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PLANAR ARRAY SYNTHESIS WITH SIDELOBE REDUCTION AND NULL CONTROL USING INVASIVE WEED OPTIMIZATION

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Abstract—Planar antenna array design is one of the most important electromagnetic optimization problems of current interest. This paper introduces a recently developed metaheuristic algorithm, known as the Invasive Weed Optimization (IWO), to the pattern synthesis of planar antenna arrays with desired pattern nulls and sidelobe level by amplitude-only and position-only optimization. The steps in the problem formulation are presented along with a design example that illustrates the performance of the IWO algorithm. Three examples have been presented and solved. Simulation results are proposed to compare with published ones to verify the effectiveness of the IWO algorithm for planar arrays.

1. INTRODUCTION

An Antenna array may be regarded as a spatial filter, which allows signal from a certain direction to pass through while rejecting all other signal sources (from other directions) impinging on the array. The goal in antenna array synthesis is to determine the physical layout that produce a radiation pattern as close to a desired pattern as possible. The desired pattern can be different for different applications. Many synthesis techniques are concerned with sidelobe level (SLL) reduction while preserving the gain of the main beam. Other methods deal with null control to reduce the effects of interference and jamming. For a planar array, this can be done by determining the positions of the elements for uniform excitation. Other methods of controlling the array pattern employ non-uniform excitation and phased array [1].

Array pattern synthesis usually involves several parameters which are non-linearly related to the objective functions. It is well known

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that the classical optimization methods need a starting point that is reasonably close to the final solution, or they are likely to be stuck in a local minimum. The quality of the solution strongly depends on the region of the solution space, where all local solutions are poor. A local search is limited to finding the best of these poor solutions. Since there are disadvantages to classic optimization techniques, heuristic optimizations techniques, such as genetic algorithm (GA) [2, 3], simulated annealing (SA) [4], differential evolution (DE) [5–8], particle swarm optimization (PSO) algorithms [9–12], have been proposed to solve planar antenna array problems.

In 2006, Mehrbian and Lucas proposed a derivative-free, metaheuristic algorithm, known as the Invasive Weed Optimization, mimicking the ecological behavior of colonizing weeds [13]. Since its inception, IWO has found successful application in many electromagnetic problems such as the design of Printed Yagi Antenna [14], E-shaped MIMO antenna [15], multi-feed reflector antennas [16], Broadband Patch Antenna [17], Conformal Phased Arrays [18], Circular Antenna Arrays [19], time modulated antenna array synthesis [20]. In this paper, IWO is used to optimize the amplitude and position of elements of planar array to produce a radiation pattern with minimum SLL and null placement control. Simulation results are proposed to compare with published ones obtained by other algorithms. The present comparative analysis is based on the "no free lunch theorem" [21]. This theorem says that [21, 22]: all algorithms that search for an extreme of a cost function perform exactly the same, when averaged over all possible cost functions. In particular, if algorithm A outperforms algorithm B on some cost functions, then loosely speaking there must exist exactly as many other functions where B outperforms A. In [22], a comparative analysis of classical and modern heuristics (such as PSO, GA, SA) have been proposed justified by the "no free lunch theorem".

This paper is organized as follows. Section 2 describes the principle of the IWO. The theoretical formulations for the planar arrays is presented in Section 3. Numerical results for planar arrays are given and analyzed in Section 4 while the conclusions are discussed in Section 5.

2. INVASIVE WEED OPTIMIZATION ALGORITHM

Invasive Weed Optimization (IWO) is a meta-heuristic algorithm that mimics the colonizing behavior of weeds. The IWO algorithm may be summarized as four steps, and more details can be found in [13]:

(I) Initialization: solutions are initialized and dispersed in the given

n dimensional search space uniformly and randomly.

(II) Reproduction: each member of the population is allowed to produce seeds depending on its own, as well as the colony's lowest and highest fitness, so that the number of seeds produced by a weed increases linearly from lowest possible seed for a weed with worst fitness to the maximum number of seeds for a plant with best fitness.

(III) Spatial distribution: the generated seeds are randomly scattered over the d-dimensional search space by perturbing them with normally distributed random numbers with zero mean and a variable variance. The standard deviation for a particular iteration can be given as in Equation (1):

$$\delta_{cur} = \frac{(iter_{\max} - iter)^n}{iter_{\max}^n} \left(\delta_{initial} - \delta_{final}\right) + \delta_{final} \tag{1}$$

The position of the new seed can be given as in Equation (2):

$$x_{son} = x_{parent} + sd = x_{parent} + randn(0,1) * \delta_{cur} \tag{2}$$

This step ensures that the probability of dropping a seed in a distant area decreases nonlinearly with iterations, which results in grouping fitter plants and elimination of weaker plants.

(IV) Competitive Exclusion: some kind of competition between plants is needed for limiting maximum number of plants in a colony. Initially, the plants in a colony will reproduce fast, and all the produced plants will be included in the existing colony, until the number of plants in the colony reaches a maximum value $p_{\max}$. The steps (1) to (4) are repeated until the maximum number of iterations has been reached, i.e., the colony size is fixed from thereon to $p_{\max}$.

3. PROBLEM STATEMENT

3.1. Planar Array

Figure 1 shows a planar array structure of N elements. The normalized array factor is given by:

$$AF(\theta) = \frac{1}{AF_{\max}} \times \sum_{n=1}^{N} I_n \left(\exp\left(\frac{j2\pi(real(z(i))\sin\theta\cos\varphi + imag(z(i))\sin\theta\sin\varphi)}{\lambda}\right)\right) \tag{3}$$

where the complex number $z(i)$ represents the position of element i, and $real(z(i))$ and $imag(z(i))$ are the locations of elements in x and y direction, respectively. θ is the elevation angle with respect to the z-axis and φ the azimuth angle with respect to x-axis. I_n is the amplitude of excitation. Since the desired pattern is symmetric about x-axis and y-axis, a quarter of the aperture is considered to reduce the number of optimization parameters to the quarter of the array elements.

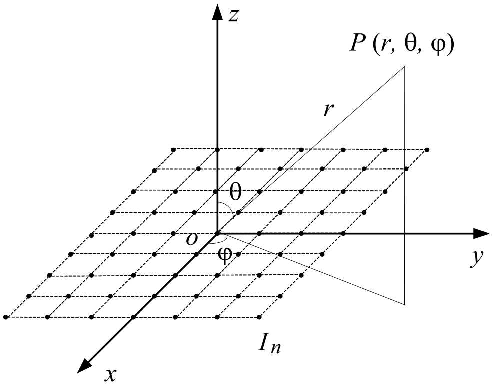

Figure 1. Geometry of a N elements symmetric planar array.

3.2. Fitness Function

The most important parameters in antenna pattern synthesis are the normalized sidelobe level that is desired to be as low as possible and null control in specific directions.

For null control, the objective function is:

$$f_{Null} = \left[\left(\sum_{k=1}^{K} w_k(|AF_k| - ND_k)^2\right)\right] \tag{4}$$

AF_k and ND_k are the calculated desired null level respectively, and K is the number of null directions, w_k the weight factor. We assume that if $|AF_k| >= |ND_k|$(dB), $w_k = 0$, else $w_k = 1$.

For side lobe suppression, the objective function is:

$$f_{\mathrm{SLL}} = w_s\left|\mathrm{SLL} - \mathrm{SLL}_{\max}\right|^2 \tag{5}$$

SLL and $\mathrm{SLL}_{\max}$ are the calculated and desired null levels, respectively. We also assume that if $|\mathrm{SLL}| >= |\mathrm{SLL}_{\max}|$, $w_s = 0$, else $w_s = 1$.

The distance between any two elements in the given aperture should be greater than or equal to the minimum value in order to reduce the mutual coupling and eliminate the grating lobe of a sparse array. We establish a penalty function f_d of each seed (represents a sparse array) in the algorithm, f_d is represented by follow equation:

$$f_d = \sum_{m}^{N(N-1)} w_{dm} \left| d_m - d_{\min} \right| \tag{6}$$

where d_m is the distance between any two elements in a sparse array, and $d_{\min}$ is the minimum spacing. d_m can be calculated by Equation (7).

$$d_m = \sqrt{(real(z_i) - real(z_j))^2 + (imag(z_i) - imag(z_j))^2} \tag{7}$$

If $d_m >= d_{\min}$, $w_{dm} = 0$; else $w_{dm} = 1$.

Combining all these objectives, one can formulate a final cost fitness is:

$$f = \sqrt{f_{Null} + f_{\mathrm{SLL}} + f_d} \tag{8}$$

4. NUMERICAL RESULTS

In this section, numerical results for three optimization problems about planar array are presented. In the first example, a planar array of 6×6 elements which is symmetrical with respect to the origin is placed on the x-y plane. The elements are assumed to be isotropic and the distance between the element to be half wavelength. And the aperture size is $-1.5\lambda \times 1.5\lambda$. The amplitude of each element is optimized by IWO algorithm to obtain a desired pattern with three nulls at $(\theta = 40°, \varphi = 30°)$, $(\theta = 60°, \varphi = 30°)$, $(\theta = 80°, \varphi = 30°)$. And the desired null depth level is set to -100 dB and desired maximum sidelobe for the overall pattern $|\mathrm{SLL}_{\max}|$ set to -20 dB. The parameters of the IWO algorithm are taken as follows: dimension of the problem $D = 36$; maximum number of seed $s_{\max} = 5$; Nonlinear index $n = 3$; maximum number of plants $p_{\max} = 20$; minimum number of seed $s_{\min} = 0$; initial standard deviation $\delta_{initial} = 0.3$; final standard deviation $\delta_{final} = 0.000001$; maximum number of iterations is 1000. The solutions are average value after running IWO algorithm for 10 times.

Optimized element amplitudes are shown in Table 1, and φ pattern for $\varphi = 30°$ is shown in Fig. 2. The maximum sidelobe level obtained for the pattern is -38.85 dB. As seen from Fig. 2, all nulls are below the level of -100 dB.

As the second example, a planar array of 200 isotropic elements with uniform excitation and zero phase is considered, with the aperture of $9.5\lambda \times 4.5\lambda$. The position of each element and the number of elements are optimized by IWO algorithm to minimize the maximum SLL in the $\varphi = 0°$ and $\varphi = 90°$ planes. Since the desired pattern is symmetric about coordinate axis, a quarter of the aperture is considered to reduce the number of optimization parameters to quarter of the array elements. The parameters of the IWO algorithm are taken as follows: initial dimension of the problem $D = 50$; maximum number of plants $p_{\max} = 20$; initial standard deviation $\delta_{initial} = 2$; final standard deviation $\delta_{final} = 0.000001$; the maximum number of iterations is 1000. The solutions are the average value after running IWO algorithm for 10 times.

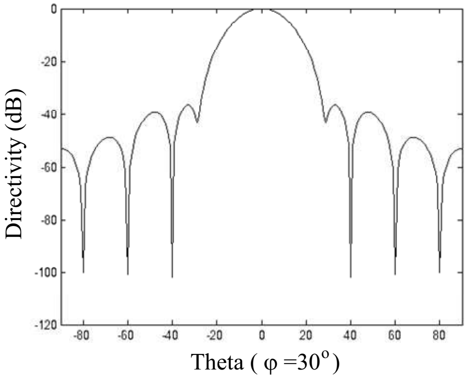

Figure 2. Array pattern for amplitude-only optimized 36 elements planar antenna array ($\varphi = 30°$).

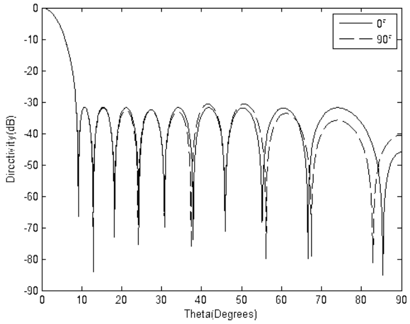

Figure 3. Radiation patterns in $\varphi = 0°$ and $\varphi = 90°$ planes.

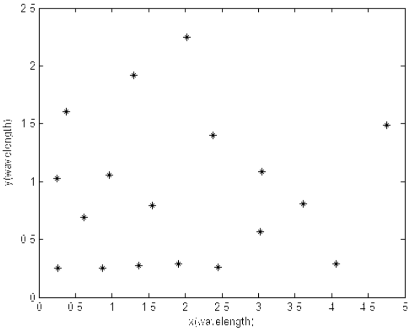

Figure 4. A quadrant configuration of the array with 72 elements.

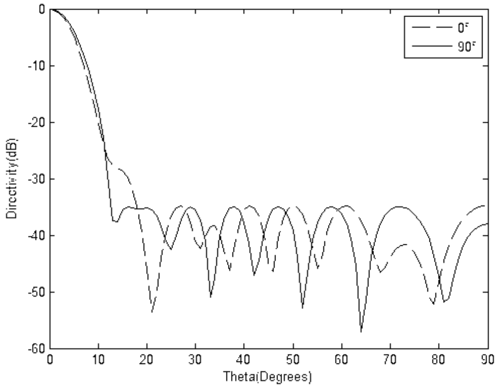

Figure 5. Radiation patterns in $\varphi = 0^\circ$ and $\varphi = 90^\circ$ planes after amplitude optimization.

Table 1. Element positions in wavelength and normalized current for 36 elements planar antenna array.

Element	dx	dy	amplitudes
1	−1.2500	−1.2500	0.311515
2	−1.2500	−0.7500	0.813955
3	−1.2500	−0.2500	0.424823
4	−1.2500	0.2500	0.494287
5	−1.2500	0.7500	0.970249
6	−1.2500	1.2500	0.354951
7	−0.7500	−1.2500	0.125304
8	−0.7500	−0.7500	0.937171
9	−0.7500	−0.2500	0.032536
10	−0.7500	0.2500	0.946874
11	−0.7500	0.7500	0.989542
12	−0.7500	1.2500	0.960018
13	−0.2500	−1.2500	0.98789
14	−0.2500	−0.7500	0.574342
15	−0.2500	−0.2500	0.92819
16	−0.2500	0.2500	0.665033
17	−0.2500	0.7500	0.824481
18	−0.2500	1.2500	0.873212

Element	dx	dy	amplitudes
19	0.2500	−1.2500	0.162847
20	0.2500	−0.7500	0.750981
21	0.2500	−0.2500	0.706398
22	0.2500	0.2500	0.380149
23	0.2500	0.7500	0.58516
24	0.2500	1.2500	0.330183
25	0.7500	−1.2500	0.733121
26	0.7500	−0.7500	0.605626
27	0.7500	−0.2500	0.909939
28	0.7500	0.2500	0.705233
29	0.7500	0.7500	0.872748
30	0.7500	1.2500	0.422094
31	1.2500	−1.2500	0.411667
32	1.2500	−0.7500	0.664776
33	1.2500	−0.2500	0.351636
34	1.2500	0.2500	0.462577
35	1.2500	0.7500	0.461633
36	1.2500	1.2500	0.376933

Table 2. Comparative results in the thinned planar array.

Algorithm	The number of elements	SLL ($\varphi = 0^\circ$ plane) dB	SLL ($\varphi = 90^\circ$ plane) dB	fitness (dB)
GA in [2]	108	−20.07	−19.76	−39.83
Modified GA in [3]	108	−29.597	−15.859	−45.456
ACO in [4]	136	−25.76	−25.674	−51.434
BPSO in [11]	112	−21.39	−21.94	−43.33
BDE in [8]	108	−26.09	−25.09	−51.18
CBPSO in [12]	108	−26.39	−26.33	−52.72
IWO	72	−30.71	−31.79	−62.5

Figure 3 shows the radiation patterns of the obtained thinned array in both $\varphi = 0^\circ$ and $\varphi = 90^\circ$ planes. Compared with the literature [2–4, 8, 11, 12], array element number decreases by more than 33.3% in the same aperture size. In literature [2], the fitness value of the optimal solution is defined as the sum of maximum SLLs in both planes. The fitness value obtained by IWO algorithm is $-62.5\,\mathrm{dB}$, the specific data as shown in Table 2. Comparing these results with those in [2–4, 8, 11, 12], much lower SLLs at both planes are achieved with lower number of elements by employing this algorithm. The array configuration of the thinned array for the upper right quarter of the aperture is shown in Fig. 4.

The third simulation example is the optimization of element amplitudes of the thinned array in the second example, to get a lower sidelobe in both $\varphi = 0^\circ$ and $\varphi = 90^\circ$ planes. The parameters are same to example one. The solutions are average value after running IWO algorithm for 10 times.

Figure 5 shows the radiation patterns of the obtained thinned array in both $\varphi = 0^\circ$ and $\varphi = 90^\circ$ planes after amplitude optimization

Table 3. The locations and amplitudes of elements in sparse array antenna.

locations	amplitudes	locations	amplitudes	locations	amplitudes	locations	amplitudes
(3.025 ,0.568)	0.7294	(-3.025 ,0.568)	0.110046	(-3.025 ,-0.568)	0.607666	(3.025 ,-0.568)	0.041181
(0.369,1.602)	0.561938	(-0.369,1.602)	0.427116	(-0.369,-1.602)	0.759573	(0.369,-1.602)	0.291195
(0.251,1.027)	0.040078	(-0.251,1.027)	0.729644	(-0.251,-1.027)	0.000284	(0.251,-1.027)	0.660291
(3.051,1.089)	0.534903	(-3.051,1.089)	0.763923	(-3.051,-1.089)	0.337797	(3.051,-1.089)	0.875852
(1.370,0.271)	0.728121	(-1.370,0.271)	0.267863	(-1.370,-0.271)	0.861034	(1.370,-0.271)	0.235688
(0.261,0.252)	0.635207	(-0.261,0.252)	0.354125	(-0.261,-0.252)	0.907554	(0.261,-0.252)	0.735832
(2.022,2.250)	0.573799	(-2.022,2.250)	0.039211	(-2.022,-2.250)	0.860188	(2.022,-2.250)	0.994408
(0.956,1.056)	0.909797	(-0.956,1.056)	0.182651	(-0.956,-1.056)	0.901941	(0.956,-1.056)	0.745316
(0.618,0.687)	0.373942	(-0.618,0.687)	0.994364	(-0.618,-0.687)	0.78644	(0.618,-0.687)	0.474973
(1.903,0.291)	0.613818	(-1.903,0.291)	0.825892	(-1.903,-0.291)	0.565953	(1.903,-0.291)	0.918763
(3.615,0.811)	0.36663	(-3.615,0.811)	0.719523	(-3.615,-0.811)	0.408667	(3.615,-0.811)	0.461037
(0.870,0.255)	0.601893	(-0.870,0.255)	0.412689	(-0.870,-0.255)	0.465393	(0.870,-0.255)	0.545361
(2.384,1.402)	0.236796	(-2.384,1.402)	0.982507	(-2.384,-1.402)	0.785246	(2.384,-1.402)	0.771376
(1.547,0.791)	0.952832	(-1.547,0.791)	0.519005	(-1.547,-0.791)	0.891503	(1.547,-0.791)	0.508321
(4.062,0.291)	0.888691	(-4.062,0.291)	0.065081	(-4.062,-0.291)	0.998092	(4.062,-0.291)	0.513088
(4.750,1.491)	0.775437	(-4.750,1.491)	0.180885	(-4.750,-1.491)	0.968456	(4.750,-1.491)	0.666248
(2.448,0.257)	0.125079	(-2.448,0.257)	0.97933	(-2.448,-0.257)	0.82968	(2.448,-0.257)	0.981819
(1.291,1.922)	0.660901	(-1.291,1.922)	0.806319	(-1.291,-1.922)	0.346261	(1.291,-1.922)	0.513

by IWO algorithm. The maximum sidelobe level in $\varphi = 0^\circ$ plane is $-34.95\,\mathrm{dB}$ and $-34.79\,\mathrm{dB}$ in $\varphi = 90^\circ$ plane. Compared with results obtained by uniform amplitude excitation, current amplitudes optimized by IWO algorithm can achieve a lower sidelobes in both $\varphi = 0^\circ$ and $\varphi = 90^\circ$ planes. The locations and amplitudes of elements in sparse array antenna are shown in Table 3.

5. CONCLUSIONS

Numerical and iterative approaches based on the IWO algorithm for pattern synthesis of planar array with prescribed pattern nulls and sidelobe reduction by optimizing the amplitudes-only and position-only are presented. In position-only optimization the coupling effects between elements are reduced by setting the desired minimum distance between array elements. Comparisons of the IWO and other techniques, the ACO, GA, binary PSO, Boolean DE and CBPSO, show the efficiency of the proposed technique.

REFERENCES

1. *Handbook of Antennas in Wireless Communications*, L. C. Godara, Editor, CR Boca Raton, FL, 2002.
2. Haupt, R. L., "Thinned arrays using genetic algorithms," *IEEE Transactions on Antennas and Propagation*, Vol. 42, No. 7, 993–999, 1994.
3. Chen, K., Z. He, and C. Han, "A modified real GA for the sparse linear array synthesis with multiple constraints," *IEEE Transactions on Antennas and Propagation*, Vol. 54, No. 7, 2167–2173, 2006.
4. Quevedo-Teruel, O. and E. Rajo-Iglesias, "Ant colony optimization in thinned array synthesis with minimum sidelobe level," *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, No. 1, 349–352, 2006.
5. Lin, C., A. Qing, and Q. Feng, "Synthesis of unequally spaced antenna arrays by using differential evolution," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 8, 2553–2561, 2010.
6. Kurup, D. G., M. Himdi, and A. Rydberg, "Synthesis of uniform amplitude unequally spaced antenna arrays using the differential evolution algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 51, No. 9, 2210–2217, 2003.

7. Guo, J.-L. and J.-Y. Li, "Pattern synthesis of conformal array antenna in the presence of platform using differential evolution algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 57, No. 9, 2615–2621, 2009.
8. Zhang, L., Y.-C. Jiao, Z.-B. Weng, and F.-S. Zhang, "Design of planar thinned arrays using a boolean differential evolution algorithm," *IET Microwaves, Antennas & Propagation*, Vol. 4, No. 12, 2172–2178, 2010.
9. Khodier, M. M. and C. G. Christodoulou, "Linear array geometry synthesis with minimum sidelobe level and null control using particle swarm optimization," *IEEE Transactions on Antennas and Propagation*, Vol. 53, No. 8, 2674–2679, 2005.
10. Ismail, T. H. and Z. M. Hamici, "Array pattern synthesis using digital phase control by quantized particle swarm optimization," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 6, 2142–2145, 2010.
11. Kennedy, J. and R. C. Eberhart, "A discrete binary version of the particle swarm algorithm," *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, 4104–4108, Orlando, FL, USA, 1997.
12. Wang, W.-B., Q.-Y. Feng, and D. Liu, "Synthesis of thinned linear and planar antenna arrays using binary PSO algorithm," *Progress In Electromagnetics Research*, Vol. 127, 371–387, 2012.
13. Mehrabian, A. R. and C. Lucas, "A novel numerical optimization algorithm inspired from weed colonization," *Ecological Informatics*, Vol. 1, 355–366, 2006.
14. Sedighy, S. H., et al., "Optimization of printed Yagi antenna using invasive weed optimization (IWO)," *IEEE Antennas and Wireless Propagation Letters*, Vol. 9, 1275–1278, 2010.
15. Karimkashi, S., A. A. Kishk, and D. Kajfez, "Antenna array optimization using dipole models for MIMO applications," *IEEE Transactions on Antennas and Propagation*, Vol. 59, No. 8, 3112–3116, 2011.
16. Foudazi, A. and A. R. Mallahzadeh, "Pattern synthesis for multi-feed reflector antennas using invasive weed optimisation," *IET Microwaves, Antennas & Propagation*, Vol. 6, No. 14, 1583–1589, 2012.
17. Monavar, F. M., N. Komjani, and P. Mousavi, "Application of invasive weed optimization to design a broadband patch antenna with symmetric radiation pattern," *IEEE Antennas and Wireless Propagation Letters*, Vol. 10, 1369–1372, 2011.

18. Bai, Y.-Y., et al., "A hybrid IWO/PSO algorithm for pattern synthesis of conformal phased arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 61, No. 4, 2328–2332, 2013.
19. Roy, G. G., et al., "Design of non-uniform circular antenna arrays using a modified invasive weed optimization algorithm," *IEEE Transactions on Antennas and Propagation*, Vol. 59, No. 1, 110–118, 2011.
20. Basak, A., et al., "A modified invasive weed optimization algorithm for time-modulated linear antenna array synthesis," *IEEE Congress on Evolutionary Computation (CEC)*, 1–8, 2010.
21. Wolpert, D. H. and W. G. Macready, "No free lunch theorems for optimization," *IEEE Transactions on Evolutionary Computation*, Vol. 1, 67–83, 1997.
22. Fulginei, F. R. and A. Salvini, "Comparative analysis between modern heuristics and hybrid algorithms," *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, Vol. 26, No. 2, 259–268, 2007.