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DEVELOPMENT OF ACCURATE MULTI-POLE DEBYE FUNCTIONS FOR ELECTROMAGNETIC TISSUE MODELLING USING A GENETIC ALGORITHM

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Abstract—The development of anatomically and dielectrically representative tissue models is key to the design and refinement of electromagnetic based diagnostic and therapeutic technologies. An important component of any such model are accurate and efficient Debye models which allow for the incorporation of the frequency dependent properties of biological tissues. The establishment of multi-pole Debye models is often a compromise between accuracy and computational cost. Furthermore, some finite difference time domain schemes impose constraints on the minimum Debye pole time-constant. In this study, the authors have developed an optimised genetic algorithm to establish Debye coefficients with minimal yet sufficient Debye poles for several different biological tissues. These Debye coefficients are fitted to existing Cole-Cole models and their accuracy is compared to previously fitted Debye models.

1. INTRODUCTION

Electromagnetic modelling of biological tissues must incorporate the frequency dependent nature of the dielectric properties. These properties consist of the permittivity and conductivity. Extensive studies have been completed to determine these properties for a variety of tissues across a wide frequency-range [1].

The Finite Difference Time Domain (FDTD) method is a powerful electromagnetic simulation tool to model the propagation of electromagnetic signals. Frequency Dependent FDTD (FD^2TD) methods have also been developed [2–5] to allow for the inclusion of frequency dependent dielectric properties using Cole-Cole, Lorentz or

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Debye models. The Debye model in particular can be easily expressed both in the frequency and time domain [2], and is the most widely used of the parametric dielectric models.

A comparison between various FD^{2}TD schemes [2] has shown that the choice of parameters in a multi-pole Debye model can impact the simulation in numerous ways:

- Choosing a Debye pole time-constant too low can make Auxiliary Differential Equation (ADE-2) [5] simulations unstable;
- Choosing a Debye pole time-constant too low will give inaccurate simulation results using the ADE-2 and Laplace Transform Piecewise Constant Recursive Convolution (LT-PCRC) [2, 6] methods;
- Increasing the number of Debye poles will increase the computation time required for all FD^{2}TD schemes.

However, accurately fitting Debye coefficients to a set of dielectric measurements across a very large frequency-range is problematic. Various fitting methods have been investigated previously including:

- Genetic algorithms (GA) [7];
- Particle-swarm least squares optimization [8];
- Iterative elimination and back-substitution [9];
- Weighted least squares fitting method [10];
- Debye properties estimation using a filter design process [11].

While previous studies investigated the trade-off between the accuracy of Debye models and the number of poles employed, the compromise between the accuracy of Debye models and the minimum Debye pole time-constant has not been investigated. Despite the fact that a limitation on time-constants can be easily implemented in most fitting methods, it is important to examine the effect of such a limitation on the resulting model accuracy. This becomes particularly critical when using FDTD schemes which place strict limits on the allowed Debye pole time-constants such as the Auxiliary Differential Equation method 2 (ADE-2) [5]. The ADE-2 method requires time-constants above a certain threshold in order to be stable and accurate for any given simulation time-step [2]. When designing such simulations this limitation must be taken into account during the Debye model fitting process. Therefore, in this paper the authors present an improved GA to find minimal yet highly accurate sets of Debye poles for muscle, fat and cortical bone across a broad frequency-range (10 Hz to 100 GHz). Significantly, the presented algorithm is the first to apply a limitation on Debye pole time-constants for more efficient ADE-2 FD^{2}TD simulations. Additionally this algorithm is

the first to apply a two-stage GA to the problem of fitting Debye models. The resulting Debye models are shown to be more accurate than previously reported results.

The remainder of the paper is organised as follows: Section 2 introduces the multi-pole Debye model and describes some potential problems when incorporating these models into FD^{2}TD schemes; Section 3 proposes an improved GA to find models with minimal Debye poles; Section 4 presents the performance of the resulting Debye models and illustrates the relationship between minimum Debye pole time-constants and dielectric model accuracy; finally Section 5 discusses the outcomes of this study.

2. MULTI-POLE DEBYE MODELS FOR FDTD SIMULATIONS

The complex permittivity of a material $\hat{\epsilon}$ consists of the relative permittivity ϵ' and conductivity σ, which both vary significantly with frequency. Measurements of tissue dielectrics at different frequencies were made by Gabriel et al. [1] and parametric Cole-Cole models [12] were fitted to these measurements. However, since Cole-Cole models cannot be easily expressed in the time domain [8], Debye models are often used instead in FD^{2}TD simulations.

The multi-pole Debye model is defined as:

$$\hat{\epsilon} = \epsilon_\infty + \sum_{i=1}^{n} \frac{\Delta\epsilon_i}{(1 + (j\omega\tau_i))} + \frac{\sigma_s}{j\omega\epsilon_0} \tag{1}$$

where ϵ_∞ is the permittivity at very high frequencies, σ_s is the static ionic conductivity and ϵ_0 is the permittivity of free space. $\Delta\epsilon_i$ is the change in permittivity and τ_i is the time-constant of the ith dispersion.

Various methods have been proposed to integrate multi-pole Debye models into FDTD simulations. Recently a subset of the newest methods have been analysed in terms of their stability, accuracy and computational complexity [2]. These include:

- Kelley-Luebbers Piecewise Linear Recursive Convolution (KL-PLRC) method [3];
- Laplace Transform Piecewise Constant Recursive Convolution (LT-PCRC) method [2, 6];
- Laplace Transform Piecewise Linear Recursive Convolution (LT-PLRC) [2];
- Auxiliary Differential Equation methods (ADE-1) [4] and (ADE-2) [5];

- A convolution-less method based on the analogy of a resistive-capacitive circuit termed the CIRC method [2].

Importantly, Feliziani et al. [2] also showed that the ADE-2 and LT-PCRC are the most computationally efficient FDTD methods. Feliziani also detailed multiple constraints on the relationship between the simulation time-step Δt and the lowest Debye pole time-constant τ_0 used in the simulation for the ADE-2 method. The ratio $\frac{\tau_0}{\Delta t}$ must be greater than 0.5 for ADE-2 to be stable, and $\frac{\tau_0}{\Delta t}$ must be greater than 5 for ADE-2 to be as accurate as the other FD^{2}TD methods for 1D problems.

The Courant-Friedrichs-Lewy (CFL) stability condition is an additional constraint which limits the value of the time-step in the FDTD method:

$$k = \frac{u\Delta t}{\Delta x} <= 1 \tag{2}$$

where Δx is the space interval and Δt is the time-step and u is the speed of light. This limits the value of the time-step depending on the desired spatial resolution of the FDTD simulation. The limitation on the value of Δt further limits the value of the smallest Debye pole time-constant τ_0.

Overall, a usable Debye model must have the following properties:

- Stable and accurate with an FDTD time-step defined by the Courant-Friedrichs-Lewy (CFL) condition;
- Computationally efficient;
- Accurate in terms of modelling the frequency dependent nature of the tissue.

The Debye model should have as many poles as is required to be accurate, while also having as few poles as possible for improved computationally efficiency. Additionally, the minimum Debye pole time-constant should be as large as possible to allow for the FDTD time-step to also be as large as possible, shortening the FDTD computation time.

Therefore, the novel GA presented in this study aims to use the minimum number of Debye poles required and the maximum possible minimum pole time-constant to find highly accurate multi-pole Debye models for fat, muscle and cortical bone tissue.

3. IMPROVED TWO STAGE GENETIC ALGORITHM FOR DEBYE POLE FITTING

Genetic Algorithms (GAs) are a search heuristic which are commonly used to search for parameters in a large solution space [13]. GAs first

evaluate each set of parameters using a user specified cost function and then apply selection, crossover and mutation operations on the individual sets of parameters in a population. In this study a GA is used to select the best multi-pole Debye model parameters to fit the Cole-Cole models developed by Gabriel et al. for muscle, fat and cortical bone [14].

The parameters to be evolved by the GA include $\log_{10}(\epsilon_\infty)$, $\log_{10}(\sigma_s)$, and for each Debye pole $\log_{10}(\Delta\epsilon_i)$ and $\log_{10}(\tau_i)$, as previously used by Clegg and Robinson [7]. Using the logarithm of the parameters reduces the otherwise huge search space, as each parameter can vary by many magnitudes between different tissues. The population is made up of 1,000 individuals as per the GA developed by Clegg and Robinson.

Significantly, the authors have developed a two-stage GA using two distinct cost functions. First a logarithmic cost function 3 is used until the GA fails to decrease the cost function output by more than 0.1% in the previous 100 generations.

$$C_{GA} = \sum_{f=10\times 2^0}^{f=10\times 2^{33}} \left(\log_{10}(c_r(f)) - \log_{10}(d_r(f))\right)^2 + \left(\log_{10}(c_i(f)) - \log_{10}(d_i(f))\right)^2 \quad (3)$$

where $d_i(f)$ and $d_r(f)$ are the imaginary and real parts of the Debye model at frequency f, and $c_i(f)$ and $c_r(f)$ are the imaginary and real parts of the Cole-Cole model at frequency f. The logarithm is taken of each imaginary and real point corresponding to the conductivity and permittivity respectively at each frequency. This sets both values on a similar magnitude scale, resulting in a cost function that reflects the accuracy of current Debye model equally in terms of both permittivity and conductivity. Note: the cost function presented in Eq. (3) uses 34 logarithmically spaced points, used here to allow for a direct comparison to the GA previously developed by Clegg and Robinson which used the same logarithmic cost function [7].

Once the error fails to further decrease (as defined previously), a linear cost function is used as described in Eq. (4). This approach is based on the assumption that the permittivity and conductivity are now in the correct range, such that a linear function may further decrease the error on both the conductivity and permittivity equally. This linear cost function cannot be used as the initial cost function as the permittivity is often magnitudes higher than the conductivity over the first 10 decades, which would result in the GA fitting the permittivity while neglecting to equally fit the conductivity.

$$C_{GA} = \sum_{f=10\times 2^0}^{f=10\times 2^{33}} \left|\frac{c_r(f) - d_r(f)}{c_r(f)}\right| + \left|\frac{c_i(f) - d_i(f)}{c_i(f)}\right| \quad (4)$$

This two-stage GA is compared to a one-stage GA as used by Clegg and Robinson [7]. To allow for a fair comparison, both GAs are run for 4,000 generations after the logarithmic cost function fails to decrease the error by the above criteria. Across all tissues and number of poles, the number of generations required to reach this point varied between 300 and 15,000.

The cost function is evaluated at 34 frequency points over the full range of the Cole-Cole models [14]. Since the employed GA uses a random starting population and random mutation at each generation, the GA does not always the same minimal cost value. Therefore each GA run is repeated 10 times and the lowest final cost function is chosen as the best model.

The GA was used to find the optimal multi-pole Debye model with the minimum allowed Debye pole time-constant constrained. The upper limit of the parameter $\log_{10}(\tau_i)$ was kept constant at -1 while the lower limit was varied from -12.5 to -10.5 in steps of 0.1. The resulting improved Debye models and their accuracies are reported in the next section.

4. PERFORMANCE METRICS AND RESULTS

The two-stage GA was used to fit multi-pole Debye models to the Cole-Cole models of muscle, fat and cortical bone, as established by Gabriel et al. [14]. All results were directly compared to the one-stage GA [7]. Evolved Debye models were compared to the Cole-Cole models using the average fractional error calculated as follows:

$$Error_{avg} = \sum_{f=10\times 2^0}^{f=10\times 2^{33}} \frac{\left|\frac{c_r(f)-d_r(f)}{c_r(f)}\right| + \left|\frac{c_i(f)-d_i(f)}{c_i(f)}\right|}{68} \tag{5}$$

Figure 1 very clearly illustrates that adding a fractional cost function after the logarithmic function reduces the final fractional error in almost all cases. This results in a more accurate Debye model compared to models developed using the GA proposed by Clegg and Robinson.

Furthermore, Figure 1 shows that, across all tissues, the error decreased monotonically as the number of poles increased to 12. Also, a decrease in the error could be seen as the number of poles increased to 18 for muscle and cortical bone. This is in contrast to previous studies which suggested that using more than 10 Debye poles did not result in a decreased error [7]. This discrepancy could be attributed to the larger number of GA generations used in this study compared to previous investigations. To further illustrate this point, the error function in

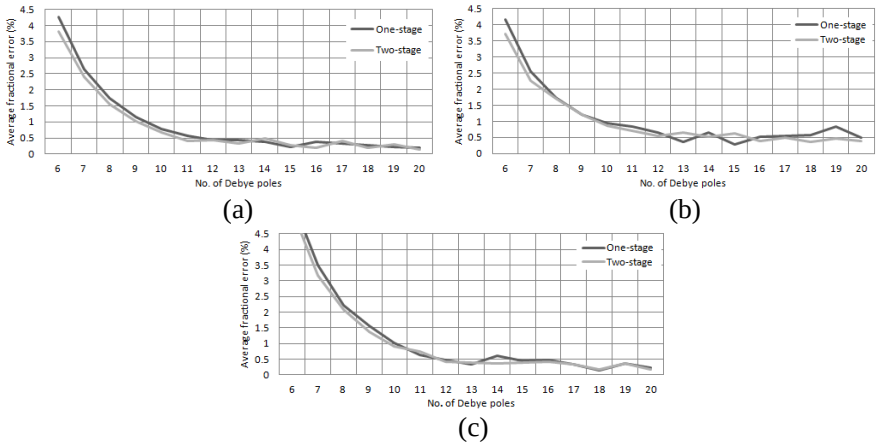

Figure 1. Average fractional error of one-stage and two-stage multi-pole Debye models for (a) muscle, (b) fat, and (c) cortical bone.

Figure 2 continues to decrease after more than 2,000 generations when using 11 poles.

Direct comparisons of the proposed GA to alternative fitting algorithms are difficult, as the accuracy of one method can be specific to the problem under consideration, including the tissue type and the frequency range being approximated. A comparison of the proposed GA to the GA by Robinson and Clegg was completed and results demonstrated that the error can be decreased by using the proposed GA even over a large number of generations. Another highly accurate method is the PSO-LS method developed by Kelley et al. [8] which yielded errors of only a few percent using 5 Debye poles over a smaller frequency range of 10 MHz to 100 GHz (4 decades). One advantage of the GA approach over the PSO-LS method is that it optimizes both the real and imaginary parts of the Debye model while the PSO-LS method only optimizes the real part which results in slightly higher errors in the imaginary part. Other methods include the weighted least squares fitting method [10] which achieved RMS errors of 1.9% for Fat, 3% for Bone and 2.8% for Muscle with 3 pole models over the frequency ranges of 100 kHz to 10 GHz (5 decades). The weighted least squares fitting method requires a manual starting point for the optimization procedure, which is not required for the proposed GA solution.

Finally, the two-stage GA was also run using different values for the minimum Debye pole time-constant, in order to find the maximum possible lower limit of the time-constant that resulted in an accurate Debye model over the frequency-range of 10 Hz to 100 GHz.

Figure 3 shows that the muscle and cortical bone tissue dielectrics are modelled most efficiently with a minimum time-constant of −11.3, while fat requires a minimum time-constant of approximately −11.5 in logarithmic terms. This corresponds to an upper limit on the FDTD time-step used for ADE-2 methods to approximately 1 ps and 0.6 ps respectively. If the lower limit on the Debye pole time-constant was not constrained, this would result in a much lower permitted FDTD time-step, and therefore a much longer computation time for ADE-2. Therefore, the benefit of the additional constraint imposed on the

Table 1. Table of parameters of 12 pole Debye models for muscle, fat and cortical bone.

	Muscle	Fat	Bone
ϵ_∞	5.9218	2.6182	2.8841
σ_s	0.20001	0.010029	0.02
$\Delta\epsilon_1$	26157	9770400	268.31
τ_1	0.000027651	0.0078503	0.00001194
$\Delta\epsilon_2$	41.595	0.61324	17.853
τ_2	4.8239E-09	7.76E-10	1.1764E-08
$\Delta\epsilon_3$	5632.6	64750	97650
τ_3	4.6119E-06	0.0015704	0.015439
$\Delta\epsilon_4$	5405.3	615.18	3.6385
τ_4	4.7578E-07	0.000015513	5.0785E-12
$\Delta\epsilon_5$	2015.8	1.6207	2645.3
τ_5	1.6424E-07	4.32E-12	0.00025856
$\Delta\epsilon_6$	219.56	43.651	1.4712
τ_6	3.0945E-08	9.19E-07	1.6751E-10
$\Delta\epsilon_7$	853670	7897.8	1675.1
τ_7	0.00030525	0.000078725	0.000067761
$\Delta\epsilon_8$	25159000	10.179	65.363
τ_8	0.0022595	1.10E-07	1.6122E-06
$\Delta\epsilon_9$	132130	1.1901	75.92
τ_9	0.00010931	2.10E-11	4.847E-08
$\Delta\epsilon_{10}$	6.5738	147.12	80.664
τ_{10}	4.7144E-10	0.000004131	1.6365E-07
$\Delta\epsilon_{11}$	33.186	13.055	5.2283
τ_{11}	5.4054E-12	1.28E-08	2.1138E-11
$\Delta\epsilon_{12}$	15.141	30416	4.0471
τ_{12}	1.7889E-11	0.00019463	1.8641E-09

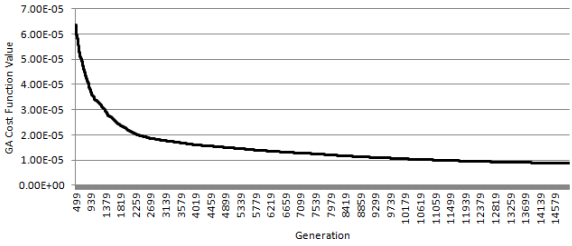

Figure 2. Two-stage GA cost function over the evolution of an 11 pole Debye model for muscle dielectric properties.

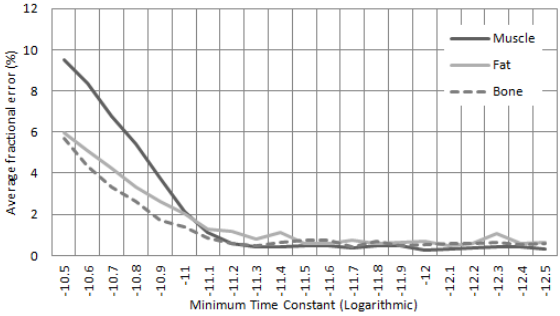

Figure 3. Two-stage GA average fractional error for 12 pole Debye models. The minimum time-constant τ_i was varied in each evolution run. All evolutions were repeated 10 times and the best is shown.

two-stage GA presented in this paper is quite clear.

The resulting best fit Debye model parameters using 12 poles for muscle, fat and cortical bone tissues are presented in Table 1.

5. CONCLUSIONS AND FUTURE WORK

The authors have developed an improved two-stage GA to find multi-pole Debye models for tissue dielectrics. This algorithm was used to find the most accurate multi-pole Debye models for muscle, fat and cortical bone tissue dielectrics.

Furthermore, the authors have shown that adding an extra GA constraint on the lower limit on the Debye pole time-constant can result in more computationally efficient Debye models for ADE-2 FDTD simulations.

Counter-intuitively, in a previous study by Clegg and Robinson, their GA failed to produce an improvement in accuracy with an

increasing number of Debye poles. However, in this study, the two-stage GA did in fact provide an improvement in accuracy with an increased number of poles, as would be expected.

The proposed method has the advantage that it can also be applied to the fitting of Cole-Cole models to measured data and also to fit Lorentz models to Cole-Cole models or to measured data. The GA cost functions do not depend on the model used, only on the approximation the model produces.

Finally, it could be argued that the improved accuracy gained by using the presented method could be more than counteracted by the inaccuracy of measured data and by dispersion and dissipation errors introduced by the discretization of time and space in FDTD schemes. However, this method will offer greater benefits where more accurate dielectric datasets exist, such as those established by Lazebnik et al. [15] and by Halter et al. [16]. Future work will focus on applying this method to such datasets and also on quantifying the errors caused by dispersion and dissipation errors.

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