DESIGN OF EVEN-ORDER SYMMETRIC BANDPASS FILTER WITH CHEBYSHEV RESPONSE

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Abstract—This paper proposes a method to design an even-order symmetric bandpass filter with Chebyshev response. The alternative Jinverters and $\lambda/4$ short-ended resonators are used in the filter design. It is well known that a conventional even-order Chebyshev bandpass filter prototype can be designed by using Jinverters. However, to achieve the Chebyshev response, a problem is that the output admittance Y_L is unequal to the input admittance Y_0 since normalized g_{n+1} is not equal to the g_0 . Thus, for the symmetrical structure, an additional impedance transform can be installed at the output port to solve this problem, thus the network of even-order symmetric bandpass filter with a Chebyshev response should be modified with new *J*-inverters. In this work, all J-inverters of the symmetric bandpass filter with Chebyshev response are extracted and described as curves to determine the circuit dimensions of the proposed structure. Two even-order Chebyshev bandpass filters with the second- and fourth-order are designed with the proposed method as its application examples. Finally, the fourthorder filter is fabricated and measured at center frequency of 2.5 GHz with the fractional bandwidth 25%. The measured result is in good agreement with the simulated one.

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1. INTRODUCTION

The resonator is an important part to constitute a microwave bandpass filter (BPF). Many different types of resonators have been proposed in recent years, e.g., multiple-mode resonator to form a UWB bandpass filter [1–3], $\lambda/4$ resonators and $\lambda/2$ resonators to constitute a narrow band bandpass filter [4–6], and many other types of resonators [7–9]. The $\lambda/4$ resonators are commonly used due to their several attractive features such as compact size, relaxed interresonator coupling, and wide upper stopband. In [10], a prominent $\lambda/4$ resonator interdigital bandpass filter was developed where each resonator had its own ground. A design method for a novel directcoupled balanced BPF based on coupled-resonator theory by utilizing $\lambda/4$ resonators and $\lambda/2$ resonators has been proposed in [11], and two transmission zeros were obtained from the delicate coupled structure. The Chebyshev polynomial is often used as a synthesis method of the filter design [12, 13]. A substrate integrated waveguide diplexer with Chebyshev response is presented in [12], and the Chebyshev lowpass prototype is used in its synthesis method. A complex derivation is given in [13] to achieve an even order Chebyshev response. Another synthesis method for dual-band impedance transformers with the Chebyshev response is presented in [14]. The J/K-inverters are often used in constituting the network of proposed structures, and two second-order microstrip bandpass filters were proposed based on $\lambda/4$ resonators in the alternative J-K-J form in [15, 16]. In [15], the Kinverter was implemented with a shunt open-end stub with a length shorter or longer than the $\lambda/4$, whereas a via-hole was employed to function as a simple K-inverter in [16]. Some compact fourth-order microstrip bandpass filters were reportedly constituted with quasielliptic functionality [17] and widened upper stopband [18], but for the Chebyshev response, the even-order filter is inapplicable because the output admittance Y_L is unequal to the input admittance Y_0 , since normalized g_{n+1} is not equal to g_0 . However, the symmetrical even-order Chebyshev filter prototype with equal I/O admittance is constructed in [19], and based on this, a detailed synthesis method for Chebyshev bandpass filter with J/K inverters and $\lambda/4$ resonator is proposed in [20].

In this work, based on [19], we propose two symmetrical even-order Chebyshev filters with N = 2 and N = 4 based on $\lambda/4$ short-ended resonators. Their transmission line models are shown in Figs. 1(a) and 2(a). The conventional even-order Chebyshev filter prototypes are shown in Figs. 1(b) and 2(b) corresponding to the N = 2 and N = 4. The conventional even-order Chebyshev filter prototype is



Figure 1. (a) The schematic of proposed second-order Chebyshev bandpass filter; (b) The conventional second-order Chebyshev bandpass filter prototype; (c) The symmetrical second-order Chebyshev bandpass filter prototype.

not symmetrical, and the admittance Y_L is not equal to unity, thus, it is difficult to implement. We propose a structure with symmetrical topology. The prototypes of even-order Chebyshev filters can be modified as shown in Figs. 1(c) and 2(c). All *J*-inverters can be calculated with the Chebyshev lowpass element values. The relation between *J* and external quality factor (*Q*) and the relation between *J* and coupling coefficient (*K*) [21] are also given in this paper. By using this approach, extracting complicate equation derivation is not required. Only by slightly modifying the existing *J*-inverter equations, the existing even-order Chebyshev lowpass prototype element values can be applied directly. The extracted values of the *J*-inverter are used to determine the circuit dimensions because the *J*-inverter is equal



Figure 2. (a) The proposed fourth-order Chebyshev bandpass filter; (b) The conventional fourth-order Chebysheve bandpass filter prototype; (c) The symmetrical fourth-order Chebyshev bandpass filter prototype.

to the even and odd impedance of parallel coupled line properly. The even and odd impedance can be converted to physical dimension easily. Two classes of even-order Chebyshev bandpass filters with second- and fourth-orders are discussed and designed. Finally, the fourth-order filter is fabricated and measured at center frequency of 2.5 GHz with fractional bandwidth of 25%. The measured result is in good agreement with simulation one.

2. THEORETICAL VALUES OF J-INVERTER

The *J*-inverter is often used as an equivalent circuit to the parallel coupled line as shown in Fig. 3(a). The equivalent *J*-inverter circuit is



Figure 3. (a) The parallel coupled line; (b) Equivalent circuit of the parallel coupled line with *J*-inverters circuit.

shown in Fig. 3(b). In order to derive from the formulas for calculating the *J*-inverter, the coupled length is often chosen as $\lambda/4$.

The *ABCD*-matrix of the parallel coupled line as shown in Fig. 3(a) can be easily obtained [22] and marked as F_a , and the proposed equivalent circuit with *J*-inverters as shown in Fig. 3(b) is derived as F_b . F_a and F_b are given below:

$$F_{a} = \begin{pmatrix} \frac{Z_{0e} + Z_{0o}}{Z_{0e} - Z_{0o}} \cos \theta_{c} & j \frac{(Z_{0e} - Z_{0o})^{2} + (Z_{0e} + Z_{0o})^{2} \cos^{2} \theta_{c}}{2(Z_{0e} - Z_{0o}) \sin \theta_{c}} \\ j \frac{2 \sin \theta_{c}}{Z_{0e} - Z_{0o}} & \frac{Z_{0e} + Z_{0o}}{Z_{0e} - Z_{0o}} \cos \theta_{c} \end{pmatrix}$$
(1a)
$$F_{b} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos \theta_{c} & j Z_{0} \sin \theta_{c} \\ \frac{j \sin \theta_{c}}{Z_{0}} & \cos \theta_{c} \end{bmatrix} \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix} \begin{bmatrix} \cos \theta_{c} & j Z_{0} \sin \theta_{c} \\ \frac{j \sin \theta_{c}}{Z_{0}} & \cos \theta_{c} \end{bmatrix}$$
$$= \begin{bmatrix} \left(J Z_{0} + \frac{1}{J Z_{0}} \right) \sin \theta_{c} \cos \theta_{c} & j \left(J Z_{0}^{2} \sin^{2} \theta_{c} - \frac{\cos^{2} \theta_{c}}{J} \right) \\ j \left(\frac{1}{J Z_{0}^{2}} \sin^{2} \theta_{c} - J \cos^{2} \theta_{c} \right) & \left(J Z_{0} + \frac{1}{J Z_{0}} \right) \sin \theta_{c} \cos \theta_{c} \end{bmatrix}$$
(1b)

When $F_a = F_b$ and $\theta_c = \pi/2$, the Z_{0e} and Z_{0o} are written as follows:

$$Z_{0e} = Z_0 \left[1 + JZ_0 + (JZ_0)^2 \right]$$
(2a)

$$Z_{0o} = Z_0 \left[1 - JZ_0 + (JZ_0)^2 \right]$$
(2b)

From (2), the extracted values of the J-inverter are used to determine the circuit dimensions because J-inverter is equal to the even and odd impedance of parallel coupled line.

Generally, for a bandpass filter formed by cascading several parallel coupled lines, the theoretical values of the J-inverter can be

easily obtained [20] as follows:

$$J_{01} = \frac{1}{Z_0} \sqrt{\frac{\pi B}{4g_0 g_1}}$$
(3a)

$$J_{n-1,n} = \frac{1}{Z_0} \frac{\pi B}{4\sqrt{g_{n-1}g_n}} \quad n = 1, 2, \dots, N$$
(3b)

On the other hand, the alternative quality factor (Q) and coupling coefficient (K) [21] have been given as in (4), where *B* represents the fractional bandwidth of the proposed filter, and $g_0, g_1, \ldots, g_{n+1}$ are the first N + 1 prototype elements for the proposed even-order Chebyshev bandpass filter.

$$Q = \frac{g_0 g_1}{B} \tag{4a}$$

$$K_{n-1,n} = \frac{B}{\sqrt{g_{n-1}g_n}}$$
 $n = 1, 2, \dots, N$ (4b)

From Q and K expressions, the expressions of J-inverters can be rewritten as follows:

$$J_{01} = \frac{1}{Z_0} \sqrt{\frac{\pi}{4Q}}$$
(5a)

$$J_{n-1,n} = \frac{1}{Z_0} \frac{\pi}{4} K_{n-1,n} \quad n = 1, 2, \dots, N$$
 (5b)

3. THE DESIGN PROCEDURE OF EVEN-ORDER CHEBYSHEV BANDPASS FILTER

In this section, the detailed procedures for designing the even-order Chebyshev bandpass filter are studied and given. The formulas of *J*-inverters studied in Section 2 will be the theoretical guidance in designing the bandpass filter. Two even-order Chebyshev bandpass filters with second- and fourth-orders are designed as examples to prove the derived theory in Section 2. These two filter prototype examples are designed and simulated, and the detailed design procedures and curves are given and discussed.

According to the coupled resonator theory, the external quality factor can be extracted by the following:

$$Q_E = \frac{f_0}{\Delta f_{\pm 90^\circ}} \tag{6}$$

where the f_0 is the resonance frequency of the excitation port of the first resonator, and the value of $\Delta f_{\pm 90^\circ}$ is determined from the

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frequency at which the phase shifts $\pm 90^{\circ}$ with respect to the absolute phase at f_0 .

On the other hand, the coupling coefficient can be extracted from EM simulation results of the coupled resonators using the following formula:

$$K = \left| \frac{f_{p2}^2 - f_{p1}^2}{f_{p2}^2 + f_{p1}^2} \right| \tag{7}$$

where f_{pi} (i = 1, 2) denotes the split resonator frequencies when two identical resonators couple to each other.

From (6) and (7) and the previous discussion about J-inverter, the J-inverters can be easily extracted from EM simulation results using the following formula:

$$J_{01} = \frac{1}{Z_0} \sqrt{\frac{\pi \cdot \Delta f_{\pm 90^\circ}}{4f_0}}$$
(8a)

$$J_{n-1,n} = \frac{1}{Z_0} \frac{\pi}{4} \left| \frac{f_{p2}^2 - f_{p1}^2}{f_{p2}^2 + f_{p1}^2} \right|$$
(8b)

Now, we use the theoretical value of J-inverter and extracted value of J inverter to determine the circuit dimensions. The design procedures are outlined below:

Step 1: Select a low-pass prototype with order N. Then we have the prototype elements g_i , i = 0, ..., N + 1. Herein, a low-pass prototype with passband ripple of 0.5 dB is selected, and the element values are $g_0 = 1$, $g_1 = 1.4029$, $g_2 = 0.7071$, $g_3 = 1.9841$ under N = 2and $g_0 = 1$, $g_1 = 1.6703$, $g_2 = 1.1926$, $g_3 = 2.3661$, $g_4 = 0.8419$, $g_5 = 1.9841$ under N = 4. In this way, we design a bandpass filter at center frequency of 2.5 GHz with fractional bandwidth of 25%. The fractional bandwidth is defined as $B = \Delta/f_0$, where Δ is the bandwidth of the passband and f_0 the central frequency of the passband.

Step 2: Calculate the value of the *J*-inverters using the proposed Equation (5). Table 1 tabulates the synthesis parameters of Step 1 and Step 2.

Step 3: Use the extract values of *J*-inverters with Equation (8). The J_{01} versus physical dimension is depicted in Fig. 4(a) as well as the $J_{12} (J_{23})$ versus physical dimensions depicted in Fig. 4(b). From Fig. 4(a), J_{01} is a function of S_0 . Obviously, J_{01} decreases slowly as the gap (s) increases, which is between the feed-line and the first resonator while the curve of $J_{12} (J_{23})$ has the same trend with the gap (s) between two adjacent resonators.

\mathbf{Ta}	ble	1.

Second-order $(N=2)$		Fourth-order $(N = 4)$	
Parameter	Value	Parameter	Value
В	25%	В	25%
g_0	1.0	g_0	1.0
g_1	1.4029	g_1	1.6703
g_2	0.7071	g_2	1.1926
g_3	1.9841	g_3	2.3661
		g_4	0.8419
		g_5	1.9841
J_{01}	$7.49 * 10^{-3}$	J_{01}	$6.85 * 10^{-3}$
J_{12}	$3.94 * 10^{-3}$	J_{12}	$2.78 * 10^{-3}$
		J_{23}	$2.34 * 10^{-3}$



Figure 4. (a) The extracted J_{01} versus the gap (s); (b) The extracted $J_{12}(J_{23})$ versus the gap (s).

Step 4: Determine the dimensions of the proposed structure. First, the parallel coupled line length should be $\lambda/4$, namely $L_2 \approx 20.1 \text{ mm}$ corresponding to the center frequency of 2.5 GHz and the substrate with a dielectric constant of 2.55 and a thickness of 0.8 mm. Then the theoretical values of *J*-inverter are used to determine the gaps between the feed-line and the first resonator as well as the gaps between adjacent resonators by looking at the design curve in the figures which plot the extracted *J*-inverter versus gap (*s*) in Figs. 4(a) and (b).

For the second-order Chebyshev bandpass filter, the theoretical

values of J-inverters are: $J_{01} = 7.49 * 10^{-3}$, $J_{12} = 3.94 * 10^{-3}$, then look up at Figs. 4(a) and (b) for $J_{01} = 7.49 * 10^{-3}$ and $J_{12} = 3.94 * 10^{-3}$. The corresponding values of physical dimensions of gaps are about: $S_0 = 0.5 \text{ m}$, $S_1 = 0.7 \text{ mm}$. Thus, the dimensions of the second-order filter layout as shown in Fig. 1(a) are: $L_0 = 3.4 \text{ mm}$, $L_1 = 0.5 \text{ mm}$, $L_2 = 20.4 \text{ mm}$, $W_0 = 2 \text{ mm}$, $W_1 = 0.4 \text{ mm}$, R = 1.2 mm, r = 0.6 mm, $S_0 = 0.50 \text{ mm}$, $S_1 = 0.7 \text{ mm}$. The simulated result is given in Fig. 5. There are two modes in the passband, but the inband performance is a little bad, because the theoretical values are all derived based on the ideal lumped circuit elements. The curves are extracted by full-wave EM simulator used as design guideline to determine the initial values of filter dimensions. Thus, a small difference is inevitable. By fine turning the filter dimensions, a good performance can be achieved with $L_0 = 3.4 \text{ mm}$, $L_1 = 0.5 \text{ mm}$, $L_2 = 20.4 \text{ mm}$, $W_0 = 2 \text{ mm}$, $W_1 = 0.4 \text{ mm}$, R = 1.2 mm, r = 0.6 mm, $S_0 = 0.52 \text{ mm}$, $S_1 = 0.8 \text{ mm}$.

For the fourth-order case, the theoretical values of J-inverters are: $J_{01} = 6.85 * 10^{-3}$, $J_{12} = 2.78 * 10^{-3}$, $J_{23} = 2.34 * 10^{-3}$, then look up at the design curves in Figs. 4(a) and (b). The values of J-inverters are $J_{01} = 6.85 * 10^{-3}$, $J_{12} = 2.78 * 10^{-3}$ and $J_{23} = 2.34 * 10^{-3}$. The corresponding values of the gaps are about: $S_0 = 0.52 \text{ mm}$, $S_1 = 0.95 \text{ mm}$, $S_2 = 1.05 \text{ mm}$, and the simulated result is given in Fig. 5(b). Four modes can also be observed in the passband, while the in-band performance is also not good enough. By fine tuning the filter, a good performance is obtained with $L_0 = 3.4 \text{ mm}$, $L_1 = 0.5 \text{ mm}$, $L_2 = 20.4 \text{ mm}$, $W_0 = 2 \text{ mm}$, $W_1 = 0.4 \text{ mm}$, R = 1.2 mm, r = 0.6 mm, $S_0 = 0.48 \text{ mm}$, $S_1 = 0.95 \text{ mm}$, $S_2 = 1.05 \text{ mm}$. It is noteworthy that



Figure 5. (a) Simulated result of the second-order Chebyshev bandpass filter; (b) Simulated result of the fourth-order Chebyshev bandpass filter.

there is a transmission zero in the upper stopband at about 3.25 GHz, caused by the coupling between non-adjacent resonators.

4. EXPERIMENTAL RESULTS

To verify the design, a fourth-order Chebyshev bandpass filter which operates at 2.5 GHz is fabricated and measured. The substrate used herein has a dielectric constant of 2.55 and thickness of 0.8 mm. Fig. 6(a) shows a photograph of the fabricated circuit. Fig. 6(b) gives a comparison between the simulated and the measured results.

The measured insertion loss is better than $1.975 \,\mathrm{dB}$ in the passband, and the return loss is better than $12 \,\mathrm{dB}$. The $3 \,\mathrm{dB}$ bandwidth is about $0.58 \,\mathrm{GHz}$, ranging from $2.2 \,\mathrm{GHz}$ to $2.78 \,\mathrm{GHz}$, while the fractional bandwidth *B* is about 23.3%, which is close to the theoretical fractional bandwidth of 25% as shown in Table 1. A good agreement is obtained between the simulated and the measured results.



Figure 6. (a) Photograph of the fabricated circuit; (b) Comparison between the measured result and simulated result.

5. CONCLUSION

In this paper, we present a method for the synthetic design of an evenorder Chebyshev bandpass filter with $\lambda/4$ short-ended resonators and J-inverters. The method is simple without complicate mathematical derivation. Only a slight modification of the *J*-inverter equations is required to apply the existing even order Chebyshev element values for filter design. The bandpass filter with a symmetrical topology is proven to achieve an even-order Chebyshev response. For a bandpass filter formed by cascading several parallel coupled lines, if the electrical parameters of the equivalent *J*-inverter are obtained, the electrical parameters of the parallel coupled line are achieved, and the characteristics of the parallel coupled line are also determined. this way, the theoretical values of J-inverter are calculated. The Jinverters of the symmetric bandpass filter with Chebyshev response are extracted and described as curves. At last, the theoretical values of J-inverter are used as a design guidance to determine the gaps between the feed-line and the first resonator as well as the gaps between Thus, the circuit dimensions of the proposed adjacent resonators. structure can be determined. To verify the proposed method, two even-order Chebyshev bandpass filters with the second- and fourthorders are proposed and designed using the proposed method. The simulated results show that the method is valid and can be used as a synthesis method to design even-order bandpass filters. At last, the fourth-order bandpass filter is fabricated and measured. The measured insertion loss is better than 1.975 dB in the passband, while the return loss is better than 12 dB. The fractional bandwidth B is about 23.3%. which is close to the theoretical value of 25%. The measured and the simulated results are in good agreement.

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