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Parametric Instability of Surface Electron Cyclotron TM-Modes

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Abstract—Excitation of waves at harmonics of electron cyclotron frequency due to utilization of an external alternating electric field is under the consideration. It is proved that they are eigen modes of plasma-dielectric-metal structures in both long (as compared with electron Larmor radius) wavelength range and short wavelength range if an external steady magnetic field is oriented perpendicularly to the plasma interface. It is assumed that uniform external electric field operates at the frequency, which belongs to the range of electron cyclotron frequencies. The problem is solved theoretically using kinetic Vlasov-Boltzmann equation for description of the plasma particles motion and Maxwell equations for description of TM-polarized field of these modes. Non-linear boundary condition for tangential magnetic field of these TM-modes is formulated using conception of non-linear surface electric current. Infinite set of equations for harmonics of their tangential electric field is derived due to this condition. This set is solved using approach of the wave packet consisting of the basic harmonic and two nearest satellite harmonics. Simple analytical expression for growth rate of surface electron cyclotron TM-modes' parametric instability is obtained and analyzed numerically.

1. INTRODUCTION

At present time, theory of bulk cyclotron waves is developed sufficiently well [see e.g., 1, 2], which is represented in a wide utilization of bulk electron cyclotron waves in nuclear fusion investigations [3–5] for additional plasma heating and plasma diagnostics. These waves are also applied to the development of new high frequency and high power electronic devices [6, 7]. A utilization of restricted plasma volumes for different practical purposes makes it possible to excite both bulk and surface types of waves [8]. In our previous articles [9, 10], we have studied the cases of surface electron cyclotron waves with extraordinary and ordinary polarization, correspondingly, and also their propagation under conditions, when an external magnetic field was assumed to be oriented parallel to a plasma-dielectric interface. Unlike these cases, here we study the case of perpendicular orientation of an external steady magnetic field and do not restrict our consideration by the wavelength range located near a limit of long wavelengths compared with Larmor radius of electron, as it has been done in [9, 10]. To derive a set of equations describing parametric excitation of surface electron cyclotron TM-modes (SECTM-modes) the non-linear boundary condition, which determines discontinuity of tangential magnetic field of studied modes, is formulated as done in papers [9, 10]. This discontinuity is determined by surface electric current, which is induced by external alternating electric field on the plasma interface.

Since cyclotron surface waves can be adequately analyzed only by kinetic approach and kinetic approach is more difficult than fluid (hydro-dynamical) approximation, surface waves are studied not as well as other waves described in fluid approximation, which can be studied with magneto-hydro-dynamical approach. Results of the SWs parametric excitation studied in magneto-hydro-dynamical approach are presented in, e.g., [11–13].

The goal of the present paper is to study parametric instability of surface electron cyclotron TM-modes under the influence of alternating electric field and compare obtained results with previous one

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devoted to studying cyclotron SW of other polarization. Thus it can be considered as the next step in development of general theory of cyclotron SWs parametric instabilities.

SECTM-modes were found theoretically to propagate along plane plasma-dielectric interface, when an external steady magnetic field is perpendicular to the plasma boundary, and penetration depth of the modes into plasma is much larger than their wavelength. These features distinguish them from surface cyclotron modes of other polarization (X- and O-modes). Eigen frequency of SECTM-modes decreases with increase of their wave vector oriented along the plasma interface, and their damping is determined by both collisional (interaction between plasma particles) and kinetic (interaction between particles and plasma interface) mechanisms. We suppose that they can propagate in a divertor region of fusion devices with magnetic confinement and can be applied as well for sustaining gas discharges of magnetron type [14].

The paper is structured as follows. The basic equations including the nonlinear boundary condition for the SECTM-modes magnetic field are presented in Section 2. Analytical expressions for growth rates of their parametric instability are derived in Section 3. Influence of plasma parameters and amplitude of the external alternating electric field on the SECTM-modes growth rates values is analyzed in Section 4. The summary of the obtained results is represented in Section 5.

2. THE BASIC EQUATIONS

Let us consider uniform magneto-active plasma, which occupies area $0 \leq z$ and is bounded by vacuum. An external constant magnetic field $\vec{B}_0$ is oriented along axis $\vec{z}$. Along $\vec{z}$ axis the spatial dispersion of plasma is supposed to be weak $k_3 \upsilon_{T\alpha} \ll |\omega - s\omega_\alpha|$, where k_3 is component of the SECTM-modes oriented along $\vec{z}$ axis, $\upsilon_{T\alpha}$ the thermal velocity of plasma particles, ω the modes frequency, s the series number of cyclotron harmonic, and ω_α the cyclotron frequency of α-type of plasma species ($\alpha = e$ for electrons and $\alpha = i$ for ions). The plasma is also affected by an external alternating electric field $\vec{E}_0 \cos(\omega_0 t)$, which is directed across axis $\vec{z}$. Frequency of the external alternating electric field ω_0 is of the same order as electron cyclotron frequency ω_e value. The alternating field $\vec{E}_0$ is assumed to be uniform that can be realized in the case of small value of gas-kinetic pressure of plasma.

The plasma particles motion is described by Vlasov-Boltzmann kinetic equation with Maxwellian non-perturbed plasma particles distribution function. Its solution for the indicated case of an external magnetic field orientation in respect to plasma-vacuum interface is the same as that realized for non-bounded plasma [15], if interaction between plasma particles and its surface is described by a mirror model. A set of Maxwell equations for the studied modes can be solve by Fourier method $\vec{E}$, $\vec{H} \propto \exp[i(k_1 x + k_3 z) - it\omega]$, then the complete set can be separated into two sets. One of them describes just TM-mode with the following components [16] E_x, H_y, E_z.

Taking into account of the influence of an external alternating electric field $E_0 \cos(\omega_0 t)$, one can solve the kinetic Vlasov-Boltzmann equation by the method of trajectories. Fourier coefficients of the electric current density j_1, j_3 and electric field of the SECTM-modes (E_1 and E_3) are connected by the following components of plasma conductivity tensor σ_{jk} calculated in the case of weak spatial dispersion of the plasma along the direction, which is perpendicular to its surface:

$$\sigma_{11}^{(n)} = \sum_{\alpha} \sum_{s,m,l} \frac{is^2 \Omega_\alpha^2 \exp\left(-y_\alpha\right) I_s\left(y_\alpha\right)}{4\pi y_\alpha \left(\omega_{n+m} - s\omega_\alpha\right)} J_m\left(a_E\right) J_{m-l}\left(a_E\right), \tag{1}$$

$$\sigma_{13}^{(n)} = \sigma_{31}^{(n)} = \sum_{\alpha} \sum_{s,m,l} \frac{isk_3 \omega_\alpha \Omega_\alpha^2 \exp(-y_\alpha) I_s(y_\alpha)}{4\pi k_1 \left(\omega_{n+m} - s\omega_\alpha\right)^2} J_m(a_E) J_{m-l}(a_E), \tag{2}$$

$$\sigma_{33}^{(n)} = \sum_{\alpha} \sum_{s,m,l} \frac{i\Omega_\alpha^2 \exp\left(-y_\alpha\right) I_s\left(y_\alpha\right)}{4\pi \left(\omega_{n+m} - s\omega_\alpha\right)} J_m\left(a_E\right) J_{m-l}\left(a_E\right), \tag{3}$$

here Ω_α and ω_α are plasma and cyclotron frequencies of the plasma particles. Subscript α is applied for designation type of plasma particles ($\alpha = e$ for electrons, $\alpha = i$ for ions), $a_E^2 = \frac{e_\alpha^2 k_1^2 (\omega_0^2 E_{0x}^2 + \omega_\alpha^2 E_{0y}^2)}{m_\alpha^2 \omega_0^2 (\omega_0^2 - \omega_\alpha^2)^2}$. ρ_α is Larmor radius of the plasma particles, $y_\alpha = k_1^2 \rho_\alpha^2 / 2$. $J_m(x)$, $I_n(z)$ and $I_m'(y)$ are Bessel function of the first kind, modified Bessel function and its derivative over the argument [17], respectively. Process of summarizing over subscripts s, m, l in these expressions can be executed independently from each other in the limits from $-\infty$ to $+\infty$, $\omega_{n+m} = \omega + (n+m)\omega_0$. Since operating frequency of the applied electric field is $\omega_0 \sim |\omega_e|$, one can estimate ratio of the arguments of the Bessel functions of the first kind in expressions for σ_{ik} tensor: $a_E(\alpha = e)/a_E(\alpha = i) \approx m_i/m_e \gg 1$. Therefore, in this case, ion terms can be neglected in the applied components of the plasma conductivity tensor σ_{ik}.

Solving algebraic set of equations for Fourier coefficients of the SECTM-modes fields in approximation of slow waves (it means that the wave phase velocity is much less than light velocity), one can obtain expression for n-th harmonic $E_1^{(n)}$ of Fourier coefficient $E_1 = \sum_{n=-\infty}^{+\infty} E_1^{(n)} \exp(-in\omega_0 t)$ of the tangential electric field in the plasma region:

$$\frac{2ck_1^2}{i\omega_n} H_y^{(n)}(+0) + \left(k_1^2 \psi_1^{(n)} + k_3^2 \psi_2^{(n)}\right) E_1^{(n)} = \sum_{\alpha} \sum_{s,m,l,l\neq 0} \left\{ k_1^2 \left\{ \frac{s^2}{y_\alpha} + \frac{k_3^2 (\omega_{n+m} + s\omega_\alpha)}{k_1^2 (\omega_{n+m} - s\omega_\alpha)} \right\} \times \frac{\Omega_\alpha^2 \exp(-y_\alpha) I_s(y_\alpha)}{\omega_n (\omega_{n+m} - s\omega_\alpha)} J_m(a_E) J_{m-l}(a_E) \right\} E_1^{(n+l)}, \quad (4)$$

here

$$\psi_1^{(n)} = 1 - \sum_{\alpha} \sum_{s,m} \frac{s^2 \Omega_\alpha^2 \exp(-y_\alpha) I_s(y_\alpha)}{y_\alpha \omega_n (\omega_{n+m} - s\omega_\alpha)} J_m^2(a_E),$$

$$\psi_2^{(n)} = 1 - \sum_{\alpha} \sum_{s,m} \frac{\Omega_\alpha^2 (\omega_{n+m} + s\omega_\alpha) \exp(-y_\alpha) I_s(y_\alpha)}{\omega_n (\omega_{n+m} - s\omega_\alpha)^2} J_m^2(a_E),$$

subscript literal indexes $\{x, y, z\}$ and subscript numerical indexes $\{1, 2, 3\}$ relate to functions and their Fourier coefficients, respectively. $H_y^{(n)}(+0)$ is the meaning of the SECTM-mode tangential magnetic field on the plasma interface. Presence of an addendum, which is proportional to $H_y^{(n)}(+0)$ in Equation (4), is connected with peculiarity of application the Fourier transform in the studied case of semi-bounded plasma. Analyzing Maxwell equations for the SECTM-modes' fields, one can make a conclusion that they have different symmetries with respect to changing sign of the normal coordinate z, namely $E_x^{(n)}(+z) = E_x^{(n)}(-z)$; $E_z^{(n)}(+z) = -E_z^{(n)}(-z)$ and $H_y^{(n)}(+z) = -H_y^{(n)}(-z)$. Therefore conducting Fourier transformation over z coordinate for the following derivative $dH_y^{(n)}/dz$, one can obtain: $\int_{-\infty}^{+\infty} dH_y^{(n)}/dz \cdot \exp(-ik_3 z) dz = -2H_y^{(n)}(+0)$.

Then by the aid of reverse Fourier transform, one can derive equation for n-th harmonic of the SECTM-modes tangential electric and magnetic fields on the plasma interface. This transform has been carried out using Jordan's lemma, which allows one to apply theory of residuals for calculation of these integrals. There is one imaginary root of the denominator of the right-hand side of the expression (4) that is located in the upper complex semi-plane of the k_3, namely:

$$k_3 = i\,|k_2| \sqrt{|\varepsilon_{11}/(\varepsilon_{33} + A)|}, \quad (5)$$

here $A \approx -2\Omega_e^2 I_S(y_e)/\ [\exp(y_e)\omega^2 h^2]$, $\quad h = 1 - s\omega_e/\omega$. According to the analysis made in [15, 16], the absolute value of $|\varepsilon_{11}|$ is much less than that of denominator of the expression (5). Therefore, the studied modes penetrate into plasma region sufficiently well, i.e., penetration depth is larger than these TM-modes' wavelength.

The value of magnetic field $H_y^{(n)}(+0)$ on the plasma interface, which is presented in expression (4), can be replaced by non-linear surface electric current using boundary conditions for tangential fields of this mode. Thus let us consider the problem of boundary conditions for SECTM-modes affected by an external electric field $\vec{E}_0 \cos(\omega_0 t)$, which is oriented perpendicularly to magnetic field $\vec{B}_0$ in details. There are two boundary conditions for the SECTM-modes fields on the plasma-vacuum ($z = 0$) interface. The first one is a well-known linear condition for tangential electric field of the wave, which means continuity of the SECTM-modes tangential electric field: $E_x^{(n)}(z = +0) = E_x^{(n)}(z = -0)$. The other boundary condition is a non-linear one, which describes the flowing of a surface electric current along the plasma boundary. This surface electric current is induced by the external alternating electric field oriented across the utilized steady magnetic field:

$$\left|H_y^{(n)}(z=+0)-H_y^{(n)}(z=-0)\right|$$
$$=\int_{-0}^{+0}\frac{4\pi}{c}j_z^{(n)}dz=\sqrt{\psi_1^{(n)}\psi_2^{(n)}}\sum_{\alpha}\sum_{s,m,l,l\neq 0}\frac{s\omega_\alpha\Omega_\alpha^2\exp(-y_\alpha)I_s(y_\alpha)}{ic\,|k_1|\,(\omega_{n+m}-s\omega_\alpha)^2}J_m(a_E)J_{m-l}(a_E)E_x^{(n+l)}(0). \tag{6}$$

Application of these boundary conditions allows one to derive the infinite set of equations for n-th harmonics of SECTM-modes tangential electric field on the plasma interface in the form, which is analogous to those obtained in [9, 10]. Let us write it below:

$$D_n(\omega,k_1)E_x^{(n)}(0)-\sum_{l,l\neq 0}F_{n,l}(\omega,k_1)E_x^{(n+l)}(0)=0, \tag{7}$$

here

$$D_n(\omega,k_1)=1-\left(\psi_1^{(n)}\psi_2^{(n)}\right)^{-1/2}, \tag{8}$$

$$F_{n,l}(\omega,k_1)=\sum_{\alpha}\sum_{s,m}\frac{s\omega_\alpha\Omega_\alpha^2\exp(-y_\alpha)I_s(y_\alpha)}{\omega_n(\omega_{n+m}-s\omega_\alpha)^2}J_m(a_E)J_{m-l}(a_E). \tag{9}$$

3. RESULTS OF ANALYTICAL RESEARCH

Structure of the infinite set of Equation (7) can be represented as equality to zero of the product of the matrix, which is composed by diagonal D_n and non-diagonal $F_{n,l}$ elements (from one side), and corresponding harmonics of tangential electric field of the SECTM-modes on the plasma-vacuum interface (from another one). Thus, the location of every element of this matrix is determined by a row index, which is equal to superscript n, and a column index, which is equal to superscript $n+l$. To simplify the consideration, one can assume that the main harmonic of the SECTM wave packet is the harmonic with $n=0$, then the equality $D_{n=0}(\omega,k_2)=0$ is dispersion equation of these modes. Coefficients $F_{n,l}$ (non-diagonal elements of the indicated matrix) describe influence of an external alternating electric field on these modes. Therefore, if a uniform set of Equation (7) has solution, then the determinant composed by elements of the indicated matrix can be equal to zero.

As one can see from analysis of expressions (9), absolute values of coefficients nearby satellite harmonics $E_x^{(n+l)}(0)\,|_{n,l\neq 0}$ decrease very quickly with increasing values $|n|$ and $|l|$ (in other words if $n\to\pm\infty$ and/or $l\to\pm\infty$, then $F_{n,l}\to 0$). Thus to obtain approximate analytical solution of the set (7), one can take into account of only the main harmonic and two of the nearest satellite harmonics. Then parametric excitation of the SECTM-modes can be described by the following reduced equation:

$$\begin{vmatrix} D_{-1} & F_{-1;+1} & F_{-1;+2} \\ F_{0;-1} & D_0 & F_{0;+1} \\ F_{+1;-2} & F_{+1;-1} & D_{+1} \end{vmatrix}=0. \tag{10}$$

Let us assume that for the main harmonic of these modes, the resonant condition $\omega=s|\omega_e|+\Delta_T+\gamma-n\omega_0$ is realized. Here the correction γ to the SECTM-modes frequency is supposed to be of small value: $|\gamma|\ll s|\omega_e|$, s is the series number of electron cyclotron harmonic (arbitrary natural number), and $\Delta_T=\Delta_T(s,y_e)$ is their frequency shift in respect to the electron cyclotron frequency [15, 16]. In the limiting case of weak amplitude of an external alternating electric field $a_E\ll 1$, one can derive the following equation:

$$D_0-\left\{\frac{\Omega_e^2\exp(-y_e)I_s(y_e)}{(\gamma+\Delta_T)^2}\right\}^2\frac{s\omega_e\omega}{\omega^2-\omega_0^2}\cdot\frac{a_E^2}{2}=0, \tag{11}$$

Its analytical solution has the following approximate forms in different ranges of these modes' wavelength. In the range of long wavelengths $y_i\ll 1$:

$$\mathrm{Im}\gamma\approx|\Delta_T|\left\{\frac{y_e^2\cdot a_E^2}{I_s(y_e)\,2\,(s^2-1)^2}\right\}^{1/5}. \tag{12}$$

If the range of intermediate wavelengths $y_e \ll 1 \ll y_i$ is under consideration, then growth rate of the SECTM-modes parametric excitation can also be described by formula (12). But in the range of short wavelengths ($y_e \gg 1$) it becomes larger than that in the previous case:

$$\mathrm{Im}\gamma \approx |\Delta_T| \left[\sqrt{2\pi} a_E^2 \frac{\omega_e^2}{\Omega_e^2}\right]^{1/5} \sqrt{y_e}. \tag{13}$$

Analysis of these expressions testifies that the growth rates of the SECTM-modes parametric instabilities $\mathrm{Im}\gamma$ are larger than analogous growth rates of the bulk quasi-potential electron cyclotron waves [18]. Values of the SECTM-modes growth rates increase with decrease of their wavelengths and series number of cyclotron harmonic.

4. RESULT OF NUMERICAL RESEARCH

For completeness of analysis of the equations, which describe the initial stage of the SECTM parametric excitation, Equation (10) is solved numerically. Results of the numerical analysis are represented in Figs. 1–4, which demonstrate enough good coincidence with results of analytical investigation for the SECTM-modes parametric excitation. Numerical investigation allows one to obtain some additional information on the initial stage of their parametric instability.

Figure 1 is devoted to the illustration of dispersion properties of the SECTM-modes. One can see that $\mathrm{Re}(\omega/|\omega_e|)$ increases with increase of $Z = \Omega_e^2/\omega_e^2$. Also for $Z = 1.5$, 10 there are pronounced maximums, and their positions shift to higher values of $x = k_1\rho_e$ with increasing cyclotron harmonics number S and/or parameter Z value, but for $Z = 100$ a dispersion curve only becomes less steep without changing the maximum value and its position. Parameter Z is proportional to the plasma density value; that is why curves in Fig. 1 show influence of the plasma density on the form of their dispersion curves and on the SECTM-modes frequency. Dependence of the modes' dispersion properties upon relative value of a plasma density is not unique feature of the studied SWs.

Comparison of the SECTM-modes dispersion curves with schematic dispersion curves, presented in [1], allows one to conclude that they are similar to longitudinal (quasi-potential) bulk electron cyclotron waves, which are described by dispersion equation $\varepsilon_{11}(\omega, k_1) = 0$. One can see that increasing these bulk cyclotron waves' frequency divided on electron plasma frequency leads to deformation of their dispersion curves from a step-like shape to the shape with a maximum of their frequency located in the range of intermediate values of their wave vector k_1 [1]. Appearance of the situation when SECTM-modes frequency value corresponds to two different meanings of the parameter $x = k_1\rho_e$ can be explained

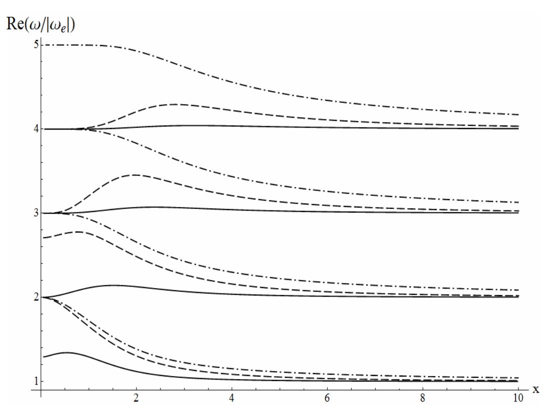

Figure 1. Eigen frequencies of the SECTM-modes vs normalized wave number $x = k_1\rho_e$; $b_E = 0.5$; $\omega_0 = |\omega_e|/2$. Solid, dashed, dot dashed curves relate to $Z = \Omega_e^2\omega_e^{-2} = 1.5$, 10 and 100, correspondingly.

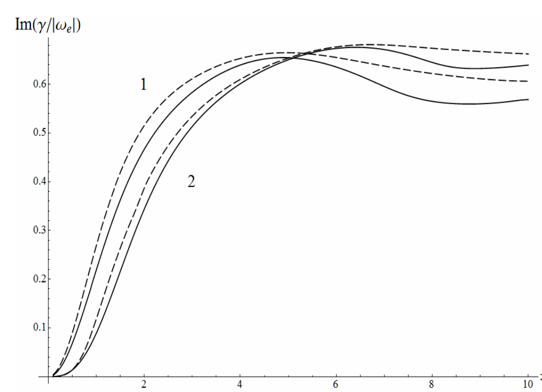

Figure 2. Growth rate of the SECTM-modes vs product $k_1\rho_e$; $b_E = 0.5$; $\omega_0 = |\omega_e|/2$. Solid and dashed curves relate to $Z = \Omega_e^2\omega_e^{-2} = 1.5$ and 10, correspondingly. Numerals 1 and 2 denote the series number of cyclotron harmonic $s = 1$, 3, correspondingly.

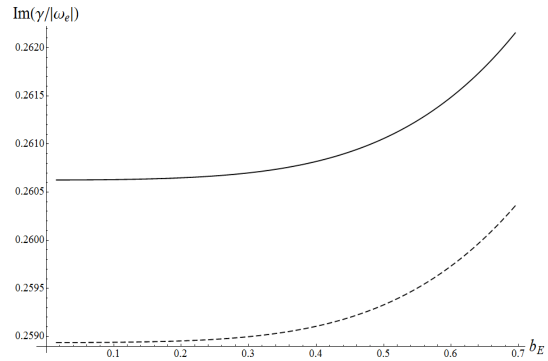

Figure 3. Growth rate of the SECTM-mode at the first frequency band $1 < \omega/|\omega_e| < 2$ vs dimensionless amplitude of a pumping electric field b_E; $\omega_0 = |\omega_e|/2$; $k_1\rho_e = 1$. Solid and dashed curves relate to $Z = \Omega_e^2\omega_e^{-2} = 50$ and 100, correspondingly.

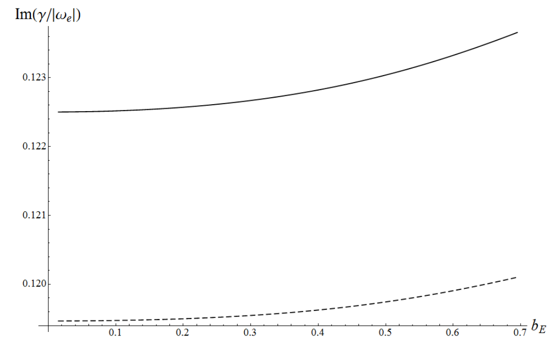

Figure 4. Growth rate of the SECTM-mode at the second frequency band $2 < \omega/|\omega_e| < 3$ vs dimensionless amplitude of a pumping electric field b_E; $\omega_0 = |\omega_e|/2$; $k_1\rho_e = 1$. Solid and dashed curves relate to $Z = \Omega_e^2\omega_e^{-2} = 50$ and 100, correspondingly.

mathematically by their dispersion properties. Eigen frequencies of both bulk longitudinal electron cyclotron waves and SECTM-modes are dependent on the product $I_s(y_e)\exp(-y_e)$, here $y_e = x^2/2$. So that in the range of long wave lengths the eigen frequency values of the studied surface modes increase with increasing parameter $x = k_1\rho_e$, and in the range of short wave lengths they decrease with increasing x. But in spite of equality of the eigen frequency values, they are two different wave perturbations: they have different meanings of wave vector k_1 and different values of phase and group velocities. It should be emphasized that the SECTM-modes power transfers in mutually opposite directions in the ranges of short and long wave lengths. Similar situations are realized as well for bulk cyclotron modes with ordinary and extraordinary polarizations [1], but their dispersion curves are characterized by the presence of minimums in the range of intermediate values of $x = k_1\rho_e$.

Analyzing dispersion curves presented in Fig. 1, one can see that decreasing the plasma density (parameter Z) leads to decrease of the SECTM-modes eigen frequency's shift in respect with corresponding electron cyclotron harmonic (module of the parameter h turns to zero). As indicated in [15, 16], this means that collisional damping δ_{col} of these modes becomes stronger if the plasma density becomes less, because $\delta_{col} \propto 1/|h|$. Therefore, decreasing the plasma density makes conditions of SECTM-modes' existence worse from physical point of view. But for step-like form of dispersion curves decreasing $|h|$ for intermediate values of k_1 is impossible, which is why these curves change their forms as shown in Fig. 1. Such forms (with maximum of the frequency in the intermediate range of wave lengths) of dispersion curves can describe strengthening the SECTM-modes damping in the whole diapason of the possible values of the $x = k_1\rho_e$.

Dependence of these modes' growth rate $\mathrm{Im}(\gamma/|\omega_e|)$ upon the wave numbers value and values of an external magnetic field (in other words, on dimensionless parameter $Z = \Omega_e^2/\omega_e^2$) is presented in Fig. 2. In the tested cases, $\mathrm{Im}(\gamma/|\omega_e|)$ increases with increase of the product $x = k_1\rho_e$, but after $x \approx 5$, this dependence becomes non-monotonous character. Decreasing Z value increases growth rate values $\mathrm{Im}(\gamma/|\omega_e|)$. This result coincides with analogous result obtained for surface electron cyclotron O-modes [10]. Increasing electron cyclotron harmonics number shifts curves $\mathrm{Im}(\gamma/|\omega_e|)$ as a whole to the side of higher values of parameter x. In case of harmonics numbers $S = 2$ and 4, the difference of the corresponding curves from the represented curves is very small, thus it will be difficult to distinguish them from each other on the common plot, which is why in this figure, we have not drawn the curves concerning the above cases.

Dependences of the SECTM growth rates parametric instability upon dimensionless amplitude of an external alternating electric field b_E are shown in Figs. 3 and 4 for the first and second electron cyclotron harmonics, respectively. Regardless of the settings, $\mathrm{Im}(\gamma/|\omega_e|)$ increases with increase of parameter b_E. Also on both figures one can see that changing Z does not essentially influence the shape of the curves. Unlike this changing electron cyclotron harmonic number S from 1 to 2 leads to decreasing absolute value of the parametric instability growth rates more than two times and to decreasing the rate of enlarging of the $\text{Im}(\gamma/|\omega_e|)$ curves generally.

5. CONCLUSIONS

In the present paper, the infinite set of equations for harmonics of SECTM-modes tangential electric field propagating along the interface between uniform semi-bounded plasma and vacuum is derived. Its solution describes an initial stage of SECTM-modes parametric instability. Analytical expressions for SECTM-modes' growth rate are obtained in the limiting case of a weak plasma spatial dispersion along normal direction relative to the plasma interface. Unlike the results of our previous papers [9, 10], the present results can be applied to arbitrary serial number of cyclotron harmonic and for the both long and short wavelength's ranges. It differs from expressions obtained for the bulk electron cyclotron waves [18].

Amplitude of an external alternating electric field, wavelength of the studied modes and their series numbers of electron cyclotron harmonics exert the main influence on the initial stage of this parametric instability. Decreasing SECTM-modes wavelength and increasing amplitude of an external alternating electric field lead to increasing values of growth rates of these modes.

The obtained results can be useful, first, for development of plasma technologies based on utilization of surface electron cyclotron waves, because application of surface waves has many advantages compared with the case of bulk waves application for sustaining gas discharges with large operating surface and uniform plasma production [19]. Second, SECTM-mode's application can be useful for diagnostics of periphery of fusion plasma [20] and for searching possibility to decrease plasma periphery heating. Third, surface waves are prospective for developing electronic devices based on application of new plasma-like meta-materials, which allows one to construct miniaturized electronic devices [21, 22]. Fourth, they can also be used in the development of electronic devices, which apply gaseous plasma [23].

And finally, one cannot miss a chance to point out a new field of applying theoretical knowledge obtained in the branch of classical electrodynamics of a bounded plasma, and we mean plasmonics (see [24] and references therein). This branch of physics studies collective motion of conductivity electrons in a metal nano-structure excited by electromagnetic waves, whose wave length belongs to the visible light spectrum. Due to this influence, these electrons can oscillate near the nano-structures' interface, which allows one to develop electromagnetic generators operating in THz frequency range and to construct highly effective solar cells structures and even devices, which can be used for investigation of some biological interaction and some biomaterials [25].

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REFERENCES

1. Akhiezer, A. I., I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, *Plasma Electrodynamics*, Pergamon Press, Oxford, 1975.
2. Stix, T. H., "Waves in plasmas: Highlight from the past and present," *Physics of Fluids B*, Vol. 2, 1729–1743, 1990.
3. Litvak, A. G., *High-frequency Plasma Heating*, American Institute of Physics, NY, 1992.
4. Ram, A. K., K. Hizanidis, and Y. Kominis, "Scattering of ECRF waves by edge density blobs and fluctuations in tokamak plasmas," *Proceedings of 17th Joint Workshop on Electron Cyclotron Emission and Electron Cyclotron Resonance Heating*, The Netherlands, 01003_p1–01003_p10, 2012.

5. Prater, R., R. J. Buttery, J. De Boo, et al., "Application of ECH on the DIII-D tokamak and projections for future ECH upgrades," *Proceedings of 17th Joint Workshop on Electron Cyclotron Emission and Electron Cyclotron Resonance Heating*, 02010_p1–02010_p6, 2012.
6. Kern, S., J-P. Hogge, S. Alberti, et al., "Experimental results and recent developments on the EU 2 MW 170 GHz coaxial cavity gyrotron for ITER," *Proceedings of 17-th Joint Workshop on Electron Cyclotron Emission and Electron Cyclotron Resonance Heating*, 04009_p1–04009_p6, 2012.
7. Kartikeyan, M. V., E. Borie, and M. K. A. Thumm, *Gyrotrons: High-power Microwave and Millimeter Wave Technology*, Springer-Verlag, Berlin, 2004.
8. Landau, L. D. and Y. M. Lifshits, *Course of Theoretical Physics. Electrodynamics of Continuous Media*, Vol. 8, Pergamon Press, Oxford, 1960.
9. Girka, V. O. and O. E. Sporov, "Parametrical instability of the surface waves on the second harmonic of electron cyclotron frequency in plasma layer," *Contributions to Plasma Physics*, Vol. 37, No. 6, 511–520, 1997.
10. Girka, V. O., A. V. Girka, and V. V. Yarko, "Parametric excitation of surface electron cyclotron O-modes by an external alternating electric field," *Physica Scripta*, Vol. 84, 045503, 2011.
11. Dragila, R. and S. Vucovic, "Excitation of surface waves by an electromagnetic wave packet," *Phys. Rev. Lett.*, Vol. 61, 2759–2761, 1988.
12. Stenflo, L. and G. Brodin, "Parametric decay of whistler waves in electron magneto-hydrodynamics," *Physica Scripta*, Vol. 83, 035503, 2011.
13. Besson, T. and W. S. Edwards, "Two-frequency parametric excitation of surface waves," *Physical Review E*, Vol. 54, No. 1, 507–513, 1996.
14. Margot-Chaker, J., M. Moisan, and J. Teichmann, "A new approach to the development of ECR plasma sources," *Proceedings of the 1st International Workshop "Strong Microwaves in Plasmas"*, Vol. 1, 473–478, Suzdal, Russian Federation, 1991.
15. Girka, V. and A. Kondratenko, "Surface cyclotron waves in a semilimited gyrotropic plasma," *Radiophysics and Quantum Electronics*, Vol. 23, No. 12, 938–941, 1980.
16. Girka, V. O., I. O. Girka, and I. V. Pavlenko, "HF surface cyclotron waves in planar waveguide structures with nonuniform plasma filling," *Journal of Plasma Physics*, Vol. 58, Part 1, 31–39, 1997.
17. Abramowitz, M. and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, National Bureau of Standards Applied Mathematics, Series 55, NY, 1964.
18. Kitsenko, A. B., J. G. Lominadze, and K. M. Stepanov, "Parametric excitation of electron cyclotron oscillations of plasma in alternating electric field," *Journal of Experimental and Theoretical Physics*, Vol. 66, 611–621, 1974 (in Russian).
19. Margot, J. and M. Moisan, "Surface wave sustained plasmas in static magnetic fields for study of ECR discharge mechanisms," *Microwave Excited Plasmas*, M. Moisan and J. Pelletier (eds.), Elsevier, Amsterdam, 1992.
20. Dine, S., J.-P. Booth, G. A. Curley, et al., "A novel technique for plasma density measurement using surface-wave transmission spectra," *Plasma Sources Science and Technology*, Vol. 14, 777–786, 2005.
21. Ederra, I., J. C. Iriarte, R. Gonzalo, et al., "Surface waves of finite size electromagnetic band gap woodpile structures," *Progress In Electromagnetics Research B*, Vol. 28, 19–34, 2011.
22. Xiong, J., H. Li, B. Z. Wang, et al., "Theoretical investigation of rectangular patch antenna miniaturization based on the DPS-ENG bi-layer super-slow TM wave," *Progress In Electromagnetics Research*, Vol. 118, 379–396, 2011.
23. Vlasov, A. N., B. Z. Shkvarunets, J. C. Rodgers, et al., "Overmoded GW-class surface-wave microwave oscillator," *IEEE Transactions on Plasma Science*, Vol. 28, No. 3, 505–560, 2000.
24. Maier, S. A., *Plasmonics: Fundamentals and Applications*, Springer, New York, 2007.
25. Green, R. J., R. A. Frazier, and K. M. Shakesheff, "Surface plasmon resonance analysis of dynamic biological interactions with biomaterials," *Biomaterials*, Vol. 21, 1823–1835, 2000.