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EXACTLY CALCULABLE FIELD COMPONENTS OF A HORIZONTAL ELECTRIC DIPOLE IN BOUNDARY BETWEEN ISOTROPIC AND ONE-DIMENSIONALLY ANISOTROPIC MEDIA

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Abstract—In this paper, the integrated formulas for the electromagnetic field in the planar boundary between isotropic and one-dimensionally anisotropic media due to a horizontal electric dipole situated on the interface are treated in detail, and the calculable field components are given in terms of series that involve confluent hypergeometric functions, namely, the Fresnel and exponential integrals. The expressions are more complex than the isotropic case, and the exact expressions and simplified formulas can be easily reduced to the corresponding isotropic case. The results are useful to study the propagation of the electromagnetic waves on the boundary of one-dimensionally anisotropic earth or sediments.

1. INTRODUCTION

The electromagnetic (EM) fields from a vertical electric dipole or a horizontal electric dipole near the interface between two different media, such as earth and air or sea water and rock, have been known in terms of general integrals for many years [1–15]. Following Sommerfeld [1], the EM fields were expressed in terms of the derivatives of Hertz potential for sufficiently large distance between the dipole

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source and the observation point. Many investigators, especially, Baños [2], Wait [3], and King [4, 5], have re-visited these problems aiming at the asymptotic evaluation on these integrals.

When studying the EM waves in stratified medium including the earth, the earth is not an isotropic medium and can be approximated by one-dimensionally anisotropic half space [4, 12], the properties of the EM field radiated by dipoles in this case have been investigated by several investigators, especially Li [13] and Pan [14]. Pan [14] had investigated the EM fields when the horizontal dipole and observation point are both located in the isotropic medium. In 2001, Margetis and Wu [15] examined the Sommerfeld integrals for EM fields in the planar boundary between air and a homogeneous, isotropic medium due to a horizontal electric dipole lying along the interface, and some integrals of the EM fields are exactly evaluated by series that involve the exponential and Fresnel integrals.

In the present study, the integrated formulas for the EM field in the planar boundary between isotropic and one-dimensionally anisotropic media due to a horizontal electric dipole situated on the interface are treated in detail, and the calculable field components are given in terms of series that involve hypergeometric functions, and the expressions are more complex than the isotropic case. The results can be reduced to the corresponding isotropic case and are useful to study the propagation of the electromagnetic waves upon the boundary of one-dimensionally anisotropic earth or sediments. The time dependence $e^{-i\omega t}$ is used throughout the whole text.

2. EXACT FIELD COMPONENTS OF HORIZONTAL ELECTRIC DIPOLE

2.1. Integrated Representations for the Field Components of a Horizontal Electric Dipole

The relevant geometry and Cartesian coordinate system are illustrated in Fig. 1, where a unit horizontal electric dipole in the $\hat{x}$ direction is located at $(0, 0, d)$. The upper half-space is Region 1 ($z \geq 0$) occupied with isotropic medium characterized by permittivity ε_1 and conductivity σ_1, the rest half-space is one-dimensionally anisotropic medium ($z \leq 0$) with $\varepsilon_x = \varepsilon_y = \varepsilon_T$, $\varepsilon_z = \varepsilon_L$, and $\sigma_x = \sigma_y = \sigma_T$, $\sigma_z = \sigma_L$, which leads to the following representation for the complex permittivity [5, 14]

$$\widetilde{\varepsilon}_1 = \begin{bmatrix} \varepsilon_1 + i\sigma_1/\omega & 0 & 0 \\ 0 & \varepsilon_1 + i\sigma_1/\omega & 0 \\ 0 & 0 & \varepsilon_1 + i\sigma_1/\omega \end{bmatrix} \tag{1}$$

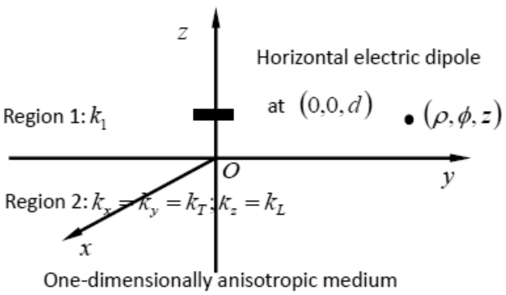

Figure 1. The geometry and Cartesian coordinate system for a unit horizontal dipole between the sea water and one-dimensionally anisotropic rock.

$$\widetilde{\varepsilon}_2 = \begin{bmatrix} \varepsilon_T + i\sigma_T/\omega & 0 & 0 \\ 0 & \varepsilon_T + i\sigma_T/\omega & 0 \\ 0 & 0 & \varepsilon_L + i\sigma_L/\omega \end{bmatrix} \tag{2}$$

It is assumed that both Region 1 and Region 2 are nonmagnetic so that $\mu_1 = \mu_2 = \mu_0$.

When both the dipole source and the observation point approach boundary ($d \to 0^+,\ z \to 0^+$), the Fourier-Bessel representations for the EM field in the cylindrical coordinates (ρ, ϕ, z) with $x = \rho\cos\phi$ and $y = \rho\sin\phi$ ($0 \le \phi \le 2\pi$) had been addressed in [5] and [14], they are

$$E_{1z} = -\frac{i\omega\mu_0\cos\phi}{4\pi k_1^2}\int_0^\infty \frac{k_1^2\gamma_e - k_T^2\gamma_1}{k_1^2\gamma_e + k_T^2\gamma_1} J_1(\lambda\rho)\lambda^2 d\lambda \tag{3}$$

$$\begin{aligned} E_{1\rho} = & -\frac{\omega\mu_0\cos\phi}{4\pi}\int_0^\infty \left\{ \frac{1}{\gamma_T + \gamma_1}[J_0(\lambda\rho) + J_2(\lambda\rho)] \right. \\ & \left. + \frac{\gamma_e\gamma_1}{k_1^2\gamma_e + k_T^2\gamma_1}[J_0(\lambda\rho) - J_2(\lambda\rho)] \right\} \lambda d\lambda \end{aligned} \tag{4}$$

$$\begin{aligned} E_{1\phi} = & \frac{\omega\mu_0\sin\phi}{4\pi}\int_0^\infty \left\{ \frac{1}{\gamma_T + \gamma_1}[J_0(\lambda\rho) - J_2(\lambda\rho)] \right. \\ & \left. + \frac{\gamma_e\gamma_1}{k_1^2\gamma_e + k_T^2\gamma_1}[J_0(\lambda\rho) + J_2(\lambda\rho)] \right\} \lambda d\lambda \end{aligned} \tag{5}$$

$$B_{1z} = \frac{i\mu_0\sin\phi}{2\pi}\int_0^\infty \frac{1}{\gamma_T + \gamma_1} J_1(\lambda\rho)\lambda^2 d\lambda \tag{6}$$

$$B_{1\rho} = -\frac{\mu_0 \sin\phi}{8\pi}\int_0^\infty \left\{\frac{\gamma_T-\gamma_1}{\gamma_T+\gamma_1}[J_0(\lambda\rho)-J_2(\lambda\rho)] -\frac{k_1^2\gamma_e-k_T^2\gamma_1}{k_1^2\gamma_e+k_T^2\gamma_1}[J_0(\lambda\rho)+J_2(\lambda\rho)]\right\}\lambda d\lambda \tag{7}$$

$$B_{1\phi} = -\frac{\mu_0 \cos\phi}{8\pi}\int_0^\infty \left\{\frac{\gamma_T-\gamma_1}{\gamma_T+\gamma_1}[J_0(\lambda\rho)+J_2(\lambda\rho)] -\frac{k_1^2\gamma_e-k_T^2\gamma_1}{k_1^2\gamma_e+k_T^2\gamma_1}[J_0(\lambda\rho)-J_2(\lambda\rho)]\right\}\lambda d\lambda \tag{8}$$

where

$$\begin{aligned}
\gamma_T^2 &= k_T^2-\lambda^2,\ \mathrm{Im}\{\gamma_T\}>0\\
\gamma_e^2 &= \frac{k_T^2}{k_L^2}\left(k_L^2-\lambda^2\right),\ \mathrm{Im}\{\gamma_e\}>0\\
\gamma_1^2 &= k_1^2-\lambda^2,\ \mathrm{Im}\{\gamma_1\}>0\\
\lambda^2 &= k_x^2+k_y^2\\
k_j &= \omega\sqrt{\mu_0\left(\varepsilon_j+i\sigma_j/\omega\right)},\ j=1,T,L.
\end{aligned} \tag{9}$$

The above results can be reduced to the appropriate isotropic results in [5] and [15] when $k_T = k_L = k_2$. It is interesting that the integral expressions for the one-dimensionally anisotropic case are no more complicated than the corresponding isotropic case. Next, the integrals in Eq. (4)–Eq. (6) will be evaluated exactly in terms of series.

The first Riemann sheet is such that $\mathrm{Im}\sqrt{k_T^2-\lambda^2} \geq 0$, $\mathrm{Im}\sqrt{k_L^2-\lambda^2} \geq 0$, $\lambda > 0$, with the branch-cut configuration similar to [15], where $|\,k_1\,|<|\,k_T\,|$, $|\,k_1\,|<|\,k_L\,|$. Note that the denominator

$$D(\lambda) = k_1^2\gamma_e + k_T^2\gamma_1 \tag{10}$$

has four simple zeros in the Riemann surface. These are located at

$$\lambda = \pm k_s = \pm\frac{k_1 k_L\sqrt{k_1^2-k_T^2}}{\sqrt{k_1^4-k_T^2k_L^2}} \tag{11}$$

and are not present in the first Riemann sheet.

In order to carrying out the integrations of the above calculable field components, consider the following replacements

$$k_j = iq_j,\quad \sqrt{k_j^2-\lambda^2} = i\sqrt{\lambda^2+q_j^2},\quad (j=1,T,L) \tag{12}$$

The Sommerfeld pole corresponds to $q_s = -ik_s$.

2.2. The z-component of the Magnetic Field

Comparing the formula Eq. (6) here and Eq. (2.4) in [15], the result for B_{1z} can be easily got

$$B_{1z} = -\frac{\mu_0}{2\pi}\frac{1}{(k_1^2-k_T^2)\rho^2}\left[e^{ik_1\rho}k_1^2\left(1+\frac{3i}{k_1\rho}-\frac{3}{k_1^2\rho^2}\right)\right.$$
$$\left.-e^{ik_T\rho}k_T^2\left(1+\frac{3i}{k_T\rho}-\frac{3}{k_T^2\rho^2}\right)\right]\sin\phi \tag{13}$$

2.3. The ϕ-component of the Electric Field

With the definition $q_s = -ik_s$, taking into account the following decompositions

$$\frac{1}{\gamma_T+\gamma_1} = -i\frac{\sqrt{\lambda^2+q_1^2}-\sqrt{\lambda^2+q_T^2}}{q_1^2-q_T^2} \tag{14}$$

and

$$\frac{\gamma_e\gamma_1}{k_1^2\gamma_e+k_T^2\gamma_1} = -i\left[\frac{q_1^2\sqrt{\lambda^2+q_1^2}-q_Tq_L\sqrt{\lambda^2+q_L^2}}{q_1^4-q_T^2q_L^2}\right.$$
$$+\frac{q_1^2q_Tq_L}{q_1^4-q_T^2q_L^2}\cdot\frac{1}{q_1^2+q_Tq_L}\cdot\frac{q_1^2-q_L^2}{q_1^2-q_Tq_L}$$
$$\left.\cdot\left(\frac{q_Tq_L\sqrt{\lambda^2+q_1^2}-q_1^2\sqrt{\lambda^2+q_L^2}}{\lambda^2+q_s^2}\right)\right] \tag{15}$$

First, we examine the following integrals

$$\widetilde{w}(\rho) = \int_0^\infty\frac{q_Tq_L\sqrt{\lambda^2+q_1^2}-q_1^2\sqrt{\lambda^2+q_L^2}}{\lambda^2+q_s^2}J_1(\lambda\rho)d\lambda \tag{16}$$

Following [15] and [16], the radical in the integral is given as follows

$$\frac{q_Tq_L\sqrt{\lambda^2+q_1^2}-q_1^2\sqrt{\lambda^2+q_L^2}}{\lambda^2+q_s^2}$$
$$= \frac{q_1q_T}{\lambda^2+q_s^2}\frac{\sqrt{q_1^2-q_L^2}}{\sqrt{q_1^2-q_T^2}}\cdot\left.\frac{\sqrt{\lambda^2+q_s^2x^2}}{\sqrt{x^2-1}}\right|_{x=q_L/q_s}^{q_1/q_s}$$
$$= \frac{q_1q_T\sqrt{q_1^2-q_L^2}}{\sqrt{q_1^2-q_T^2}}\cdot\int_{q_L/q_s}^{q_1/q_s}\frac{d(x^2-1)^{-1/2}}{\lambda^2+q_s^2x^2} \tag{17}$$

then

$$\widetilde{w}(\rho) = \frac{q_1 q_T \sqrt{q_1^2 - q_L^2}}{\sqrt{q_1^2 - q_T^2}} \cdot \int_{q_L/q_s}^{q_1/q_s} d\left(\frac{1}{\sqrt{x^2-1}}\right) \cdot \int_0^\infty \frac{J_1(\lambda\rho)}{\sqrt{\lambda^2 + q_s^2 x^2}} d\lambda \tag{18}$$

Finally, Eq. (5) can be re-written as follows

$$E_{1\phi} = -\frac{i\omega\mu_0 \sin\phi}{2\pi} \cdot \left\{ \frac{1}{q_1^2 - q_T^2} \frac{d}{d\rho}\left[\frac{I_e(q_1\rho) - I_e(q_T\rho)}{\rho^2}\right] + \frac{1}{q_1^4 - q_T^2 q_L^2} \frac{1}{\rho}\left[\frac{q_1^2 I_e(q_1\rho) - q_T q_L I_e(q_T\rho)}{\rho^2}\right] + \frac{q_1^2 - q_L^2}{\left(q_1^2 - q_T q_L\right)^2} \frac{q_1^3 q_T^2 q_L}{\left(q_1^2 + q_T q_L\right)^2} \frac{\sqrt{q_1^2 - q_L^2}}{\sqrt{q_1^2 - q_T^2}} \frac{1}{\rho} \cdot w(\rho) \right\} \tag{19}$$

where

$$I_e(\alpha) = \int_0^\infty \sqrt{x^2 + \alpha^2} J_1(x) dx = \alpha + e^{-\alpha} \tag{20}$$

$$w(\rho) = \int_{q_L/q_s}^{q_1/q_s} d\left(\frac{1}{\sqrt{x^2-1}}\right) \cdot \int_0^\infty \frac{J_1(\lambda\rho)}{\sqrt{\lambda^2 + q_s^2 x^2}} d\lambda \tag{21}$$

The following integration is used as [17]

$$\int_0^\infty \frac{J_1(\lambda\rho)}{\sqrt{\lambda^2 + q_s^2 x^2}} d\lambda = \frac{1}{q_s \rho x}\left(1 - e^{-q_s \rho x}\right) \tag{22}$$

$w(\rho)$ can be divided as follows

$$w(\rho) = -\frac{1}{q_s \rho}\left[\frac{q_1 - q_L}{q_s} - \frac{q_1 e^{-q_L\rho} - q_L e^{-q_1\rho}}{q_s} + \rho \mathtt{w}(\rho)\right] \tag{23}$$

The similar integral of Eq. (21) has been addressed in [15] for the isotropic case. It is noted that, for the corresponding one-dimensionally anisotropic case, q_s is more complex, the result for $\mathtt{w}(\rho)$ can be written readily.

$$\mathtt{w}(\rho) = q_s K_1(q_s\rho) - \frac{e^{-q_1\rho}}{\rho} \sum_{m=0}^{\infty} U_m(\rho) + q_s e^{-q_L\rho} \sum_{m=0}^{\infty} V_m(\rho) \tag{24}$$

Then

$$w(\rho) = -i\frac{k_1 - k_L}{k_s^2\rho} + i\frac{k_1 e^{ik_L\rho} - k_L e^{ik_1\rho}}{k_s^2\rho} + \frac{\pi}{2}H_1^{(1)}(k_s\rho) - \frac{e^{ik_1\rho}}{ik_s\rho}\sum_{m=0}^{\infty}U_m(\rho) - e^{ik_L\rho}\sum_{m=0}^{\infty}V_m(\rho) \tag{25}$$

where

$$U_m(\rho) = \begin{cases} 1, \ m = 0 \\ \frac{\left(\frac{1}{2}\right)_m\left(-k_s^2\rho^2\right)^m}{m!(2m-1)!}\frac{d^{2m-1}}{dz^{2m-1}}\left[e^z E_i(-z)\right]_{z=-ik_1\rho}, \ m = 1, 2, 3\dots \end{cases} \tag{26}$$

$$V_m(\rho) = \sqrt{2\pi}i^m \frac{\left(\frac{1}{2}\right)_m}{m!}\frac{m+\frac{1}{2}}{m-\frac{1}{2}}\left(\frac{k_L - k_s}{2k_s}\right)^{m+\frac{1}{2}} \cdot \left\{e^{-iz}\frac{d^m}{dz^m}\left[z^{-1/2}F_0(z)\right]\right\}_{z=(k_L-k_s)\rho} \tag{27}$$

In Eq. (26) and Eq. (27), $E_i(-z)$ is the exponential integral, $F_0(z) = \int_0^z \frac{e^{it}}{\sqrt{2\pi t}}dt = C(z) + iS(z)$, $C(z)$ and $S(z)$ are the Fresnel integrals as in [18]. Finally, the expression for $E_{1\phi}$ is

$$\begin{aligned} E_{1\phi} = -\frac{i\omega\mu_0\sin\phi}{2\pi}\Bigg\{ & e^{ik_1\rho}\left[\frac{2}{k_1^2 - k_T^2}\frac{1}{\rho^3} - \frac{ik_1}{k_1^2 - k_T^2}\frac{1}{\rho^2}\right. \\ & \left. - \frac{k_1^2}{k_1^4 - k_T^2 k_L^2}\frac{1}{\rho^3} - \frac{ik_1 k_T^2}{k_1^4 - k_T^2 k_L^2}\frac{\left(k_L^2 - k_1^2\right)^{3/2}}{\left(k_T^2 - k_1^2\right)^{3/2}}\frac{1}{\rho^2}\right] \\ & + e^{ik_T\rho}\frac{1}{k_T^2 - k_1^2}\left[\frac{2}{\rho^3} - \frac{ik_T}{\rho^2}\right] + e^{ik_L\rho}\frac{k_T^2 k_L}{k_1^4 - k_T^2 k_L^2} \\ & \times\left[\frac{1}{k_T\rho^3} + \frac{k_1^2}{k_L^2}\frac{\left(k_L^2 - k_1^2\right)^{3/2}}{\left(k_T^2 - k_1^2\right)^{3/2}}\frac{i}{\rho^2}\right] + \frac{ik_1}{k_T^2 - k_1^2}\frac{1}{\rho^2} \\ & - \frac{ik_T}{k_T^2 - k_1^2}\frac{1}{\rho^2} + \frac{k_1^3 - k_T k_L^2}{k_1^4 - k_T^2 k_L^2}\frac{i}{\rho^2} - \frac{k_1 k_T^2\left(k_1 - k_L\right)}{k_L\left(k_1^4 - k_T^2 k_L^2\right)}\frac{\left(k_L^2 - k_1^2\right)^{3/2}}{\left(k_T^2 - k_1^2\right)^{3/2}}\frac{i}{\rho^2} \\ & + \frac{k_s^2 k_1 k_T^2}{k_L\left(k_T^2 k_L^2 - k_1^4\right)}\frac{\left(k_L^2 - k_1^2\right)^{3/2}}{\left(k_T^2 - k_1^2\right)^{3/2}}\frac{i}{\rho^2}\times\left[\frac{i\pi}{2}H_1^{(1)}(k_s\rho)\right. \\ & \left. - \frac{e^{ik_1\rho}}{k_s\rho}\sum_{m=0}^{\infty}U_m(\rho) - ie^{ik_L\rho}\sum_{m=0}^{\infty}V_m(\rho)\right]\Bigg\} \end{aligned} \tag{28}$$

2.4. The ρ-component of the Electric Field

The integral for $E_{1\rho}$ is evaluated via the interchange of $1/\rho$ and the operator $(d/d\rho)$ in Eq. (19). We can easily get

$$E_{1\rho} = -\frac{i\omega\mu_0\cos\phi}{2\pi}\cdot\left\{\frac{1}{q_1^2-q_T^2}\frac{1}{\rho}\left[\frac{I_e(q_1\rho)-I_e(q_T\rho)}{\rho^2}\right]\right.$$
$$+\frac{1}{q_1^4-q_T^2q_L^2}\frac{d}{d\rho}\left[\frac{q_1^2I_e(q_1\rho)-q_Tq_LI_e(q_T\rho)}{\rho^2}\right]$$
$$\left.+\frac{q_1^2-q_L^2}{\left(q_1^2-q_Tq_L\right)^2}\frac{q_1^3q_T^2q_L}{\left(q_1^2+q_Tq_L\right)^2}\frac{\sqrt{q_1^2-q_L^2}}{\sqrt{q_1^2-q_T^2}}\frac{d}{d\rho}w(\rho)\right\} \tag{29}$$

Following the same steps as in [15], the result of $E_{1\rho}$ is given as

$$E_{1\rho} = -\frac{\omega\mu_0\cos\phi}{2\pi}\cdot\left\{\left[\frac{1}{k_1+k_T}+\frac{k_Tk_L^2-k_1^3}{k_1^4-k_T^2k_L^2}\right.\right.$$
$$\left.+\frac{k_1k_T^2\left(k_1-k_L\right)}{k_L\left(k_1^4-k_T^2k_L^2\right)}\frac{\left(k_L^2-k_1^2\right)^{3/2}}{\left(k_T^2-k_1^2\right)^{3/2}}\right]\frac{1}{\rho^2}$$
$$+\left[\frac{1}{k_1^2-k_T^2}\frac{i}{\rho^3}-\frac{1}{k_1^4-k_T^2k_L^2}\left(\frac{2ik_1^2}{\rho^3}+\frac{k_1^3}{\rho^2}\right)\right.$$
$$\left.-\frac{k_1k_T^2\left(k_s-k_L\right)}{k_L\left(k_1^4-k_T^2k_L^2\right)}\frac{\left(k_L^2-k_1^2\right)^{3/2}}{\left(k_T^2-k_1^2\right)^{3/2}}\left(\frac{1}{\rho^2}-\frac{ik_1}{\rho}\right)\right]e^{ik_1\rho}$$
$$+\left[\frac{k_1^2k_T^2}{k_L^2}\frac{\left(k_L^2-k_1^2\right)^{3/2}}{\left(k_T^2-k_1^2\right)^{3/2}}\left(\frac{ik_L}{\rho}-\frac{1}{\rho^2}\right)\right.$$
$$\left.+k_T\left(\frac{k_L}{\rho^2}+\frac{2i}{\rho^3}\right)\right]\frac{k_L}{k_1^4-k_T^2k_L^2}e^{ik_L\rho}-\frac{i}{\rho^3\left(k_1^2-k_T^2\right)}e^{ik_T\rho}$$
$$-\frac{k_1k_T^2k_s^3}{k_L\left(k_1^4-k_T^2k_L^2\right)}\frac{\left(k_L^2-k_1^2\right)^{3/2}}{\left(k_T^2-k_1^2\right)^{3/2}}\left[\frac{i\pi}{2}H_1^{(1)'}(k_s\rho)\right.$$
$$\left.\left.+e^{ik_1\rho}\sum_{m=0}^{\infty}\widetilde{U}_m(\rho)-e^{ik_L\rho}\sum_{m=0}^{\infty}\widetilde{V}_m(\rho)\right]\right\} \tag{30}$$

where

$$\widetilde{U}_m(\rho)=\frac{\left(\frac{1}{2}\right)_{m+1}\left(-k_s^2\rho^2\right)^m}{(2m)!(m+1)!}\frac{d^{2m}}{dz^{2m}}\left[e^zE_i(-z)\right]_{z=-ik_1\rho},\ m=0,1,2\ldots \tag{31}$$

$$\widetilde{V}_m(\rho) = \sqrt{2\pi} i^m \frac{\left(\frac{1}{2}\right)_m}{m!} \frac{m^2+3/4}{(m-1/2)(m-3/2)} \left(\frac{k_L-k_s}{2k_s}\right)^{m+\frac{1}{2}} \cdot \left\{ e^{-iz} \frac{d^m}{dz^m} \left[z^{-1/2} F_0(z) \right] \right\}_{z=(k_L-k_s)\rho} \quad (32)$$

3. SIMPLIFIED FORMULAS UNDER THE SPECIAL CONDITIONS

When $|k_1| \ll |k_T|$, $|k_1| \ll |k_L|$ and $|k_j\rho| \gg 1$, $j = 1, L, T$, with the similar approximations and replacements as Appendix C in [15], the asymptotic results can be got and written as

$$B_{1z,2} \sim e^{ik_T\rho} \frac{\mu_0}{2\pi} \frac{k_T^4}{k_1^2} \sqrt{\frac{2}{\pi k_T \rho}} \int_0^\infty \sqrt{2t} e^{-k_T \rho t} dt \sin\phi = e^{ik_T\rho} \frac{\mu_0}{2\pi} \frac{k_T^2}{k_1^2 \rho^2} \sin\phi \quad (33)$$

$$E_{1\rho,2} \sim \frac{\omega\mu_0}{4\pi} \left[\frac{4ik_T^2}{k_1^2 \rho} \frac{e^{ik_T\rho}}{\sqrt{\pi k_T \rho}} \int_0^\infty \sqrt{t} e^{-k_T\rho t} dt + 2\frac{k_T k_L^2}{k_1^2} \times \frac{e^{ik_L\rho}}{\sqrt{\pi k_L \rho}} \int_0^\infty \frac{\sqrt{t}}{it + (1/2)k_T^2/k_1^2} e^{-k_L\rho t} dt \right] \cos\phi \sim e^{ik_L\rho} \frac{\omega\mu_0 k_T^2 k_L^2}{2\pi k_1^3} \sqrt{\frac{\pi}{k_L\rho}} \left[F(p) - i(2\pi p)^{-1/2} \right] \cos\phi \quad (34)$$

where

$$p = \frac{k_L k_T^2 \rho}{2k_1^2} \quad (35)$$

$$E_{1\phi,2} \sim \frac{\omega\mu_0}{4\pi} \left[\frac{4k_T^3}{k_1^2} \frac{e^{ik_T\rho}}{\sqrt{\pi k_T\rho}} \int_0^\infty \sqrt{t} e^{-k_T\rho t} dt + \frac{2ik_T k_L}{k_1^2 \rho} \times \frac{e^{ik_L\rho}}{\sqrt{\pi k_L\rho}} \int_0^\infty \frac{\sqrt{t}}{it + (1/2)k_T^2/k_1^2} e^{-k_L\rho t} dt \right] \sin\phi = \frac{\omega\mu_0 k_T}{2\pi k_1^2} \left[e^{ik_L\rho} \frac{ik_L k_T}{k_1 \rho} \sqrt{\frac{\pi}{k_L\rho}} \left(F(p) - i(2\pi p)^{-1/2} \right) + e^{ik_T\rho} \frac{1}{\rho^2} \right] \sin\phi \quad (36)$$

$$E_{1z,2} \sim \frac{\omega\mu_0}{2\pi} \frac{k_T k_L^3}{k_1^3} \frac{e^{ik_L\rho}}{\sqrt{\pi k_L\rho}} \int_0^\infty \frac{\sqrt{t}}{it + (1/2)k_T^2/k_1^2} e^{-k_L\rho t} dt \cos\phi = e^{ik_L\rho} \frac{\omega\mu_0 k_T^2 k_L^3}{2\pi k_1^4} \sqrt{\frac{\pi}{k_L\rho}} \left[F(p) - i(2\pi p)^{-1/2} \right] \cos\phi \quad (37)$$

$$
\begin{aligned}
B_{1\rho,2} &\sim \frac{\mu_0}{2\pi}\Bigg[-\frac{2k_T^3}{k_1}\frac{e^{ik_T\rho}}{\sqrt{\pi k_T\rho}}\int_0^\infty \sqrt{t}e^{-k_T\rho t}dt \\
&+\frac{ik_Tk_L}{k_1\rho}\frac{e^{ik_L\rho}}{\sqrt{\pi k_L\rho}}\int_0^\infty \frac{\sqrt{t}}{it+(1/2)k_T^2/k_1^2}e^{-k_L\rho t}dt\Bigg]\sin\phi \\
&=\frac{\mu_0 k_T}{2\pi k_1^2\rho^2}\left[ik_Te^{ik_L\rho}\sqrt{\pi k_L\rho}\left(F(p)-i(2\pi p)^{-1/2}\right)-e^{ik_T\rho}k_1\right]\sin\phi \quad (38)
\end{aligned}
$$

$$
\begin{aligned}
B_{1\phi,2} &\sim -\frac{\mu_0}{2\pi}\frac{k_Tk_L^2}{k_1}\frac{e^{ik_L\rho}}{\sqrt{\pi k_L\rho}}\int_0^\infty \frac{\sqrt{t}}{it+(1/2)k_T^2/k_1^2}e^{-k_L\rho t}dt\cos\phi \\
&= -e^{ik_L\rho}\frac{\mu_0k_T^2k_L^2}{2\pi k_1^2}\sqrt{\frac{\pi}{k_L\rho}}\left[F(p)-i(2\pi p)^{-1/2}\right]\cos\phi \quad (39)
\end{aligned}
$$

4. DISCUSSION AND CONCLUSION

The exact expressions can be reduced to the corresponding isotropic results. As an example, let $k_L = k_T = k_2$ and with cancelations made, Eq. (30) is simplified and written as

$$
\begin{aligned}
E_{1\rho} =& -\frac{\omega\mu_0\cos\phi}{2\pi}\cdot\Bigg\{\left[\frac{1}{k_1+k_2}+\frac{k_2^3-k_1^3}{k_1^4-k_2^4}+\frac{k_1k_2^2\left(k_1-k_2\right)}{k_2\left(k_1^4-k_2^4\right)}\right] \\
&\frac{1}{\rho^2}+\Bigg[\frac{1}{k_1^2-k_2^2}\frac{i}{\rho^3}-\frac{1}{k_1^4-k_2^4}\times\left(\frac{2ik_1^2}{\rho^3}+\frac{k_1^3}{\rho^2}\right) \\
&-\frac{k_1k_2\left(k_s-k_2\right)}{\left(k_1^4-k_2^4\right)}\left(\frac{1}{\rho^2}-\frac{ik_1}{\rho}\right)\Bigg]e^{ik_1\rho} \\
&+\left[k_1^2\left(\frac{ik_2}{\rho}-\frac{1}{\rho^2}\right)+k_2\left(\frac{k_2}{\rho^2}+\frac{2i}{\rho^3}\right)\right]\frac{k_2}{k_1^4-k_2^4}e^{ik_2\rho} \\
&-\frac{i}{\rho^3\left(k_1^2-k_2^2\right)}e^{ik_2\rho}-\frac{k_1k_2k_s^3}{\left(k_1^4-k_2^4\right)}\Bigg[\frac{i\pi}{2}H_1^{(1)'}(k_s\rho) \\
&+e^{ik_1\rho}\sum_{m=0}^\infty\widetilde{U}_m(\rho)-e^{ik_2\rho}\sum_{m=0}^\infty\widetilde{V}_m(\rho)\Bigg]\Bigg\} \\
=& \frac{i\omega\mu_0\cos\phi}{2\pi}\cdot\frac{k_2^2}{k_1^4-k_2^4}\Bigg\{\Bigg[\frac{k_1^2-k_2^2}{k_2^2\rho^3}-ik_1\frac{k_1^2-k_2^2\left(1-k_s/k_2\right)}{k_2^2\rho^2} \\
&+\frac{k_1^2\left(1-k_s/k_2\right)}{\rho}\Bigg]e^{ik_1\rho}+\left(\frac{k_1^2-k_2^2}{k_2^2\rho^3}-i\frac{k_1^2-k_2^2}{k_2\rho^2}-\frac{k_1^2}{\rho}\right)e^{ik_2\rho} \\
&-i\frac{k_1}{k_2}k_s^3\left[\frac{i\pi}{2}H_1^{(1)'}(k_s\rho)+e^{ik_1\rho}\sum_{m=0}^\infty\widetilde{U}_m(\rho)-e^{ik_2\rho}\sum_{m=0}^\infty\widetilde{V}_m(\rho)\right]\Bigg\} \quad (40)
\end{aligned}
$$

It is seen that the reduced result for $E_{1\rho}$ is identical to the corresponding isotropic result Eq. (3.49) in [15]. With a similar progress,the simplified formulas in this paper can also be reduced to the corresponding isotropic results. The expressions for EM fields in the one-dimensionally anisotropic medium are much more complicated than the corresponding isotropic case. The results in this paper are useful to study the propagation of the EM waves on the boundary of one-dimensionally anisotropic earth or sediments.

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