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ON THE OUTAGE PROBABILITY PREDICTION OF TIME DIVERSITY SCHEME IN BROADBAND SATELLITE COMMUNICATION SYSTEMS

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Abstract—A novel model for the outage probability prediction in time diversity satellite communication (SatCom) systems operating above 10 GHz is proposed. Due to the migration of operating frequency at Ka band and above, atmospheric phenomena affect the signal. Rain is the dominant fading mechanism. Diversity techniques are the probable solution of the compensation of rain fading. Among the diversity techniques, time diversity has been identified as an efficient and cost effective technique. A method for the prediction of outage performance and diversity gain of time diversity SatCom systems is presented based on the physical assumptions of a well accepted dynamic stochastic model. The new method is tested against with simulated and experimental data with encouraging results.

1. INTRODUCTION

The ever increasing demand of high data rate in satellite communication and the congestion of conventional frequency bands lead to the adoption of high bandwidth and high frequency bands. A promising solution of future Satellite Communication (SatCom) systems is the adoption of the Ka frequency band or even higher such as the Q/V bands. In [1], the Terabit/s satellite system is investigated for the deliverance of high data rates via satellites through the operation of

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the system at Ka and Q/V bands. These high frequencies may be used either for backhaul application (feeder links) or Direct-to-User (DTU) and Direct-to-Home (DTH) satellite services. Since in these high millimeter wave frequency bands, rain attenuation can cause very significant losses of the power of the signal for low time percentages, the adoption of a power margin for its compensation leads to inefficient use of the sources of the SatCom system. For this reason, Propagation Impairment Mitigation Techniques have been identified and tried to be implemented into SatCom systems. Such techniques include Adaptive Coding and Modulation, Power Control and diversity techniques [2]. The most effective PIMTs are the diversity techniques. The four diversity techniques mentioned and analyzed in [2] are site diversity, orbital diversity, frequency and time diversity. In all the diversity techniques, two or more separated, in frequency, space or time, radiopaths are used for the realization of the communication.

More particularly, in time diversity satellite communications systems, the signal is retransmitted after a period of time, otherwise called waiting period, with a duration estimated by the dynamics of the atmospheric phenomenon that is degrading the system's performance. In case of rain attenuation, due to the decrease of correlation of rain rate and rain attenuation with time [3–5] the same signal is retransmitted after the time lag. The two received signals can be regarded as two signals received from uncorrelated radio paths, considering a very large time lag. Therefore, the independence of the signals leads to higher performance of the system. The signals can be then combined with various techniques such as Selection Combining (SC), Maximal Ratio Combining (MRC) or Equal Gain Combining (EGC) techniques [6] leading to a further increase of the diversity gain.

However, it must be noted that time diversity as a technique for the improvement of the quality of the link has some drawbacks and limitations. More particularly, since the signal is retransmitted after a time period, a buffer is needed at the transmitter and the receiver, i.e., two buffers in total, in order to store the first transmitted signal and therefore the cost of the system is increased. Now, considering the duration of waiting period for the retransmission of the signal, the buffer needed might be a large one in order to store all the required signals, since during a given waiting period the temporal diversity is applied to other time slots also. Furthermore, time diversity applied to satellite communication systems trying to mitigate rain impairments, can be considered as a mitigation technique for delay tolerant applications, such as Earth observation applications or data services. The latter holds due to the fact that the temporal correlation of rain rate and rain attenuation is high enough for a long time period [3–5] and so in delay intolerant applications such as phone calls or teleconferences time diversity will certainly decrease the quality of the service.

Time diversity as a technique for the compensation of rain attenuation has been studied in many papers [7–11]. In [7] an empirical relationship for calculating the exceedance probability of rain attenuation for time diversity systems at equatorial areas is presented using data derived from measurements at Malaysia. Also, the Synthetic Storm Technique (SST) has been used for the computation of diversity gain due to time diversity and an analytical expression as a function of time delay frequency of the link and attenuation level has been presented in [8]. Apart from the modelling of diversity gain, another method can be used for the computation of outage probability of time diversity systems under rain. The latter is based on the knowledge and modelling of joint exceedance probability of rain attenuation. In [12, 13], two models are proposed for the computation of joint exceedance probability of rain attenuation based on the assumption that rain attenuation follows the lognormal distribution. Following this assumption and adopting a temporal correlation coefficient the joint Complementary Cumulative Distribution Function (CCDF) of rain attenuation is calculated.

In this paper, a new methodology is presented for the computation of the joint CCDF of rain attenuation based on the well-known stochastic dynamic model in [3]. More particularly, the model is based on the derivation of the Transition Probability Density Function (TPDF) of rain attenuation from the Stochastic Differential Equation (SDE) of the Maseng-Bakken model [3]. The paper is organized as follows: the proposed model and its main assumptions are presented in Section 2, in Section 3 numerical results and comparative tests are shown and in Section 4, some useful conclusions are drawn.

2. TIME DIVERSITY OUTAGE PREDICTION MODEL

The main assumptions of the Maseng-Bakken model is that rain attenuation follows the lognormal distribution and that the rate of change of rain attenuation is proportional to the instantaneous value of rain attenuation. Therefore, we adopt that the Probability Density Function (PDF) of the rain attenuation on every time instant is:

$$p_A\left(A\right)=\frac{1}{\sqrt{2\pi}S_A A}\exp\left\{-\frac{\left(\ln A-\ln A_m\right)^2}{2\,S_A^2}\right\} \tag{1}$$

with S_A the standard deviation of $\ln(A)$ and A_m the median value of the lognormal distribution of rain attenuation.

Next, by adopting the Maseng-Bakken model [3], rain attenuation (A_t) can be modelled as a stochastic process and be described by the following Stochastic Differential Equation:

$$dA_t = A_t \beta_A \left[S_A^2 - \ln\left(A_t/A_m\right)\right] dt + \sqrt{2\beta_A} A_t S_A dW_t \tag{2}$$

where $\beta_\mathrm{A}\,(\mathrm{sec}^{-1})$ is a parameter on which the dynamics of rain attenuation depend and hereafter will be named as the dynamic parameter of rain attenuation and dW_t are the Brownian increments [14].

It is known that for a SDE of the form:

$$dA_t = f\left(A_t, t\right) dt + g\left(A_t, t\right) dW_t \tag{3}$$

with $f(A_t,\, t)$ the drift coefficient and $g(A_t,\, t)$ the diffusion coefficient, the Transition Probability Density Function (TPDF), i.e., $p\{A(t),\, t|A(t_0),\, t_0\}$ is the solution of the differential equation [15]:

$$\begin{aligned} &\frac{\partial p\{A(t), t|A(t_0), t_0\}}{\partial t} + \frac{\partial}{\partial A}[f(A(t), t) \cdot p\{A(t), t|A(t_0), t_0\}] \\ &-\frac{1}{2}\frac{\partial^2}{\partial A^2}\left[g^2(A(t), t) \cdot p\{A(t), t|A(t_0), t_0\}\right] = 0 \end{aligned} \tag{4}$$

The TPDF describes the evolution of the stochastic process by giving the probability of the stochastic process in time $t = t_0 + \Delta t$ given the initial value of the process at time t_0.

The solution of the above differential equation is [16]:

$$\begin{aligned} &p\{A(t), t|A(t_0), t_0\} \\ &= \frac{1}{\sqrt{2\pi} S_{a0}(\Delta t) A(t)} \exp\left\{-\frac{[\ln A(t) - \ln A_{m0}(\Delta t)]^2}{2S_{a0}^2(\Delta t)}\right\} \end{aligned} \tag{5}$$

where

$$A_{m0}\left(\Delta t\right) = A_m^{[1-\exp(-\beta_A \Delta t)]} A\left(t_0\right)^{\exp(-\beta_A \Delta t)} \tag{6}$$

and

$$S_{a0}\left(\Delta t\right) = S_a \sqrt{1 - \exp\left(-2\beta_A \Delta t\right)} \tag{7}$$

Therefore, the outage probability, joint exceedance probability of rain attenuation, i.e., the probability that rain attenuation on two different time instances are above a certain threshold (a_{thr}), for a time diversity system can be calculated by:

$$\begin{aligned} P_\mathrm{TD} &= \Pr\left\{A\left(t\right) \geq a_{thr}, A\left(t_0\right) \geq a_{thr}\right\} \\ &= \int_{a_{thr}}^{\infty} \int_{a_{thr}}^{\infty} p\left\{A\left(t\right), t|A\left(t_0\right), t_0\right\} p\left\{A\left(t_0\right), t_0\right\} dA\left(t\right) d\left(A\left(t_0\right)\right) \end{aligned} \tag{8}$$

with $p\{A(t), t|A(t_0), t_0\}$ derived from (5) and $p\{A(t_0), t_0\}$ from (1). Furthermore, a_{thr} is the threshold value of rain attenuation, which means the joint exceeded value of rain attenuation for both time instances for a given joint exceedance probability P_{TD}. The double integral of (8) can be reduced to a single integral, since the TPDF $p\{A(t), t|A(t_0), t_0\}$ is a lognormal PDF with median value $A_{m0}(\Delta t)$ and standard deviation of the natural logarithm $S_{a0}(\Delta t)$. Therefore, (8) becomes:

$$P_{\mathrm{TD}} = \int_{a_{thr}}^{\infty} P\left\{A\left(t\right) \geq a_{thr}, t|A\left(t_0\right), t_0\right\} p\left\{A\left(t_0\right), t_0\right\} d\left(A\left(t_0\right)\right) \tag{9}$$

where

$$P\left\{A\left(t\right) \geq a, t|A\left(t_0\right), t_0\right\} = \frac{1}{2}\mathrm{erfc}\left(\frac{\ln a - \ln A_{m0}\left(\Delta t\right)}{\sqrt{2}S_{a0}\left(\Delta t\right)}\right) \tag{10}$$

The final expression (9) for the calculation of the outage probability in a time diversity scheme is easily calculated numerically and converges very fast due to the monotonically decreasing nature of the integrand functions. The long-term parameters A_m and S_A are derived through fitting procedure on the latest version ITU-R P.618 [17] and the dynamic parameter $\beta_{\mathrm{A}}\,(\mathrm{sec}^{-1})$ is either considered by ITU-R P.1853 [18] or the physical-mathematical model in [19, 20].

3. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results are given and the proposed model is tested against experimental data. Firstly, the proposed expression given in (9) is tested against the joint CCDF derived from the time series of rain attenuation generated by the Maseng-Bakken model. For this purpose, a hypothetical Earth space link with the ground station located at Athens, Greece and operating frequency of 40 GHz and elevation angle of 40° is considered. The dynamic parameter is set equal to $2 \times 10^{-4}\,\mathrm{sec}^{-1}$ as this is recommended in [18]. In Figure 1, the single link CCDF of rain attenuation for the above link as well as the simulated and proposed joint CCDF of rain attenuation for a time diversity system are given. The time lag was set equal to 10 min and the lognormal CCDF which was given as an input to the time series was derived after the fitting of theoretical CCDF to the prediction of ITU-R. P. 618-10 model [17]. As it can be observed from Figure 1, the theoretical results coincide with the simulated ones.

In Figure 2, for the same hypothetical link, the CCDF of induced rain attenuation for a system without time delay and for a system

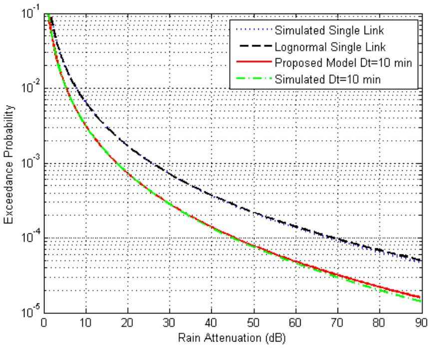

Figure 1. CCDF of rain attenuation for a single link with and without time diversity derived from the proposed model and from simulations.

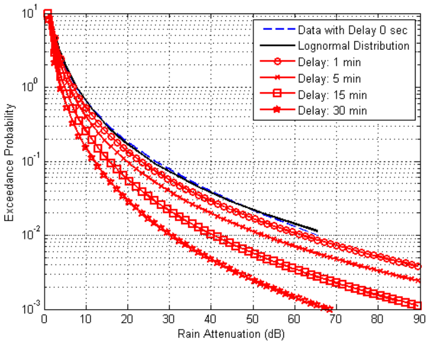

Figure 2. CCDF of rain attenuation for a link without time delay and for various delays.

with various time delays are shown. Apart from the joint exceedance probability the diversity gain is of paramount importance for the design of time diversity SatCom systems. The diversity gain $G_{\text{TD}}(\Delta t,\, p\%)$ for $p\%$ time percentage and a TD system with time lag Δt is defined as:

$$G_{\text{TD}}\left(\Delta t, p\%\right) = A_{\text{TD}}\left(\Delta t = 0, p\%\right) - A_{\text{TD}}\left(\Delta t \neq 0, p\%\right) \qquad (11)$$

where $A_{\text{TD}}(\Delta t = 0,\, p\%)$ is the attenuation exceeded for $p\%$ in case that there is no TD adopted and $A_{\text{TD}}(\Delta t \neq 0,\, p\%)$ is the attenuation exceeded for $p\%$ for a TD system with time delay Δt. In Figure 3, the achieved diversity gain is given for probability levels 1% and 0.01%. As it can be observed from Figures 2 and 3, increasing the time delay of the diversity system the joint CCDF of rain attenuation is decreasing. This is very obvious since with the increase of time delays the radiopaths are less correlated thus resulting to lower joint exceedance probability. However, the gain is increasing less and less with the increase of time delays, as this can be seen in Figure 3.

In Figures 4 and 5, for data derived from measurements at Spino d'Adda at frequencies 19 GHz and 40 GHz, respectively. The experimental curves were derived from [13]. In [13], as this is mentioned in the paper, the statistics presented refer to in-excess attenuation. However, in this paper due to the lack of measurements for time diversity systems, we compare the proposed model with these data. For the derivation of the theoretical exceedance probabilities, we computed the parameters of lognormal distribution of the system without time diversity (delay equal to 0 sec) through a non-linear regression process. The value of the dynamic parameter was set equal to $10^{-4}\,\text{sec}^{-1}$ for both experiments, as this has been measured for these experiments [12].

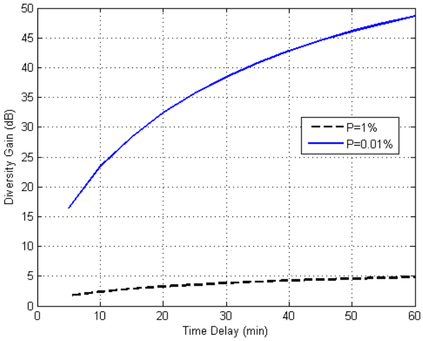

Figure 3. Gain achieved with the adoption of time diversity for 0.01% and 1% time percentages.

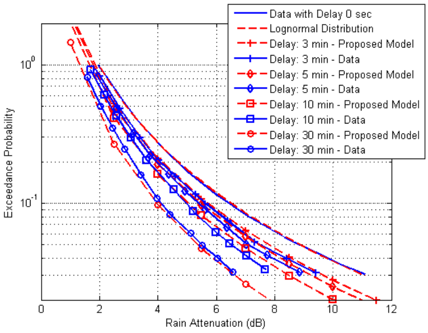

Figure 4. CCDF of rain attenuation with and without TD derived from data and the proposed model for Spino d'Adda at 19 GHz.

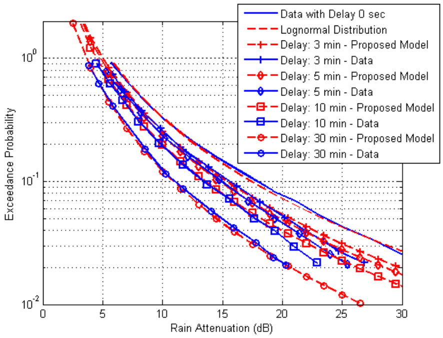

Figure 5. CCDF of rain attenuation with and without TD derived from data and the proposed model for Spino d'Adda at 40 GHz.

From the figures it can be easily observed that the method proposed can be considered accurate for both experiments.

Finally, the time diversity gain achieved for 0.01% for various elevation angles and various time delays is computed. It has been observed and shown that the dynamic parameter of rain attenuation depends on the dynamics of rain rate and the characteristics of the link, i.e., frequency, elevation angle etc. [19, 20]. Therefore, we assessed the performance of time diversity systems for various elevation angles using the methodology presented in [20] for the computation of the dynamic parameter of rain attenuation for various elevation angles. In Figure 6, the time diversity gain is shown for 0.01% of time for time

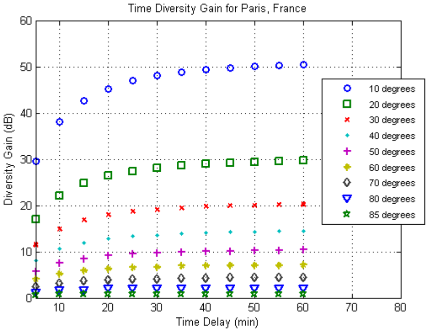

Figure 6. Time diversity gains for Paris, France for various elevation angles.

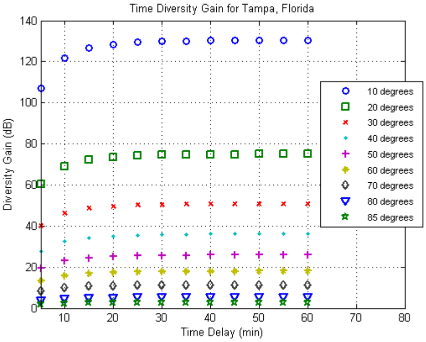

Figure 7. Time diversity gains for Tampa, Florida for various elevation angles.

delays between 5 min and 60 min for various elevation angles. The Earth station was considered to be at Paris, France and the frequency of the link was set equal to 30 GHz. In Figure 7, the same curves are shown but for an Earth station located at Tampa of Florida and 20 GHz operating frequency of the link. The dynamic parameter of rain rate was set equal to 10^{-3} sec, considering the measurements presented in [5]. It can be easily observed that higher gains are achieved as the elevation angle gets lower.

4. CONCLUSIONS

In this paper, a new simple physical-mathematical method for the estimation of outage probability of SatCom time diversity systems is proposed. The method is based on the assumptions of the Maseng-Bakken model for the static and dynamic properties of rain attenuation. Firstly, the model was evaluated with synthesized time series of rain attenuation from the Maseng-Bakken model showing a very good accuracy. It is also found that increasing the time delay the joint CCDF is decreasing less and less. Moreover, the model has been compared to experimental data measured at Spino d'Adda of Italy. From the comparative tests it is observed that the model gives accurate results. Finally, using an expression for the calculation of elevation dependent dynamic parameters of rain attenuation, the time diversity gain was calculated for various time delays and various elevation angles. It was found that increasing the elevation angle of the link the time diversity gain is decreasing. The method can be used for the prediction of the performance of time diversity SatCom systems operating at frequencies above 10 GHz.

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