values of the quadrupole electric field (2) and experimental data of relative value measurements are about 79%.

The third method directly uses the Lorentz electromagnetic field potential transformations without solving the electrostatic problem [7]. The electric field potential is determined by the direct transformations from the frame of reference rotating with the sphere to the laboratory frame

$$\varphi = \frac{(\boldsymbol{\omega} \times \mathbf{r}) \cdot \mathbf{A}}{c} = \frac{m\omega}{cr}\sin^2\theta, \tag{3}$$

where $\mathbf{A}$ is the vector potential, $r \geq R$. The potential φ (3) is valid only inside the light cylinder $\omega r \ll c$.

The radial component of the electric field intensity obtained from (3) ($r \geq R$) is given by [7]

$$E_r = \frac{m\omega}{cr^2}\sin^2\theta. \tag{4}$$

This electric field intensity decreases with distance from the center of the sphere r as $1/r^2$. The electric field potential (3) and the electric field (4) are independent from material (conductor, dielectric) and radius R of the sphere. The values of the measured electric intensity of a rotating toroidal magnet [8] are in good agreement with theoretical values of the electric field deduced using the Lorentz transformations, the discrepancies are about 10%. In the experiment [4] with the toroidal magnet approximating the sphere the discrepancy between the Equation (4) and the experimental data of relative value measurements is about 10%.

2. EXPERIMENT AND DISCUSSION

2.1. Experimental Setup

In this paper we report measurements of the potential difference of the electric field of a rotating conducting magnet and compare the experimental data with the theory. A spinning magnet from neodymium sphere with diameter 30 mm was a source of the potential electric field. The axis of rotation passed through the center of the sphere and was parallel to the magnetic moment. The magnetic moment of the sphere was calculated from the magnetic field measurements at 15 points located in the equatorial plane on different distances from the center. Magnetic field was measured by the linearized Hall probe with the instrument 1.5% error. The magnetic moment was calculated for each measured value of the magnetic field. Further, the average value of the magnetic moment and the mean square error were calculated. This method allows to take into account errors of mutual orientation of the magnetic moment and of the Hall probe, distance measurement errors between the sphere center and the Hall probe. The relative mean square error of 15 measurements and total error were 0.5% and 1.6%, respectively. The measured value of the magnetic moment of the sphere was $13638 \pm 218\,\mathrm{G{\cdot}cm^3}$. The sphere was rotated at an angular velocity of 1470 rev/min (24.5 rev/s) with an accuracy to about 1%.

In the experiment, the potential difference was measured across the cylindrical air capacitors of sheet brass (0.4 mm thick). During measurements, one of the capacitors was mounted symmetrically with respect to the axis of rotation in the equatorial plane of the sphere (Figure 1). It was repeated each time before new series of measurements with other capacitor. The first air cylindrical capacitor armatures were 44.2 mm and 56.8 mm in diameters and 22.0 mm and 25.0 mm in heights, respectively. The second capacitor was 82 mm internal diameter, 96 mm external diameter and 26 mm height. The third capacitor with diameters 117 mm, 132 mm had a height of 26 mm. Capacitor armatures were connected with insulated gates of field effect transistors (FETs) of the input differential amplifier by

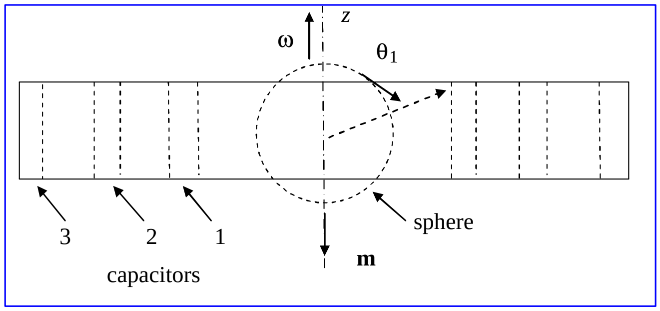

Figure 1. A scheme of the experimental setup (the rotating magnetized sphere and coaxial-cylinder capacitors).

thin wires with diameter 0.15 mm. The fabrication error of capacitors and device geometric dimensions was 0.5 mm. Air capacitors capacitance and the input capacitance of the FETs were measured by the LRC meter E7-11. The instrument error of the LRC meter E7-11 in our measurement range was $(1 + 20/C)\%$, where C is the value of measured capacitance in pF. Capacities of capacitors were 7.5 pF with an error of 3.7%, 12 pF — 2.7% and 15.5 pF — 2.3%. Input capacitances of KP305D FETs were 4.7 pF and 5.1 pF with an error of 5%. During measurements, the sphere, a capacitor, and an input amplifier were shielded from external fields by a grounded aluminum screen. An output signal of the differential amplifier was applied to the input of a direct current millivoltmeter, located outside of the shielded volume. Before measurements, the amplifier calibrating and the differential amplifier stage balancing were done, and the operating point of transistors was adjusted. The calibration voltage was 1 mV with an accuracy of 1.5%.

2.2. Measurements of the Potential Difference across Capacitors

The reading of the millivoltmeter deflection was taken. The double-deflection method [9] was used to make measurements, and the average number of divisions and the mean-square error were found. While the magnetic sphere rotates, an electrization of the sphere and capacitor occurs by an air flow. Electrical polarity of the potential difference across the capacitor, caused by the electrization, is independent from direction of rotation of the sphere, whereas polarity of the electromagnetic signal depends on direction of rotation. By subtracting the signals obtained, while the sphere spun in opposite directions, we subtracted a noise and summed useful signals. Potential differences across the capacitor were calculated by the formula

$$\Delta\varphi = nl \tag{5}$$

where n — the average number of divisions and l — the scale value of the millivoltmeter. The scale value for all measurements was 0.091 mV/div with an error of 5%. A total relative error of measurement of the potential difference across the capacitors was calculated by

$$\frac{\sigma_{\Delta\varphi}}{\Delta\varphi} = \sqrt{\frac{\sigma_n^2}{n^2} + \frac{\sigma_o^2}{n^2} + \frac{\sigma_l^2}{l^2}}, \tag{6}$$

where σ_n — the mean square error of the mean number of divisions, σ_o — the reading error, and σ_l — the error of the scale value. For the first capacitor the measured value of the potential difference was 1.84 mV with an error of 6%, for the second — 0.59 mV with an error of 11%, for the third — 0.34 mV with an error of 15%.

2.3. Calculation of the Potential Differences across Capacitors

In the potential difference calculating across capacitors some spatial distribution distortion of the electric field by the measuring circuit is taken into account. The cylindrical capacitor armatures are equipotential surfaces and ones average potential distribution along the coordinate θ (Figure 1). The average value of the potential of a capacitor armature was calculated by the formula

$$\varphi = \frac{1}{\pi/2 - \theta_1} \int_{\theta_1}^{\pi/2} \varphi\left(\theta, r\right) d\theta = \varphi\left(r_0\right) f\left(\theta_1\right), \tag{7}$$

where $\varphi(r_0)$ is the potential in the midpoint of an capacitor armature, $f(\theta_1)$ the averaging factor, and θ_1 the polar angle of the capacitor armature edge (Figure 1) with respect to the z-axis.

The input capacitance of the amplifier reduces the measured potential difference in k times, where

$$k = \frac{C_c}{C_c + C_{in}}, \tag{8}$$

here C_c is the cylindrical capacitor capacitance, C_{in} is the input capacitance of the amplifier.

Capacitances of the "sphere — an armature of a cylindrical capacitor" also affect the measured potential difference. An influence of the sphere-capacitor capacitance on the first capacitor (Figure 1) is stronger than the influence of one on a capacitor located father from the sphere center. It is difficult to estimate this effect, so it was not included in the calculation of the potential difference. This effect can be reduced by increasing radius of a capacitor, increasing its capacitance, decreasing the input capacitance of transistors.

The potential difference across the cylindrical capacitor deduced from Equation (1) for quadrupole electric field potential is given by

$$\Delta\varphi = \frac{m\omega R^2}{3c}\left(\frac{f_1}{r_1^3} - \frac{f_2}{r_2^3}\right) k. \tag{9}$$

Here f_1, f_2 are averaging coefficients, calculated from Equation (7), and k is the factor from (8).

An absolute error of the expression in parentheses was calculated by the formula

$$\sigma_{rf} = \sqrt{\sigma_1^2 + \sigma_2^2}. \tag{10}$$

Here σ_1, σ_2 are the absolute errors of the first and second terms in (9). The relative error of the potential difference (9) is

$$\frac{\sigma_{\Delta\varphi}}{\Delta\varphi} = \sqrt{\frac{\sigma_m^2}{m^2} + \frac{\sigma_\omega^2}{\omega^2} + \frac{\sigma_{rf}^2}{\left(r_1^{-3} f_1 - r_2^{-3} f_2\right)^2} + \frac{\sigma_k^2}{k^2}}. \tag{11}$$

The first term under the square root gave an error of 1.6%, the second — 1%, and the third (for the first capacitor) — 16%, the fourth (for the first capacitor) — 2.4%. Thus, the basic error of the calculated values of the potential difference is determined by an accuracy of the geometric dimensions of the capacitors.

The results of the potential difference calculated by Equation (9) and its error $\delta = \frac{\sigma_{\Delta\varphi}}{\Delta\varphi}$ calculated by the Equation (11) are $\Delta\varphi_1 = 0.41\,\text{mV}$, $\delta = 16\%$ for the first capacitor, $\Delta\varphi_2 = 0.05\,\text{mV}$, $\delta = 13\%$ for the second capacitor, $\Delta\varphi_3 = 0.02\,\text{mV}$, $\delta = 10\%$ for the third capacitor.

The potential difference due to (3) can be written in the form

$$\Delta\varphi = \frac{m\omega}{c}\left(\frac{f_1}{r_1} - \frac{f_2}{r_2}\right) k. \tag{12}$$

The results of calculations of the potential difference by the Equation (12) and errors δ are $\Delta\varphi_1 = 1.34\,\text{mV}$, $\delta = 13\%$ for the first capacitor, $\Delta\varphi_2 = 0.55\,\text{mV}$, $\delta = 11\%$ for the second capacitor, $\Delta\varphi_3 = 0.33\,\text{mV}$, $\delta = 10\%$ for the third capacitor.

All of the obtained results are presented in Table 1.

As seen from Table 1, discrepancies between the experimental values of the potential difference and the calculated values according to the Equations (9) or (1) are greater than measurement errors. For the first capacitor, the discrepancy between the experimental value of the potential difference and the theoretical value calculated according to Equations (12) or (3) is 0.5 mV, which is more than the

Table 1. Experimental and theoretical values of the potential difference across a capacitor.

No.	1	2	3
radii of the cylindrical capacitor armatures (cm)	2.21 ± 0.05 2.84 ± 0.05	4.10 ± 0.05 4.80 ± 0.05	5.85 ± 0.05 6.60 ± 0.05
experimental values of the potential difference across the capacitors (mV)	1.84 ± 0.11	0.59 ± 0.06	0.34 ± 0.05
theoretical values of the potential difference calculated by the Equation (9) (mV)	0.41 ± 0.06	0.055 ± 0.007	0.021 ± 0.002
theoretical values of the potential difference calculated by the Equation (12) (mV)	1.34 ± 0.17	0.55 ± 0.06	0.33 ± 0.03

measurement error. This discrepancy, possibly, arises due to capacity of the system: the sphere — the first capacitor. For the second and third capacitors, experimental values of the potential difference coincide within the experimental error with the values calculated by Equation (12).

What are the cause of large discrepancies between the experimental data and values deduced from the Equation (1) and the cause of small discrepancies between the experimental data and values deduced from the Equation (3)?

Theoretical methods, based on elements of the special relativity and subsequent decision of the boundary-value electrostatic problem, use the Lorentz transformations only inside the sphere [1, 5, 6]. Since the magnetic field of a magnetized sphere is distributed not only in the volume of the sphere, but also in the surrounding space, the Lorentz field transformations must be applied both inside and outside the sphere. Solutions, obtained using the Lorentz transformations inside a sphere and the Laplace equation outside the sphere, are incorrect.

It can be shown that the flow of the electric field vector (3) across a closed cylindrical surface with the generatrix parallel to the axis of sphere rotation is equal to zero inside the light cylinder ($\omega r \ll c$).

$$\oint E ds = \int_0^{2\pi} \rho d\varphi \int_{-\infty}^{\infty} E_\rho dz = 0, \tag{13}$$

where ρ, φ, z are cylindrical coordinates.

Thus, the total electric charge inside the cylindrical surface is equal to zero. This means that the Lorentz transformations applying inside and outside a magnetized sphere does not violate the charge conservation law.

It follows from Equation (3) and the equation of electrostatics that the charge is distributed in the magnetic field inside and outside the rotating magnetized sphere in the form of an apparent charge with the density

$$\rho = -\frac{\boldsymbol{\omega} \cdot \mathbf{B}}{2\pi c} \tag{14}$$

and with the surface density

$$\sigma = \frac{(\boldsymbol{\omega} \times \mathbf{R}) \cdot \mathbf{j}}{c^2}, \tag{15}$$

here $\mathbf{B}$ is the magnetic field induction and $\mathbf{j}$ the surface current density.

Indeed, it follows from the expression (14) that the charge density is independent from permittivity ε and conductivity of the sphere, and it exists in a vacuum. Thus, these apparent electrical charges do not have carriers in the form of electrons, protons or ions, and they are a manifestation of the purely relativistic first-order effect.

3. CONCLUSION

In this work we report an experiment, in which series of measurements of the potential difference across cylindrical capacitors were performed. Cylindrical capacitors were placed in the equatorial plane of a rotating conducting magnetic sphere coaxially to the axis of rotation. The experimental data allow us to state that the observed electric field is not the quadrupole field. The experimental data are essentially in accord with the theoretical Equation (3). A possible cause of discrepancies between the experimental data and the theoretical values for the quadrupole field potential is the incorrect statement of theoretical problem.

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