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CURRENT DISTRIBUTION AND INPUT IMPEDANCE OF A STRIP LOOP ANTENNA LOCATED ON THE SURFACE OF A CIRCULAR COLUMN FILLED WITH A RESONANT MAGNETOPLASMA

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Abstract—We study the electrodynamic characteristics of a strip loop antenna located on the surface of a circular column filled with a resonant magnetoplasma and surrounded by a homogeneous isotropic background medium. The antenna current is excited by a time-harmonic voltage creating an electric field with the azimuthal component in a narrow gap on the strip surface. It is shown that the current distribution and input impedance of such an antenna are strongly influenced by the presence of an infinite number of propagating quasielectrostatic modes that are guided by a column containing a resonant magnetoplasma.

1. INTRODUCTION

The current distribution on metal wire antennas located in a resonant magnetoplasma and the behavior of their input impedance have been addressed in a limited number of works. In most papers on the subject, dipole and loop antennas in a homogeneous plasma medium are considered [1–10]. By resonant magnetoplasma, we mean a cold, collisionless magnetized plasma in which the refractive index of one of the characteristic waves tends to infinity when an angle between the

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wave normal direction and an external dc magnetic field approaches a certain value determined by the plasma parameters. In this case, the classical thin-antenna theory cannot be employed readily since no matter how small the cross section of the antenna wire might be physically, it is always possible to find some wave normal direction for which one wavelength in the plasma medium will become less than the wire cross-sectional extent and the antenna conductor will appear to be "thick" [2, 3, 9, 10].

Recently, the electrodynamic characteristics of a strip loop antenna located on the surface of a plasma column have been studied in the case where the column is filled with a nonresonant magnetoplasma [11]. In the present article, we extend the analysis of [11] to the case of a resonant magnetoplasma in the column.

Our article is organized as follows. In Section 2, we present the formulation of the problem and basic equations. Section 3 deals with the solution of integral equations for the antenna current. In Section 4, we give numerical results for the current distribution and input impedance of the antenna. Section 5 presents conclusions of the performed analysis.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

As in [11], we consider an antenna having the form of an infinitesimally thin, perfectly conducting, narrow strip of half-width d, which is coiled into a circular loop of radius a ($d \ll a$). The antenna is located coaxially on the surface of a uniform plasma column surrounded by a homogeneous isotropic medium with the real dielectric permittivity $\varepsilon_{\rm out} = \epsilon_0 \varepsilon_a$, where ϵ_0 is the permittivity of free space. The column is aligned with an external static magnetic field $\mathbf{B}_0$ (see Fig. 1), which is parallel to the z axis of a cylindrical coordinate system (ρ, ϕ, z).

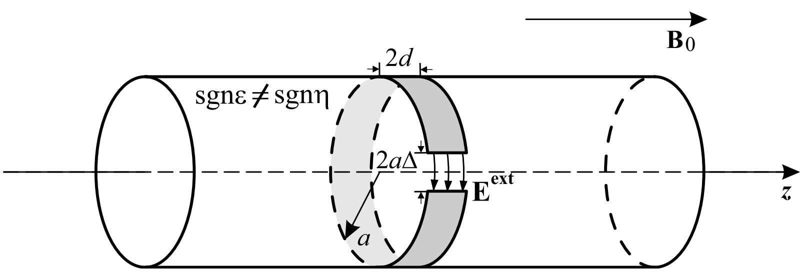

Figure 1. Geometry of the problem.

The medium inside the column is a two-component, cold, collisionless magnetoplasma described by the dielectric tensor

$$\boldsymbol{\varepsilon} = \epsilon_0 \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}. \tag{1}$$

Expressions for the tensor elements ε, g, and η can be found elsewhere [11–13]. In contrast to [11], it is assumed throughout this work that the diagonal elements ε and η of the plasma dielectric tensor have opposite signs, which corresponds to the case of a resonant magnetoplasma [13]. Recall that a two-component magnetoplasma turns out to be resonant if the angular frequency ω belongs to one of the following three frequency ranges [13]:

$$\begin{aligned} &\omega < \Omega_H, \\ &\omega_{\mathrm{LH}} < \omega < \min\{\omega_H,\, \omega_p\}, \\ &\max\{\omega_H,\, \omega_p\} < \omega < \omega_{\mathrm{UH}}. \end{aligned} \tag{2}$$

Here, Ω_H, ω_H, ω_{LH}, ω_p, and ω_{UH} are the ion and electron gyrofrequencies, the lower hybrid frequency, the electron plasma frequency, and the upper hybrid frequency, respectively.

The antenna is excited by a time-harmonic ($\sim \exp(i\omega t)$) voltage which creates an electric field with the azimuthal component E_ϕ^{ext} in a narrow angular interval $|\phi - \phi_0| \le \Delta \ll \pi$ on the surface of the strip (i.e., at $\rho = a$ and $|z| < d$):

$$\begin{aligned} E_\phi^{\mathrm{ext}}(a, \phi, z) \; = \; & \frac{V_0}{2a\Delta}[U(\phi - \phi_0 + \Delta) - U(\phi - \phi_0 - \Delta)] \\ & \times [U(z + d) - U(z - d)]. \end{aligned} \tag{3}$$

Here, V_0 is an amplitude of the given voltage, U a Heaviside function, and Δ the angular half-width of the gap centered at $\phi = \phi_0$. The excitation field E_ϕ^{ext} can be written as

$$E_\phi^{\mathrm{ext}} = \sum_{m=-\infty}^{\infty} A_m \exp(-im\phi), \tag{4}$$

where

$$A_m = \frac{V_0}{2\pi a} \frac{\sin(m\Delta)}{m\Delta} \exp(im\phi_0). \tag{5}$$

The density $\mathbf{J}$ of the electric current excited on the antenna by field (3) is sought in the form

$$\mathbf{J} = \boldsymbol{\phi}_0 I(\phi, z) \delta(\rho - a), \tag{6}$$

where $|z| < d$, δ is a Dirac function, and $I(\phi, z)$ is the surface current density which can be represented as follows:

$$I(\phi, z) = \sum_{m=-\infty}^{\infty} \mathcal{I}_m(z) \exp(-im\phi). \tag{7}$$

To find $I(\phi, z)$, one should express the azimuthal (E_ϕ) and longitudinal (E_z) components of the electric field excited by current (6) in terms of unknown quantities $\mathcal{I}_m(z)$ and use the boundary conditions for the tangential field components on the surface of the plasma column ($\rho = a$ and $-\infty < z < \infty$). Then the following boundary conditions on the antenna surface ($\rho = a$ and $|z| < d$) are applied:

$$E_\phi + E_\phi^{\text{ext}} = 0, \quad E_z = 0. \tag{8}$$

It is shown in [11] that the boundary condition for E_ϕ in (8) yields the integral equation

$$\int_{-d}^{d} K_m\left(z - z'\right) \mathcal{I}_m(z') dz' = -A_m, \tag{9}$$

whereas the boundary condition for E_z in (8) gives

$$\int_{-d}^{d} k_m\left(z - z'\right) \mathcal{I}_m\left(z'\right) dz' = 0. \tag{10}$$

It is assumed in (9) and (10) that $|z| < d$. The kernels of integral Equations (9) and (10) can be written as [11]

$$\begin{aligned} K_m(\zeta) &= \sum_n \frac{2\pi a}{N_{m,n}} E^2_{\phi;m,n}(a) \exp\left(-ik_0 p_{m,n}|\zeta|\right) \\ &\quad + \frac{ik_0}{2\pi} \int_0^\infty \frac{q}{p(q)} \sum_{l=1}^{2} \sum_{k=1}^{2} \frac{B_{mk}^{(l)}}{\Delta_m^{(l)}} \left[J_{m+1}(Q_k) + \alpha_k m \frac{J_m(Q_k)}{Q_k} \right] \\ &\quad \times \exp(-ik_0 p(q)|\zeta|) dq, \end{aligned} \tag{11}$$

$$\begin{aligned} k_m(\zeta) &= \operatorname{sgn}\zeta \Bigg\{ \sum_n \frac{2\pi a}{N_{m,n}} E_{\phi;m,n}(a) E_{z;m,n}(a) \exp\left(-ik_0 p_{m,n}|\zeta|\right) \\ &\quad + \frac{i}{2\pi a\eta} \int_0^\infty \frac{q}{p(q)} \sum_{l=1}^{2} \sum_{k=1}^{2} \frac{B_{mk}^{(l)}}{\Delta_m^{(l)}} n_k Q_k J_m(Q_k) \\ &\quad \times \exp(-ik_0 p(q)|\zeta|) dq \Bigg\}, \end{aligned} \tag{12}$$

where

$$p\,(q) = \left(\varepsilon_a - q^2\right)^{1/2}, \tag{13}$$

J_m is a Bessel function of the first kind of order m and k_0 the wave number in free space. $E_{\phi;m,n}(\rho)$ and $E_{z;m,n}(\rho)$ are functions describing the distributions over the transverse coordinate ρ of the azimuthal and longitudinal electric-field components of eigenmodes (discrete-spectrum waves) that are guided by the column and have the azimuthal and radial indices m and n, respectively ($m = 0, \pm 1, \pm 2, \ldots$ and $n = 1, 2, \ldots$), and $N_{m,n}$ is the norm of an eigenmode with the propagation constant $p_{m,n}$. Note that the function $p(q)$ has the meaning of the normalized (to k_0) propagation constant of the characteristic wave of the background isotropic medium for the transverse wave number $q = k_\perp/k_0$ and satisfies the condition $\operatorname{Im} p(q) < 0$. The integrals over q in Equations (11) and (12) describe the contribution of continuous-spectrum waves to the kernels. All the quantities entering the corresponding integrands and containing p are calculated for $p = p(q)$.

The fields of the eigenmodes supported by a magnetized plasma column as well as the dispersion relation allowing one to determine their propagation constants are given in [14]. The norm $N_{m,n}$ can be calculated as [13, 15, 16]

$$N_{m,n} = 4\pi \int_0^\infty \left[E_{\rho;m,n}(\rho) H_{\phi;m,n}(\rho) + E_{\phi;m,n}(\rho) H_{\rho;m,n}(\rho)\right] \rho\, d\rho. \quad (14)$$

When deriving Equations (11), (12), and (14), we took into account the fact that without loss of generality, the eigenmode fields can always be defined so as to satisfy the relationships [13]

$$\begin{aligned} E^{(T)}_{\rho;-m,-n}(\rho) &= -E_{\rho;m,n}(\rho), \quad E^{(T)}_{\phi,z;-m,-n}(\rho) = E_{\phi,z;m,n}(\rho), \\ H^{(T)}_{\rho;-m,-n}(\rho) &= -H_{\rho;m,n}(\rho), \quad H^{(T)}_{\phi,z;-m,-n}(\rho) = H_{\phi,z;m,n}(\rho), \end{aligned} \quad (15)$$

where the negative sign of the subscript n denotes modes propagating in the negative direction of the z axis, and the superscript (T) designates fields taken in an auxiliary ("transposed") medium that is described by the transposed dielectric tensor $\boldsymbol{\varepsilon}^T$.

The quantities $\Delta_m^{(l)}$ and $B_{mk}^{(l)}$ in Equations (11) and (12) are written in the form [11]

$$\begin{aligned} \Delta_m^{(l)} \; = \; (-1)^l \Bigg\{ & n_2 \left[\frac{\eta}{\varepsilon_a} J_m^{(1)} \tilde{J}_m^{(2)} - \left(J_m^{(1)} + \frac{\eta}{\varepsilon_a} \tilde{J}_m^{(2)} \right) \mathcal{H}_m^{(l)} \right] \\ & - n_1 \left[\frac{\eta}{\varepsilon_a} \tilde{J}_m^{(1)} J_m^{(2)} - \left(J_m^{(2)} + \frac{\eta}{\varepsilon_a} \tilde{J}_m^{(1)} \right) \mathcal{H}_m^{(l)} \right] \\ & + (n_2 - n_1) \left[\left(\mathcal{H}_m^{(l)} \right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right] \\ & + p \frac{\eta}{\varepsilon_a} \frac{m}{Q^2} \left[J_m^{(1)} - J_m^{(2)} + \tilde{J}_m^{(1)} - \tilde{J}_m^{(2)} \right] \Bigg\}, \end{aligned} \quad (16)$$

$$\begin{aligned} B_{mk}^{(l)} &= (-1)^{k+1} Z_0 \frac{k_0 a}{Q_k J_m(Q_k)} \left[\frac{\eta}{\varepsilon_a} n_{k-v} \tilde{J}_m^{(k-v)} \mathcal{H}_m^{(l)} \right. \\ &\quad \left. + p \frac{\eta}{\varepsilon_a} \frac{m}{Q^2} J_m^{(k-v)} - n_{k-v} \left(\left(\mathcal{H}_m^{(l)} \right)^2 - \frac{p^2}{\varepsilon_a} \frac{m^2}{Q^4} \right) \right], \end{aligned} \tag{17}$$

where Z_0 is the wave impedance of free space. The other quantities in Equations (11), (12), (16), and (17) are given by

$$\begin{aligned} J_m^{(k)} &= \frac{J_{m+1}(Q_k)}{Q_k J_m(Q_k)} + m \frac{\alpha_k}{Q_k^2}, \quad \tilde{J}_m^{(k)} = \frac{J_{m+1}(Q_k)}{Q_k J_m(Q_k)} - m \frac{\beta_k}{Q_k^2}, \\ \mathcal{H}_m^{(l)} &= \frac{H_{m+1}^{(l)}(Q)}{Q H_m^{(l)}(Q)} - \frac{m}{Q^2}, \quad k, l = 1, 2, \quad v = (-1)^k, \\ Q_k &= k_0 a q_k(p), \;\; Q = k_0 a q, \;\; n_k = -\frac{\varepsilon}{pg} \left[p^2 + q_k^2(p) + \frac{g^2}{\varepsilon} - \varepsilon \right], \\ \alpha_k &= \left[p^2 + q_k^2(p) - \varepsilon \right] g^{-1} - 1, \quad \beta_k = p n_k^{-1} + 1, \\ q_k(p) &= \frac{1}{\sqrt{2}} \left\{ \varepsilon - \frac{g^2}{\varepsilon} + \eta - \left(\frac{\eta}{\varepsilon} + 1 \right) p^2 - \left(\frac{\eta}{\varepsilon} - 1 \right) \right. \\ &\quad \left. \times (-1)^k \left[\left(p^2 - P_b^2 \right) \left(p^2 - P_c^2 \right) \right]^{1/2} \right\}^{1/2}, \\ P_{b,c} &= \left\{ \varepsilon - (\eta + \varepsilon) \frac{g^2}{(\eta - \varepsilon)^2} + \frac{2\chi_{b,c}}{(\eta - \varepsilon)^2} \left[\varepsilon g^2 \eta \left(g^2 - (\eta - \varepsilon)^2 \right) \right]^{1/2} \right\}^{1/2}, \end{aligned} \tag{18}$$

where $\chi_b = -\chi_c = -1$ and $H_m^{(l)}$ is a Hankel function of the lth kind of order m.

The above formulas are valid regardless of whether the column is filled with a resonant or nonresonant magnetoplasma. In the next section, we will solve the derived integral equations for the antenna current in the case where the plasma inside the column is resonant, i.e., $\operatorname{sgn} \varepsilon \neq \operatorname{sgn} \eta$.

3. SOLUTION OF THE INTEGRAL EQUATIONS FOR THE ANTENNA CURRENT

As in the case where the column is filled with a nonresonant plasma [11], the kernels of integral Equations (9) and (10) can be represented as the sums of singular and regular parts:

$$K_m(\zeta) = K_m^{(s)}(\zeta) + K_m^{(r)}(\zeta), \quad k_m(\zeta) = k_m^{(s)}(\zeta) + k_m^{(r)}(\zeta). \tag{19}$$

The singular parts $K_m^{(s)}(\zeta)$ and $k_m^{(s)}(\zeta)$ are introduced in such a way that they tend to infinity for $\zeta \to 0$, whereas the regular parts $K_m^{(r)}(\zeta)$ and $k_m^{(r)}(\zeta)$, which have no singularities at this point, can be taken for $\zeta = 0$ if the antenna is sufficiently narrow such that the following inequalities take place:

$$d \ll a, \quad d \ll a|\eta/\varepsilon|^{1/2}, \quad (k_0 d)^2 \max\{|\varepsilon_a|, |\varepsilon|, |g|, |\eta|\} \ll 1. \tag{20}$$

It is worth noting that in the case considered here, expressions for the singular and regular parts of the kernels differ significantly from those in [11]. The difference is because of the fact that the column filled with a resonant magnetoplasma supports the guided propagation of an infinite number of eigenmodes the fields of which contribute to $K_m^{(s)}(\zeta)$ and $k_m^{(s)}(\zeta)$, whereas eigenmodes of a nonresonant plasma column in [11] contribute only to the regular parts of the kernels.

The singular parts of the kernels for the column containing a resonant magnetoplasma can be written as

$$\begin{aligned} K_m^{(s)}(\zeta) &= -Z_0 \left\{ \frac{2m^2}{\pi k_0 a^2} \frac{|\varepsilon\eta|^{1/2}}{|\varepsilon\eta| + \varepsilon_a^2} \sum_{n=1}^{\infty} \frac{\exp[-i\chi(2n+m+1/2)]}{2n+m+1/2} \right. \\ &\quad + i\frac{k_0^2 a}{2} \int_0^\infty J_{m+1}^2(k_0 a q) \exp(-k_0 q|\zeta|) dq - i\frac{m^2}{\pi k_0 a^2} \frac{\varepsilon_a}{|\varepsilon\eta|} \\ &\quad \left. \times \int_0^\infty q^{-1/2} U(q) I_m\left(k_0 a |\eta/\varepsilon|^{1/2} q\right) \exp(-k_0 q|\zeta|)\, dq \right\}, \end{aligned} \tag{21}$$

$$\begin{aligned} k_m^{(s)}(\zeta) &= -\operatorname{sgn}\zeta \, \frac{Z_0}{k_0 a^2} \frac{m}{|\varepsilon\eta| + \varepsilon_a^2} \left\{ \varepsilon \lim_{\nu\to 0} \sum_{n=1}^{\infty} \exp[-(i\chi + \nu) \right. \\ &\quad \left. \times (2n+m+1/2)] - \frac{k_0 a \varepsilon_a}{\pi} \int_0^\infty \exp(-k_0 q|\zeta|) dq \right\}, \end{aligned} \tag{22}$$

where $\chi = |\varepsilon/\eta|^{1/2}(\pi|\zeta|/2a)\operatorname{sgn}\varepsilon$, I_m is a modified Bessel function of the first kind of order m, and

$$U(q) = (2\pi k_0 a)^{1/2} |\eta/\varepsilon|^{1/4} \left(1 + \varepsilon_a^2 |\varepsilon\eta|^{-1}\right)^{-1} \exp\left(-k_0 a |\eta/\varepsilon|^{1/2} q\right). \tag{23}$$

The quantities under the summation and integral signs in Equations (21) and (22) are obtained by making the limiting transitions $|p_{m,n}| \to \infty$ (see Appendix A) and $q \to \infty$, respectively, in the corresponding expressions (11) and (12) for the kernels. It should be noted that in the limit $q \to \infty$, the quantities q_1, $p(q)$, $n_{1,2}$, $\alpha_{1,2}$, and $\beta_{1,2}$ are calculated as in [11]. An exception takes place only for the quantity

q_2, which is now equal to $q_2 = i|\eta/\varepsilon|^{1/2}q$ in the considered limit. To avoid misunderstanding, we also note that in this case, the function $U(q)$ is initially found to be equal to

$$U(q) = \frac{I_m(k_0a|\eta/\varepsilon|^{1/2}q)}{q^{1/2}[I_{m+1}^2(k_0a|\eta/\varepsilon|^{1/2}q) + \varepsilon_a^2|\varepsilon\eta|^{-1}I_m^2(k_0a|\eta/\varepsilon|^{1/2}q)]}.$$

Using the large-argument approximation for I_m, we finally arrive at Equation (23).

The quantities $K_m^{(r)}(\zeta)$ and $k_m^{(r)}(\zeta)$, which are not presented here for brevity, are given by the sums over n and the integrals over q in which the terms under the summation and integral signs are determined by the differences of the respective quantities entering the rigorous expressions (11) and (12) for the kernels and the corresponding expressions, taken at $|p_{m,n}| \to \infty$ and $q \to \infty$, for the singular parts of the kernels.

The integrals in (21) are evaluated as [17]

$$\int_0^\infty J_{m+1}^2(k_0aq)\exp(-k_0q|\zeta|)dq = \frac{1}{\pi k_0 a}Q_{m+\frac{1}{2}}\left(1+\frac{|\zeta|^2}{2a^2}\right), \quad (24)$$

$$\int_0^\infty q^{-1/2}U(q)I_m(k_0a|\eta/\varepsilon|^{1/2}q)\exp\left(-k_0q|\zeta|\right)dq$$
$$= \frac{2|\varepsilon\eta|}{|\varepsilon\eta|+\varepsilon_a^2}\,Q_{m-\frac{1}{2}}\left(1+\sqrt{\frac{|\varepsilon|}{|\eta|}}\frac{|\zeta|}{a}\right), \quad (25)$$

where $Q_\mu(z)$ are Legendre functions of the second kind. With allowance for the first two inequalities in (20), these functions can be approximated using the following asymptotic formula, which is valid for $z \to 1+0$ and $\mu \neq -1, -2, \ldots$ [18]:

$$Q_\mu(z) = -\frac{1}{2}\ln\left(\frac{z}{2}-\frac{1}{2}\right) - \psi(\mu+1) - \gamma. \quad (26)$$

Here, $\psi(z) = d\ln\Gamma(z)/dz$ is the logarithmic derivative of a gamma function and $\gamma = 0.5772\ldots$ is Euler's constant. When applied to functions (24) and (25), formula (26) needs to be corrected for very large values of $|m|$. However, the terms $|m| > \tilde{m}$, where $\tilde{m} \sim \Delta^{-1}$ is a sufficiently large integer, do not contribute significantly to the current series (7) because of the properties of quantities (5) that will enter the resulting expressions for $\mathcal{I}_m$. It can be shown that the requirement for formula (26) to be valid to approximate (24) and (25) for $|m| < \tilde{m}$ reduces to the conditions $d \ll 2a\tilde{m}^{-1}$ and $d \ll 2a\tilde{m}^{-1}|\eta/\varepsilon|^{1/2}$, respectively, whence we have

$$d \ll 2a\Delta, \quad d \ll 2a\Delta|\eta/\varepsilon|^{1/2}. \quad (27)$$

Using the second inequality in (27), the series in (21) can be rewritten as

$$\sum_{n=1}^{\infty}\frac{\exp[-i\chi(2n+m+1/2)]}{2n+m+1/2}=\sum_{n=1}^{\infty}\frac{\exp[-i\chi(2n-1)]}{2n-1}-\Phi_m, \qquad (28)$$

where

$$\Phi_m=\left(m+\frac{3}{2}\right)\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+m+1/2)}\,. \qquad (29)$$

When writing (29), we put $\zeta=0$, since Φ_m remains finite (regular) in the limit $\zeta\to 0$. The series in (28) is calculated in closed form [17]:

$$\sum_{n=1}^{\infty}\frac{\exp[-i\chi(2n-1)]}{2n-1}=\frac{1}{2}\ln\cot\frac{|\chi|}{2}-i\frac{\pi}{4}\operatorname{sgn}\varepsilon, \qquad (30)$$

where $\cot(|\chi|/2)\simeq 2|\chi|^{-1}$ for $\zeta\to 0$.

Thus, we find that kernel (11) possesses the logarithmic singularity:

$$K_m(\zeta)=-iZ_0\frac{k_0}{2\pi}\frac{1}{\delta_m}\left(\ln\frac{|\zeta|}{2a}+S_m\right), \qquad (31)$$

where

$$\delta_m = -\frac{i(k_0a)^2\xi}{m^2+i(k_0a)^2\xi}, \quad \xi=\frac{|\varepsilon\eta|^{1/2}+i\varepsilon_a}{2}, \qquad (32)$$

$$\begin{aligned}S_m = \frac{1}{m^2+i(k_0a)^2\xi}\Bigg\{m^2\Bigg[\ln\sqrt{\frac{|\varepsilon|}{|\eta|}}-\frac{2i\varepsilon_a}{|\varepsilon\eta|^{1/2}-i\varepsilon_a}\left(\psi\left(m+\frac{1}{2}\right)+\gamma\right)\\ +\frac{|\varepsilon\eta|^{1/2}}{|\varepsilon\eta|^{1/2}-i\varepsilon_a}\left(\ln\frac{\pi}{2}+2\Phi_m+i\frac{\pi}{2}\operatorname{sgn}\varepsilon\right)\Bigg]\\ +i(k_0a)^2\xi\left[\psi\left(m+\frac{3}{2}\right)+\gamma-i\frac{2\pi}{Z_0k_0}K_m^{(r)}(0)\right]\Bigg\}. \qquad (33)\end{aligned}$$

As a result, Equation (9) takes the form

$$\int_{-d}^{d}\mathcal{I}_m(z')\ln\frac{|z-z'|}{2a}\,dz' = -i\frac{2\pi A_m}{Z_0k_0}\delta_m-S_m\int_{-d}^{d}\mathcal{I}_m(z')dz'. \qquad (34)$$

In turn, the quantity $k_m^{(s)}$ in (22), after some algebra, can be shown to have the Cauchy singularity: $k_m^{(s)}(\zeta)\sim m/\zeta$. Taking into account the relation $k_m^{(r)}(0) = 0$ (see [11] for details), we reduce Equation (10) to the form

$$\int_{-d}^{d} m\frac{\mathcal{I}_m(z')}{z-z'}dz' = 0, \tag{35}$$

where the integral is understood in the sense of the Cauchy principal value.

Since the solution of Equation (34) with the logarithmic kernel automatically satisfies Equation (35) with the Cauchy kernel, it is sufficient to consider only Equation (34). The solution to this equation can be found using the techniques discussed in [10] and is written as

$$\mathcal{I}_m(z) = \frac{2i}{Z_0 k_0\sqrt{d^2-z^2}}\frac{A_m\delta_m}{\ln(4a/d) - S_m}\,. \tag{36}$$

Substituting (36) into (7) and integrating the linear current density $I(\phi, z)$ over z between $-d$ and d, we obtain the total current $I_\Sigma(\phi)$ in the cross section $\phi = \text{const}$:

$$I_\Sigma(\phi) = \frac{iV_0}{Z_0k_0a}\sum_{m=-\infty}^{\infty}\frac{\sin(m\Delta)}{m\Delta}\,\frac{\delta_m\exp[-im(\phi-\phi_0)]}{\ln(4a/d)-S_m}. \tag{37}$$

Generally, the summation over m in (37) can be performed only numerically. A closed-form expression for the current distribution can be derived if the strip is so narrow that the inequality $\ln(4a/d) \gg |S_m|$ is valid for $|m| < \tilde{m}$. Then, neglecting S_m and making steps similar to those performed in [10], we deduce

$$I_\Sigma(\phi) = -\frac{iV_0\pi h}{Z_0k_0\ln(4a/d)}\frac{\cos[(\pi-\phi+\phi_0)ha]}{\sin(\pi ha)}\,, \tag{38}$$

where $0 \le \phi - \phi_0 \le \pi$ and $h = k_0(1-i)\sqrt{\xi/2}$. Approximate representation (38) evidently corresponds to the transmission-line theory with the complex current propagation constant h. Such nature of h is related to the excitation of an infinite number of propagated quasielectrostatic eigenmodes in the resonant plasma column. If $|\varepsilon\eta|^{1/2} \ll \varepsilon_a$, then $h = k_0\sqrt{\varepsilon_a/2}$. In the opposite case $|\varepsilon\eta|^{1/2} \gg \varepsilon_a$, we obtain $h = k_0(1-i)|\varepsilon\eta|^{1/4}/2$, which is a factor of $\sqrt{2}$ smaller than the corresponding quantity for a loop antenna in a resonant homogeneous magnetoplasma (see [10]).

Using the current distribution $I_\Sigma(\phi)$, the input impedance $Z = R+iX$ of the antenna can be found in a standard way: $Z = V_0/I_\Sigma(\phi_0)$. Within the framework of approximation (38), we obtain

$$Z = iZ_0k_0(\pi h)^{-1}\ln(4a/d)\tan(\pi ha). \tag{39}$$

In the case $\pi|\mathrm{Im}\,h|a \gg 1$, the input impedance given by (39) simplifies to $Z = Z_0k_0(\pi h)^{-1}\ln(4a/d)$. For $\pi|h|a \ll 1$, Equation (39) yields the inductance of an electrically small loop antenna in free space.

4. NUMERICAL RESULTS

Using the above-described approach, we have calculated the current distribution and input impedance of a loop antenna for some cases of interest. Calculations have been performed for the following values of the parameters: the angular frequency $\omega = 1.7 \times 10^8\,\mathrm{s}^{-1}$, the relative dielectric permittivity of the background medium is equal to $\varepsilon_a = 1$ (free space), the external static magnetic field $B_0 = 800\,\mathrm{G}$, and the plasma density inside the column is equal to $N = 10^{13}\,\mathrm{cm}^{-3}$. The chosen values can easily be realized under laboratory conditions and correspond to the case of a resonant plasma, for which $\omega_{\mathrm{LH}} \ll \omega < \omega_H < \omega_p$ and the diagonal elements of the dielectric tensor have the opposite signs: $\varepsilon = 1.62 \times 10^2$ and $\eta = -1.1 \times 10^6$. For computations, it was assumed that $d/a = 0.02$, the midpoint of the region to which the given voltage is supplied has the azimuthal coordinate $\phi_0 = 0$, and $\Delta = 0.05\,\mathrm{rad}$.

Figure 2 shows the magnitude $|I_\Sigma(\phi)|$ (normalized to its maximum $|I_{\Sigma\max}|$) and the phase angle $\theta(\phi) = \arctan(\mathrm{Im}\, I_\Sigma(\phi)/\mathrm{Re}\, I_\Sigma(\phi))$ of the antenna current for two values of the loop and column radius a. The solid and dashed lines in the figure correspond to the rigorously derived formula (37) and the approximate formula (38), respectively. It is seen in the figure that formula (38) describes the current behavior with acceptable accuracy, especially for small and moderate values of ϕ.

It is worth noting that in Fig. 2, the distribution given by Equation (37) demonstrates some asymmetry about the midpoint of the region to which the excitation voltage is supplied. This asymmetry

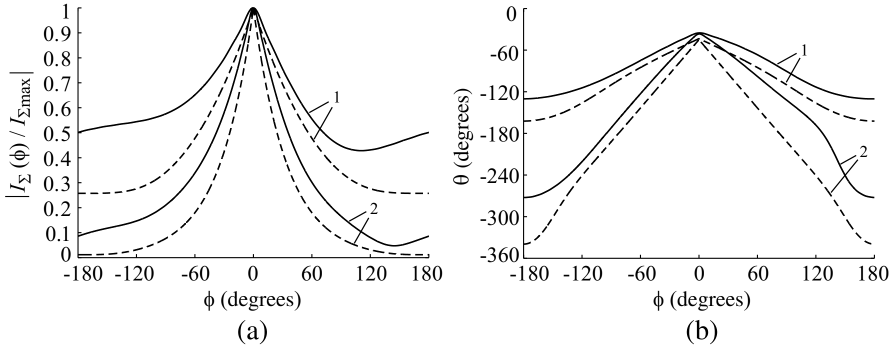

Figure 2. (a) Normalized amplitude and (b) phase of the antenna current calculated using the rigorous formula (37) (solid lines) and the approximate formula (38) (dashed lines) as functions of the angle ϕ for (1) $a = 2\,\mathrm{cm}$ and (2) $a = 5\,\mathrm{cm}$ if $d/a = 0.02$, $\Delta = 0.05$ rad, $\phi_0 = 0$, $\varepsilon_a = 1$, $N = 10^{13}\,\mathrm{cm}^{-3}$, $B_0 = 800\,\mathrm{G}$, and $\omega = 1.7 \times 10^8\,\mathrm{s}^{-1}$.

is stipulated by the gyrotropy of the plasma inside the column. The reversal of the direction of the external magnetic field results in the current- and phase-distribution changes described by the replacements $I_\Sigma(\phi) \to I_\Sigma(-\phi)$ and $\theta(\phi) \to \theta(-\phi)$, respectively.

It is evident that the dependences $|I_\Sigma(\phi)/I_{\Sigma\max}|$ and $\theta(\phi)$ for the antenna located on the surface of a resonant plasma column qualitatively resemble the corresponding distributions for a loop antenna in a homogeneous magnetoplasma with appropriate parameters [10]. If the antenna of the same radius were located in free space, it would have a quasi-uniform current distribution. Therefore, the presence of a resonant plasma column significantly affects the current distribution of the loop antenna.

Figure 3 shows the real (R) and imaginary (X) parts of the antenna input impedance Z as functions of the radius a for the previously chosen values of the parameters of the problem. These results were obtained using Equation (39). Calculations based on the rigorous formula for the antenna current give results which almost coincide with those in Fig. 3 and, therefore, are not shown for brevity. For comparison, the figure also presents similar dependences for the real (R_0) and imaginary (X_0) parts of the input impedance of the same antenna located in a homogeneous magnetoplasma the parameters of which coincide with those inside the column.

It follows from the results obtained that R and X turn out to

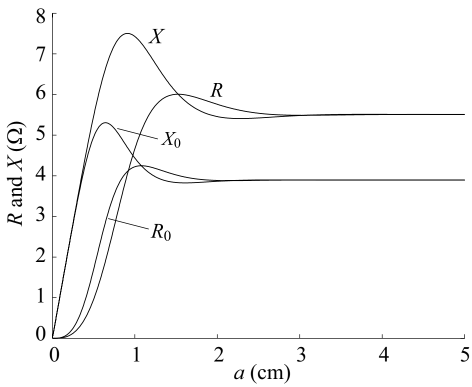

Figure 3. Real and imaginary parts of the input impedance as functions of the antenna size in the cases where the antenna is located on the surface of a plasma column (R and X) of the same radius and in a homogeneous magnetoplasma (R_0 and X_0). The values of d/a and the other parameters are the same as in Fig. 2.

be greater than the corresponding quantities R_0 and X_0 for the loop antenna in a homogeneous magnetoplasma. Although the impedance demonstrates qualitatively similar behavior with increasing antenna radius in the two cases, an important difference between them is that the input radiation resistance R is almost completely determined by the eigenmodes of the plasma column, i.e., the discrete spectrum of the antenna-excited waves, whereas in the case of a homogeneous plasma, the quantity R_0 is entirely determined by the continuous-spectrum waves.

5. CONCLUSIONS

In this paper, we obtained the solution to the problem of the current distribution of a loop antenna in the form of an infinitesimally thin, perfectly conducting, narrow strip located on the surface of an axially magnetized plasma column and operated in the resonant frequency band of a magnetoplasma. The found solution describes the distribution of the surface-current density both along and across the strip and makes it possible to study the electrodynamic characteristics of the antenna as functions of its parameters as well as the parameters of the plasma column and the surrounding medium. Finally, we note that the method used in this work can be applied to the case where the column with a loop antenna is filled with a resonant anisotropic medium of another type such as, e.g., a hyperbolic metamaterial.

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APPENDIX A. CONTRIBUTION OF EIGENMODES TO THE SINGULAR PARTS OF THE KERNELS

Although the contribution of eigenmodes to $K_m^{(s)}(\zeta)$ and $k_m^{(s)}(\zeta)$ can be obtained by calculating the corresponding terms of (11) and (12) in the limit $|p_{m,n}| \to \infty$, a simpler way is to use formulas of the quasielectrostatic approximation, which is valid in this limit. Within the framework of this approximation, the electric field is expressed as $\mathbf{E}(\mathbf{r}) = -\nabla\Psi(\mathbf{r})$, where the potential $\Psi(\mathbf{r})$ is sought in the form $\Psi(\mathbf{r}) = \Psi(\rho)\exp(-im\phi - ik_0pz)$. The function $\Psi(\rho)$ is represented as $\Psi(\rho) = \mathcal{B}J_m(k_0\tilde{q}\rho)$ for $\rho < a$, and as $\Psi(\rho) = \mathcal{D}K_m(k_0s\rho)$ for $\rho > a$, where K_m is a modified Bessel function of the second kind of order m, $\tilde{q} = (-p^2\eta/\varepsilon)^{1/2}$, and $s = |\varepsilon/\eta|^{1/2}\tilde{q}$. The coefficients $\mathcal{B}$ and $\mathcal{D}$ as well as the eigenmode propagation constants $p = p_{m,n}$ are found from the requirement that $\Psi(\rho)$ and the radial electric-displacement component are continuous at $\rho = a$. Then the dispersion relation for quasielectrostatic eigenmodes takes the form

$$\frac{J_{m+1}(k_0 a\tilde{q})}{J_m(k_0 a\tilde{q})} = \operatorname{sgn}\varepsilon \, \frac{\varepsilon_a}{|\varepsilon\eta|^{1/2}} \frac{K_{m+1}(k_0 a s)}{K_m(k_0 a s)}. \tag{A1}$$

If the propagation constants are sufficiently large, so that $k_0 a|\eta/\varepsilon|^{1/2}|p| \gg |m|$ and $k_0 a|p| \gg |m|$, one can use the large-argument approximation for the Bessel functions in (A1) and arrive at

$$p_{m,n} = (k_0 a)^{-1}|\varepsilon/\eta|^{1/2}\left[\chi_0 + \frac{\pi}{2}(2n + m + 1/2)\operatorname{sgn}\varepsilon\right], \tag{A2}$$

where $\chi_0 = \arctan(\varepsilon_a/|\varepsilon\eta|^{1/2})$. If slight losses in the plasma medium are allowed for, then each of the quantities $p_{m,n}$ acquires a small imaginary part such that $\operatorname{Im} p_{m,n} < 0$.

In the case considered, the norm $N_{m,n}$ of each quasielectrostatic eigenmode is approximately determined by integration over ρ in the limits $0 \le \rho \le a$ in (14). The magnetic field of such eigenmodes in the column is found from the equation $\nabla^2\mathbf{H}(\mathbf{r}) = -i\omega\nabla \times (\boldsymbol{\varepsilon}\cdot\mathbf{E}(\mathbf{r}))$. Then we have

$$\begin{aligned} N_{m,n} &= Z_0^{-1} 2\pi (k_0 a)^2 |\eta| p_{m,n} \mathcal{B}^2 J_m^2\left(k_0 a|\eta/\varepsilon|^{1/2} p_{m,n}\right) \\ &\quad \times \left[1 + \frac{J_{m+1}^2\left(k_0 a|\eta/\varepsilon|^{1/2} p_{m,n}\right)}{J_m^2\left(k_0 a|\eta/\varepsilon|^{1/2} p_{m,n}\right)}\right] \operatorname{sgn}\varepsilon, \end{aligned} \tag{A3}$$

where the second term in the brackets can approximately be replaced by $\varepsilon_a^2/|\varepsilon\eta|$, as is evident from (A1). In addition, for sufficiently large n when $\pi n \gg |\chi_0|$, we can neglect the term χ_0 in (A2) when substituting $p_{m,n}$ into (A3). Finally, taking into account that by virtue of the second inequality in (20), $\exp(-i\chi_0|\varepsilon/\eta|^{1/2}|\zeta|/a) \simeq 1$, we obtain the series in Equation (21).

The series in Equation (22) is derived in a similar way, but one should first allow for a small collisional loss in the plasma by introducing the term ν (see [10]) to ensure the series convergence, with subsequent passage to the weak limit $\nu \to 0$.

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