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REQUIRED NUMBER OF OTA ANTENNAS IN THE MULTI-PROBE TEST SYSTEM

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Abstract—The number of OTA antennas of the multi-probe over-the-air (OTA) test system should be large enough for accurate OTA testing yet not too large due to the increasing cost. In this work, the required number of OTA antennas is studied using the spatial correlation function. Some key issues are discussed.

1. INTRODUCTION

The multiple-input multiple-output (MIMO) system has drawn considerable attention due to its enhancement of the spectral efficiency in multipath environments [1]. Opposite to the real-life measurements, over-the-air (OTA) tests in controlled (emulated) multipath environments are fast, repeatable and cost-effective [2]. There exist three types of OTA test systems, i.e., the two-stage OTA system [3], the reverberation chamber (RC) based OTA system [4], and the anechoic chamber (AC) and the fading emulator based multi-probe system [5]. The two-stage OTA system requires measuring the antenna pattern in an AC and then using the measured antenna pattern together with the fading emulator for conductive measurement. The availability of external antenna ports on the device under test (DUT) and the assumption that the external RF (radio frequency) cable has little effect on the actual antenna of the DUT make the two-stage OTA system less preferred than the other two OTA systems. The RC based OTA system has the lowest cost among the three OTA systems, yet it is usually limited to a special reference environment (i.e., isotropic scattering environment). The multi-probe system can flexibly emulate channels with different angular distributions. Therefore, it is particularly suitable for MIMO-OTA testing [5–9].

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Nevertheless, many probes (or OTA antennas) of the multi-probe system are needed for an accurate measurement and each OTA antenna has to be connected to one port of the channel emulator, which increases the cost of the multi-probe system. As a result, there is an urgent need for determining the required number of OTA antennas for the multi-probe system with certain targeted measurement accuracy. Two common performance metrics for the multi-probe system are the spatial correlation function and the synthesized plane wave [6–8]. Unfortunately, analyses based on the two different metrics tend to result in different required numbers of OTA antennas [9].

In this work, effort is exerted in the investigation of the required number of OTA antennas. Specifically, we present an analysis for determining the required number of OTA antennas based on the spatial correlation function. The resulting required number of OTA antennas is in agreement with that obtained using the spherical wave expansion method based on the synthesized plane wave [9]. Thus, this work helps provide a unified required number of OTA antennas for the multi-probe OTA test system. Moreover, the required number of OTA antennas presented in [9] is an immediate result of applying the spherical wave expansion of the synthesized plane wave; analyses such as the decay rate of the synthesized error have been omitted. This work also provides discussions on correlations between the expanded modes (and therefore the possibility of using even fewer OTA antennas yet with different placements of the OTA antennas for emulating different angular distributions) and the decay rate of the synthesized error of the spatial correlation function.

2. ANALYSIS

Most multi-probe OTA test systems are in two-dimensional (2D) configuration due to the cost constraint. Thus, this work will focus on the 2D multi-probe system. For analysis simplicity, we first consider the single-polarization case. The obtained required number of OTA antennas can be extended to the dual-polarization case by simply doubling it [9]. Assuming the far field condition is satisfied in the testing region of the 2D multi-probe system, the (true) multipath field to be emulated can be expressed as

$$F(\mathbf{x}) = \int_0^{2\pi} \alpha(\phi) \exp(j\mathbf{x} \cdot \mathbf{k})\, d\phi \tag{1}$$

where ϕ is the angle of arrival, $\alpha(\phi)$ the random complex-valued gain in the angle of arrival, $\mathbf{x}$ the spatial position, $\mathbf{k} = (k, \phi)$ in the polar form, $k = 2\pi/\lambda$ with λ denoting the wavelength, and $\cdot$ the dot product. The power angular spectrum (PAS) is

$$P(\phi) = \frac{E[\alpha(\phi)\,\alpha*(\phi)]}{\int_0^{2\pi} E[\alpha(\phi)\,\alpha*(\phi)]\,d\phi} \tag{2}$$

where the superscript * denotes the complex conjugate. The spatial correlation function is defined as

$$\rho(\mathbf{x}_1, \mathbf{x}_2) = \frac{E[F(\mathbf{x}_1)\,F*(\mathbf{x}_2)]}{\sqrt{E[F(\mathbf{x}_1)\,F*(\mathbf{x}_1)]\,E[F(\mathbf{x}_2)\,F*(\mathbf{x}_2)]}} \tag{3}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are two spatial positions between which the spatial correlation function is evaluated. Assuming uncorrelated scattering (US) [1] and combining (1)–(3), the spatial correlation boils down to

$$\rho(\Delta\mathbf{x}) = \int_0^{2\pi} P(\phi)\exp(j\Delta\mathbf{x}\cdot\mathbf{k})\,d\phi \tag{4}$$

where the spatial distance $\Delta\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$.

The plane wave $\exp(j\Delta\mathbf{x}\cdot\mathbf{k})$ can be expanded using the Jacobi-Anger identity

$$\exp(j\Delta\mathbf{x}\cdot\mathbf{k}) = \sum_{n=-\infty}^{\infty} j^n J_n(k\Delta x)\exp[jn(\varphi-\phi)] \tag{5}$$

where $\Delta\mathbf{x}\cdot\mathbf{k} = k\Delta x\cos(\varphi-\phi)$ with φ denoting the angle between $\mathbf{x}_1$ and $\mathbf{x}_2$, and J_n is the Bessel function of the first kind with order n. Substituting (5) into (4),

$$\begin{aligned} \rho(\Delta\mathbf{x}) &= \int_0^{2\pi} P(\phi)\sum_{n=-\infty}^{\infty} j^n J_n(k\Delta x)\exp[jn(\varphi-\phi)]d\phi \\ &= \sum_{n=-\infty}^{\infty} j^n J_n(k\Delta x)\widetilde{P_n}\exp(jn\varphi) \end{aligned} \tag{6}$$

where

$$\widetilde{P_n} = \int_0^{2\pi} P(\phi)\exp(-jn\phi)\,d\phi \tag{7}$$

is the nth Fourier series coefficient of the PAS.

Similarly, using the Jacobi-Anger identity, the multipath field can be expanded as

$$F(\mathbf{x}) = \sum_{n=-\infty}^{\infty} j^n J_n(k\Delta x)\, \widetilde{\alpha_n} \exp(jn\theta) \tag{8}$$

where θ is the angle of $\mathbf{x}$ and

$$\widetilde{\alpha_n} = \int_0^{2\pi} \alpha(\phi) \exp(-jn\phi)\, d\phi \tag{9}$$

represents the nth Fourier series coefficient of the random angular gain $\alpha(\phi)$. In a 2D single-polarized multi-probe OTA system with $K = 2N + 1$ OTA antennas, the emulated multipath field can be expressed as

$$\hat{F}(\mathbf{x}) = \sum_{n=-N}^{N} j^n J_n(k\Delta x)\, \widetilde{\alpha_n} \exp(jn\theta). \tag{10}$$

Substituting (2) and (9) into (7), after a few arrangements, one obtains

$$\begin{aligned} \widetilde{P_n} &= \frac{E\left[\int_0^{2\pi} \alpha(\phi) \exp(-jm\phi)\, d\phi \int_0^{2\pi} \alpha*(\phi) \exp(-j(n-m)\phi)\, d\phi\right]}{E\left[\int_0^{2\pi} \alpha(\phi) \exp(-jm\phi)\, d\phi \int_0^{2\pi} \alpha*(\phi) \exp(jm\phi)\, d\phi\right]} \\ &= \frac{E\left[\widetilde{\alpha_m} \widetilde{\alpha_{m-n}*}\right]}{E\left[\widetilde{\alpha_m} \widetilde{\alpha_m}*\right]}. \end{aligned} \tag{11}$$

Thus, $\widetilde{P_n}$ is the correlation coefficient of $\widetilde{\alpha_n}$ in (8) or (10). This implies that $\widetilde{\alpha_n}$ are uncorrelated only for the uniform APS case and that for a particular non-uniform APS, in theory less than K OTA antennas are needed to emulate the multipath field. The latter corresponds to, e.g., the Karhunen-Loève (KL) expansion [10] of the multipath field to yield uncorrelated coefficients. However, the KL expansion for each APS requires a distinct placement of the OTA antennas, which is impractical in OTA tests where the OTA antennas are usually fixed and uniformly placed along a circle. Hence, this work uses the Jacobi-Anger expansion which does not require different placements of OTA antennas for emulating different APSs. Note that the Jacobi-Anger expansion in (8) or (10) corresponds to the 2D spherical wave expansion similar to [9]. Thus, the required number of OTA antennas based on the reflectivity level of the synthesized plane wave,

$$\varepsilon = \max\left\{\frac{\left|\hat{F}\left(\mathbf{x}\right) - F\left(\mathbf{x}\right)\right|}{\max\left\{F\left(\mathbf{x}\right)\right\}}\right\}, \tag{12}$$

will be the same as that in [9]. Instead, we focus on the required number of OTA antennas based on the spatial correlation function in this work.

Similar to (10), the emulated spatial correlation function can be expressed as

$$\hat{\rho}\left(\Delta\mathbf{x}\right) = \sum_{n=-N}^{N} j^n J_n\left(k_0\Delta x\right)\widetilde{P_n}\exp\left(jn\varphi\right). \tag{13}$$

As pointed out in [6], for the measurement-based evaluation of the spatial correlation function, different results may occur when the spatial sampling points are limited on a line or on a circle. To avoid this problem, the whole test zone with a radius of r_0 is sampled in this work. Specifically, we define the normalized mean square error of the emulated spatial correlation function as

$$\xi = \frac{E\left[\int_0^{r_0}\int_0^{2\pi}\left|\hat{\rho}\left(\Delta\mathbf{x}\right) - \rho\left(\Delta\mathbf{x}\right)\right|^2 r d\varphi dr\right]}{E\left[\int_0^{r_0}\int_0^{2\pi}\left|\rho\left(\Delta\mathbf{x}\right)\right|^2 r d\varphi dr\right]} \tag{14}$$

where the expectation is taken over the random variable $\widetilde{P_n}$. Substituting (6) and (13) into (14) and exchanging the order of integration and expectation,

$$\xi = \frac{\int_0^{r_0}\int_0^{2\pi}\sum_{|n|>N} J_n^2\left(kr\right)E\left[\left|\widetilde{P_n}\right|^2\right] r d\varphi dr}{\int_0^{r_0}\int_0^{2\pi}\sum_{n=-\infty}^{\infty} J_n^2\left(kr\right)E\left[\left|\widetilde{P_n}\right|^2\right] r d\varphi dr}. \tag{15}$$

As can be seen from (7), $\widetilde{P_n}$ is independent of φ and $E[|\widetilde{P_n}|^2] = \int_0^{2\pi} E[P(\phi)P*(\phi)]d\phi$ is independent of n. Thus, the term $E[|\widetilde{P_n}|^2]$

in both the numerator and denominator of (15) cancels each other. Note that replacing Δx with r gives no difference due to the double integration and that $\sum\limits_{n=-\infty}^{\infty} J_n^2(kr) = 1$. Therefore, (15) boils down to

$$\xi = \frac{2\int\limits_0^{r_0} \sum\limits_{n>N} J_n^2\,(kr) r dr}{r_0^2/2}. \tag{16}$$

The factor of 2 appears because $\sum_{|n|>N} J_n^2(kr) = 2\sum_{n>N} J_n^2(kr)$. As can be seen, ξ does not depend on the PAS. Hence, the required number of OTA antennas obtained based on ξ is valid for any PAS (which is desirable in that the multi-probe OTA system is able to emulate channels with different PASs).

By virtue of the properties of the Bessel function, the summation term in the integral in (16) is upper bounded by

$$\begin{aligned}\varsigma &= \sum_{n>N} J_n^2\,(kr) \le \sum_{n>N} \frac{(kr/2)^{2n}}{(n!)^2} \\ &= \frac{(kr/2)^{2(N+1)}}{[(N+1)!]^2} \sum_{n\ge 0} \frac{[(N+1)!]^2\,(kr/2)^{2n}}{[(N+n+1)!]^2} = \eta(N)\end{aligned} \tag{17}$$

where ! denotes the factorial operator. Since, for an integer n_1, $\sum\limits_{n\ge 0}[(N+1)!]^2/[(N+n+1)!]^2 \ge \sum\limits_{n\ge 0}[(N+n_1+1)!]^2/[(N+n+n_1+1)!]^2$,

$$\begin{aligned}\frac{\eta(N+n_1)}{\eta(N)} &= \frac{(kr/2)^{2n_1}[(N+1)!]^2}{[(N+n_1+1)!]^2} \frac{\sum\limits_{n\ge 0} \frac{[(N+n_1+1)!]^2 (kr/2)^{2n}}{[(N+n+n_1+1)!]^2}}{\sum\limits_{n\ge 0} \frac{[(N+1)!]^2 (kr/2)^{2n}}{[(N+n+1)!]^2}} \\ &\le \frac{(kr/2)^{2n_1}[(N+1)!]^2}{[(N+n_1+1)!]^2} = \frac{(kr/2)^{2n_1}}{[(N+2)\ldots(N+n_1+1)]^2} < \left(\frac{kr}{2N}\right)^{2n_1}.\end{aligned} \tag{18}$$

The last inequality in (18) follows because $(N+2)\ldots(N+n_1+1) > N^{n_1}$. When N is larger than kr, $\eta(N+n_1)/\eta(N) < (1/2)^{2n_1}$, namely, ζ decays exponentially.

Moreover, it can be easily checked by numerical calculation that $\sum\limits_{n>N} J_n^2(kr) = (1 - \sum\limits_{|n|\le N} J_n^2(kr))/2$ is negligible for $N = \lceil kr \rceil$ (i.e., the smallest integer that is larger than kr). This implies that the mean square error of the emulated spatial correlation function (16) is negligible.

To be accurate over the whole test zone, the required number of OTA antennas is chosen to be

$$K_{sp} = 2N + 1 = 2\left(\lceil kr_0 \rceil + n_1\right) + 1. \tag{19}$$

Note that n_1 is needed to ensure different level of accuracies and to be consistent with the spherical expansion approach in [9], according to which, n_1 ranges from 0 to 10 in practice depending o the desired accuracy.

It should be noted that the above analysis is for the single polarization case. For the dual-polarized multi-probe OTA system, the required number of OTA antennas is [9],

$$K_{dp} = 2K_{sp} = 4\left(\lceil kr_0 \rceil + n_1\right) + 2. \tag{20}$$

Equation (20) is identical to the required number of OTA antennas derived in [9] based on the spherical wave expansion of the synthesized plane wave.

3. SIMULATION

We resort to simulations for verifying the analysis. To that end, we assume single-polarized uniform angular distribution with a coherence

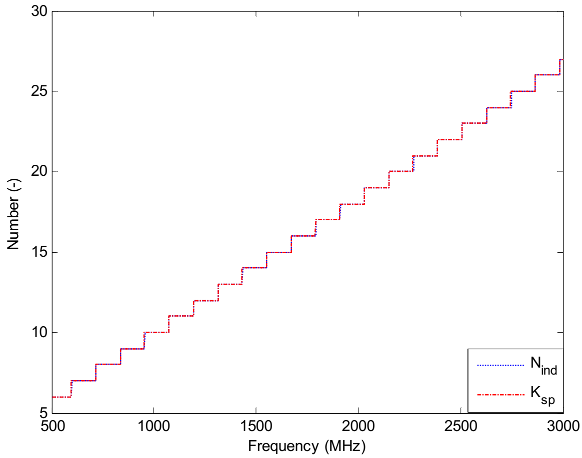

Figure 1. Comparison of the required number of OTA antennas and the number of uncorrelated samples on a circle for the single-polarized uniform angular distribution case.

distance of $\lambda/2$ [1]. Travelling on a circle with a radius of r_0, the maximum number of uncorrelated samples is

$$N_{ind} = \left\lceil \frac{2r_0 \sin(\pi/N_c)N_c}{\lambda/2} \right\rceil + 1 \tag{21}$$

where N_c is chosen to be larger than N_{ind} for a given r_0 (i.e., 0.2 m) over the whole frequency range (i.e., $500 \sim 3000$ MHz), e.g., $N_c = 50$. The derivation of (21) is quite intuitive: the number of $\lambda/2$ (uncorrelated samples) is obtained by dividing the summation of all the piece-wise linear distances between consecutive platform positions by $\lambda/2$ plus one. As mentioned in Section 2, required number of OTA antennas for the single-polarization case equals the number of uncorrelated modes (samples). Fig. 1 shows the comparison of (21) and (19) with $n_1 = 0$. As expected, the derived required number of OTA antennas equals that of uncorrelated samples on a circle (i.e., the two curves overlap each other).

4. CONCLUSION

In this work, by expanding the spatial correlation function using the Jacobi-Anger identity, the required number of OTA antennas is derived. The required number of OTA antennas is universal for the emulation of different APSs using the multi-probe OTA test system. The number of OTA antennas has been studied either using the spatial correlation function or the synthesized plane wave in literature [7–9]. However, approaches based on the two metrics tend to yield different results. The required number of OTA antennas derived in this work (based on the spatial correlation function) is in agreement with that derived by performing spherical wave expansion of the plane wave in [9]. Hence, this work provides a unified required number of OTA antennas for the multi-probe OTA test system. Moreover, the required number of OTA antennas derived in [9] is an immediate result of the spherical wave expansion; information like the decay rate of the synthesized error is not available. In this work, key issues such as correlations between the expanded modes and the decay rate of the synthesized error of the spatial correlation function are also discussed.

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