Progress In Electromagnetics Research Letters, Vol. 42, 79–88, 2013

IMPROVED ANALYTICAL METHOD FOR PLASMA ELECTRON DENSITY MEASUREMENT BY RESONANT CAVITY PERTURBATION THEORY

Junjian Mao*, Pu Tang, Yanfei Zhou, Liping Yuan, and Wuqiong Luo

University of Electronic Science and Technology of China, Chengdu 611731, P. R. China

Abstract—A method of plasma density measurement based on microwave resonant cavity perturbation (by Kornegay [14]) is described. Resonant cavity theory was analyzed and a resonant cavity with special structure was designed for measuring the low density plasma. In the middle of the closed cavity, there were cut-off tubes which were extended a little into the cavity to get through the plasma. It was found that the distribution of the electrical field intensity was the densest near the cut-off tubes when the cylindrical cavity operating with TM_{010} mode. By using the method of resonant equivalent circuit analysis, both the amplitudes and phases of the Scattering matrices (S matrices) were obtained before the plasma came and at the time of the plasma passing through. Then the electron line density (N_e) and the electron-molecule collision frequency for momentum transfer (v_m) were calculated. A modified formula was proposed based on our simulation which was conducted in HFSS and experimental results. With the comparison of our results and Kornegay's, it was found that the accuracy of the plasma dielectric constant calculation was improved about 5 percent.

1. INTRODUCTION

When a hypervelocity enters the aerosphere (below 70 km altitude), the surrounding air will be compressed and heated by the projectile which would make plasma around the hypervelocity [1–3]. The density of the generated low density plasma flow is from $10^5\,(\mathrm{e/cm^3})$ to $10^{13}\,(\mathrm{e/cm^3})$ [4–6]. The measure of the low density plasma with high-speed and non-equilibrium is much difficult [7–12].

Received 28 June 2013, Accepted 15 August 2013, Scheduled 27 August 2013

* Corresponding author: Junjian Mao (Maojunjian1988@126.com).

Buchsbaum et al. introduced the theory of microwave measuring low density plasma, and the diagnosis of waveguide coupling method was proposed [13]. Heald and Wharton of Lincoln Laboratory summarized the practical diagnosis method and system for ballistic target wake electron density in their book [10] and put emphasis on the microwave interferometer and cavity diagnosis system. In 1968, Kornegay proposed a transmission resonant cavity technique, which was used to measure electron line density and collision frequencies in the ionized wakes of hypervelocity projectiles [14]. In the seventies, James etc. analyzed reentry flying wake plasma characteristics comprehensive, put forward and solved the problem of reentry communication blackout the feasibility of the scheme, and perfected the ballistic target wake measurement scheme [9].

In practice, different methods were utilized for different ranges of electron density to achieve better accuracy. The 8 mm Microwave Interferometer Measurement System is applicable when the electron density is $10^{10}\sim10^{13}\,(\mathrm{e/cm^3})$ [15–17]; the 35 GHz Open Microwave Resonator is applicable when the electron density is $10^{9}\sim10^{11}\,(\mathrm{e/cm^3})$ [18, 19]. Everyone caring about wake electron density in the range of $10^{6}\sim10^{9}\,(\mathrm{e/cm^3})$, which belongs to the low density range, can use a closed cavity to measure [14, 20].

The main purpose of this paper is to describe the resonant cavity method as a technique for measuring electron line densities and collision frequencies in wakes. The cavity with extended cut-off tubes was designed. The modified formula applying the concept of the equivalent height was proposed to analyze the cavity. Both simulation and measurement results were discussed and compared with that of Wade's. It was found that with our method, the accuracy of the calculation of the dielectric was improved about 5%.

2. DESIGN OF MICROWAVE CAVITY

We use microwave resonator to measure the dielectric properties of the plasma, but we must consider the characteristic of high-speed ballistic wake, and microwave cavity should satisfy the following principle:

1) Microwave cavity must provide ballistic channel, and microwave cutoff tubes should be long enough to avoid electromagnetic energy leakage to the outside of the cavity.
2) The cavity must not be so small that the cavity walls will interfere with the ionized walk itself. The cavity construction must be rugged enough to withstand the mechanical deformation produced by shock waves and other stresses caused by hypervelocity flight and firing of the gun.

3) The cavity must provide enough space and time resolution.
4) The cavity should be designed for single-mode operation and meet the best perturbation terms.

The right circular cylindrical cavity is well suited to make measurements on a cylindrical plasma, such resonant cavities are used in this way (see Fig. 1) [14]. The electric field of TM_{010} mode concentrates on the center of the cavity, and the biggest electric field is in the axial center of a cavity [21]. The TM_{010} mode can provide the best perturbation, which can be applied in the ballistic target wake measurement. The cavity field is shown to be

$$\begin{cases} E_z = E_{010} J_0(2.405r/R) \\ H_\varphi = j\frac{E_{010}}{\eta_0} J_0'(2.405r/R) \\ E_\varphi = E_r = H_z = H_r = 0 \end{cases} \tag{1}$$

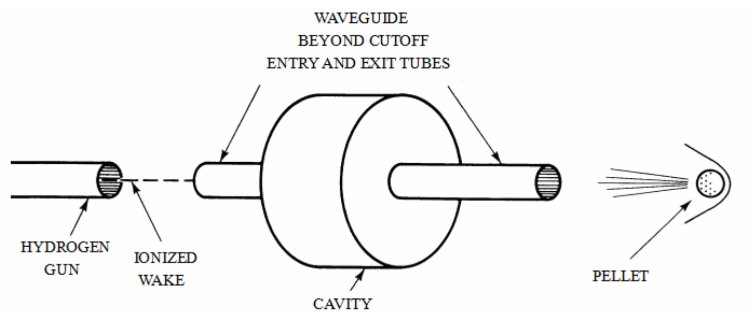


Figure 1. Disposition of hydrogen gun, resonant cavity, pellet, and ionized wake.

According to the characteristics of ballistic target wake closed resonator measurement designs closed cavity as shown in Fig. 2, R is the radius of the cavity, h the height of the cavity, L the length of cutoff tubes, and $2a$ the diameter of cutoff tubes, which can provide ballistic channel and avoid electromagnetic energy leakage to the outside of the cavity (see Fig. 3(a)). If the cavity resonance frequency and working patterns are fixed, the radius of the cavity is determined. When the cavity resonates in the TM_{010} mode at 450 MHz. R is 51.3 cm and h 20 cm long. The walls of the cavity and cutoff tubes have a thickness of 1.5 cm. To avoid electromagnetic energy leakage to the outside of the cavity, cutoff tubes are 17 cm long, and cutoff tubes diameter must be larger than wake size, which is 19.1 cm in diameter. In addition, the cut-off tubes partly stretch inside resonant cavity, so that the electric field becomes smoother, more concentrated in the central region. The smaller changes in plasma wake section electric field are, the more consistent with the measurement theory the electric field will be.

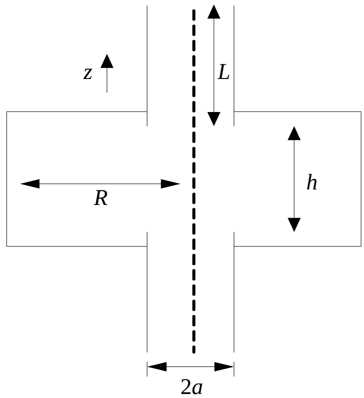
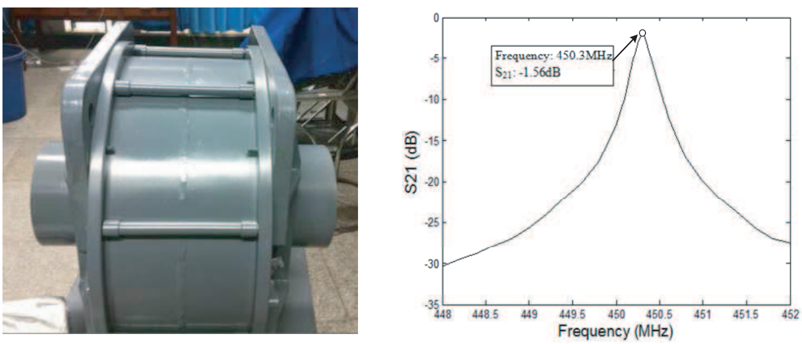


Figure 2. Structure of microwave resonant cavity.



(a) (b)

Figure 3. (a) Cavity physical figure. (b) The measured S_{21}.

Using vector network analyzer, we get measurement result, shown in Fig. 3(b).

Resonance frequency is about 450.3 MHz, quality factor about 2692, and insertion loss about -1.56 dB.

3. METHOD OF TM_{010} MODE MEASURING PLASMA DESITY

In effect, the presence of the ionized wake in the resonant cavity changes its complex admittance. The wake electron-density and electron-molecule collision frequency for momentum transfer can be determined from the real and imaginary parts of the admittance and the fixed parameters of the cavity. As a result of cut-off tubes, cavity mode is a quasi TM_{010} mode. When frequency is at the resonance point, single-mode cavity can be the equivalent lumped circuit, and the result is presented in Fig. 4.

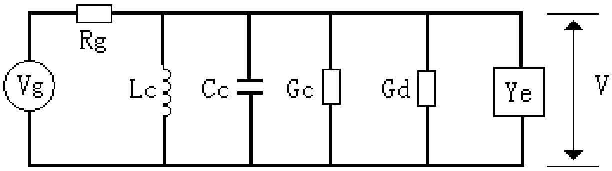


Figure 4. Equivalent circuit of the resonance cavity.

L_c, C_c are equivalent inductance and equivalent capacitance when the cavity resonance occurs; V_g, R_g are the voltage of source and series impedance; R_g is the coupling ring to match, G_d the conductance ring caused by the coupling, Y_e the admittance caused by the plasma, and G_C the real component of cavity-conductance. Cavity is in a state of balance. At some point, all energy is electric energy in the cavity, and the axial electric field is E_0. All energy is present in the capacitor at this point,

$$W = \frac{1}{2} C_c E_0^2 h^2 \tag{2}$$

Formula (1) gives the field equation of cylindrical cavity, and the resonant cavity energy in TM_{010} mode is given by

$$W = \frac{1}{2}\varepsilon_0 \int_0^R \left(E_0 J_0 \left(\frac{2.405r}{R} \right) \right)^2 h \cdot 2\pi r dr = \frac{\pi}{2}\varepsilon_0 E_0 h R^2 J_1^2 (2.405) \tag{3}$$

According to [14], for an ideal cavity, the paper points out that the electron line density (N_e) and electron-molecule collision frequency for momentum transfer (ν_m)

$$\nu_m = \omega_0 \frac{\cos\theta - S}{\sin\theta} \tag{4}$$

$$N_e = \frac{\pi m_e}{e^2} \frac{2.405^2 J_1^2 (\alpha)}{Q_L} \left[1 + \left(\frac{\cos\theta - S}{\sin\theta} \right)^2 \right] \frac{\sin\theta}{S} \tag{5}$$

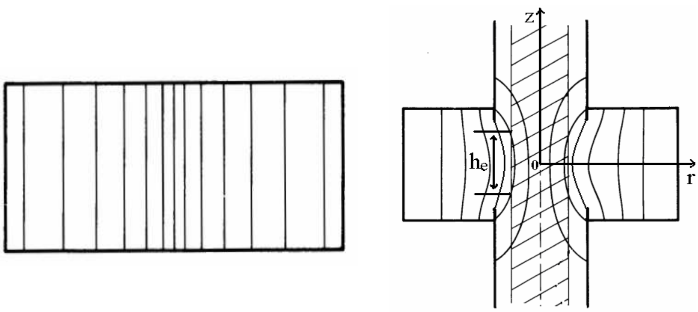
In the cavity resonance point, we can measure the transmission coefficient at two ports about the voltage ratio (S_{21}), and measure S_{21}^e at a fixed frequency when the plasma occurs perturbation, where S is given by

$$S = |S_{21}^e / S_{21}| \tag{6}$$

θ is the phase shift before the plasma came and at the time of the plasma passing through, ω_0 the angular frequency of oscillation, Q_L the loaded Q of the cavity, and $2.405^2 J_1^2(\alpha)\pi m_e/\mu_0 e^2$ a constant, numerically equal to $1.38 \times 10^{12}\,\mathrm{cm}^{-1}$. When measuring the amplitude and phase change, formulas (4) and (5) can be used to calculate N_e and ν_m.

4. FORMULA FOR CORRECTION

The derivation process of formula (2) is according to the ideal closed cavity (see Fig. 5(a)) to analysis. The actual closed cavity (see Fig. 5(b)) is the middle opening. The center axis of the electric field is changing and presented in Fig. 6.



(a) (b)

Figure 5. (a) Electric field of in ideal cavity. (b) Electric field of in actual cavity.

The center electric field of an ideal closed cavity has nothing to do with the other parameters, and it is constant. The electric field of actual closed cavity rapidly decreases from the center to both sides, and it is zero at the end of cut-off tubes. So the axial electric field is different from standard pattern TM_{010}. It is difficult to obtain closed cavity field structure, and we can use axial effective height to replace electric field change. According to perturbation theory [20], we can get that

$$\frac{\Delta f}{f_0} = -\frac{\Delta\varepsilon \left|E_0\right|^2 V_e}{2\int_V \varepsilon_0 \left|E\right|^2 dV} \tag{7}$$

Taking into account the $E \approx E_0$, formula (7) can be written as

$$\frac{\Delta f}{f_0} = -\frac{\left(\varepsilon_r' - 1\right)\varepsilon_0 E_0^2 V_e}{J_1^2(\alpha)\varepsilon_0 E_0^2 V_0} = 3.71\left(\varepsilon_r' - 1\right)\frac{r_0^2 h_e}{R^2 h} \tag{8}$$

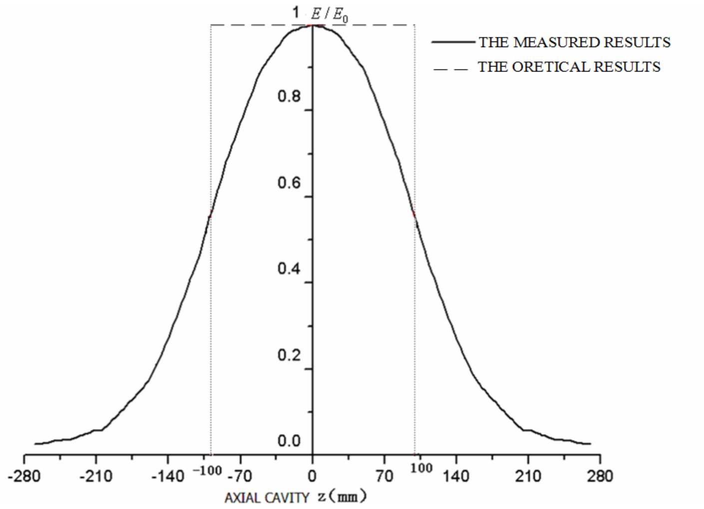


Figure 6. Measured and ideal closed cavity center axial electric field distribution.

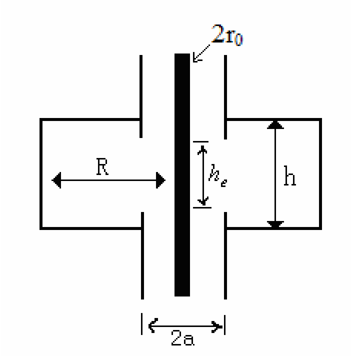


Figure 7. The perturbation center cavity dielectric materials.

where V_e is the effective volume of medium, V_0 the cavity volume, h_e the axial effective height, and the mode is as shown in Fig. 7. The medium is placed in the cavity center, and the effective distance of medium is h_e.

Using HFSS to simulate the cylindrical dielectric materials, the radius and dielectric constant of dielectric materials are changed, and frequency deviation under the condition of perturbation is obtained. By using formula (8), the results are shown in the following Table 1.

No matter how much radius of medium is, as long as no more than the size of the cut-off tubers, the effective length of the medium

Table 1. Different radius and the dielectric constant perturbation.

$r_0 = 15\,\mathrm{mm}$			$r_0 = 25\,\mathrm{mm}$		
dielectric constant	frequency deviation (MHz)	h_e/h	dielectric constant	frequency deviation (MHz)	h_e/h
1.85	1.836	0.378	1.25	1.597	0.4
2.1	2.514	0.399	1.4	2.415	0.38
2.65	3.783	0.4	1.55	3.388	0.388
$r_0 = 35\,\mathrm{mm}$			$r_0 = 45\,\mathrm{mm}$		
dielectric constant	frequency deviation (MHz)	h_e/h	dielectric constant	frequency deviation (MHz)	h_e/h
1.12	1.337	0.36	1.07	1.369	0.38
1.2	2.303	0.37	1.12	2.342	0.38
1.3	3.615	0.387	1.18	3.686	0.4

is $h_e \approx 0.38h$. Related to plasma linear density (N_e), the formula is revised as follows

$$N_e = \frac{0.38\pi m_e}{e^2}\frac{\alpha^2 J_1^2(\alpha)}{Q_L}\left[1+\left(\frac{\cos\theta - S}{\sin\theta}\right)^2\right]\frac{\sin\theta}{S} \tag{9}$$

In order to verify the correctness of the above conclusion, we choose a kind of hard foam material ($\varepsilon_r = 1.08$, rectangle size $38.5\,\mathrm{mm} \times 39.5\,\mathrm{mm}$, and length 45 cm). The vector network analyzer measures cavity resonance frequency $f_0 = 450.304\,\mathrm{MHz}$ and resonance frequency $f_1 = 449.933\,\mathrm{MHz}$ when perturbation occurs in closed cavity. We get $\varepsilon_r = 1.079$ by using formula (9) ($h_e = 0.38h$), and the error is only 0.1% with the actual dielectric constant. If we get $\varepsilon_r = 1.0304$ by using formula (5) ($h_e = h$), the error is about 5% with the actual dielectric constant, so the accuracy of the plasma dielectric constant calculation was improved about 5 percent.

This formula applicable condition is related with the cut-off tubes. The equivalent height is larger when the radius of cut-off tubes is smaller.

5. CONCLUSION

A method of plasma density measurement based on microwave resonant cavity perturbation for plasma is described. The design of resonant cavity is used to measure flight body densities, and special details of the cavity method are given. A discussion of the formula of the plasma density, which is made by carrying out the electromagnetic analysis of the electron line densities, is given. Using the equivalent height, correction formula is presented. Also, plasma dielectric constant is improved about 5 percent.

The closed cavity was proper for the measurement of low-density plasma. The resonant frequency would increase with the increase of the density, which made the resonant cavity smaller. When the resonant frequency was as high as 35 GHz, the closed cavity would be inapplicable so that the open microwave resonator should be exploited.

REFERENCES

1. Robert, M., "Analysis of electromagnetic wave propagation in a magnetized re-entry plasma sheath via the Kinetic equation," NASA/TM-2009-21096, National Aeronautics and Space Administration, Manning Glenn Research Center, Cleveland, Ohio, 2009.
2. Wilson, L. N., "The far wake behavior of hypersonic cones," *AIAA J.*, Vol. 5, No. 8, 1393–1396, 1967.
3. Vidmar, R. J., "On the use of atmospheric pressure plasma as electromagnetic reflectors and absorbers," *IEEE Transactions on Plasma Science*, Vol. 18, No. 4, 73, 1990.
4. Burkley, C. J. and M. C. Sexton, "Measurement of plasma electron distributions using microwave cavities," *J. Appl. Phys.*, Vol. 39, 5013, 1968.
5. Persson, K. B. and E. G. Johnson, "The errors in plasma measurements by the microwave cavity techniques," Technical Note 607, 1971.
6. Chang, C. H., C. H. Hsieh, H. T. Wang, et al., "A transmission-line microwave interferometer for plasma electron density measurement," *Plasma Sources Sci. Technol.*, Vol. 16, 67, 2007
7. Laroussi, M. and R. J. Vidmar, "Numerical calculation of the reflection absorption and transmission of microwaves by a non-uniform plasma," *IEEE Transactions on Plasma Science*, Vol. 2, No. 3, 733–741, 1990.
8. Gibsonw, E. and P. V. Marrone, "A similitude for non-equilibrium phenomena in hypersonic flight," *AGARD Meeting on High Temperature Aspects of Hypersonic Fluid Dynamics*, 105–131, Brussels, Belgium, Apr. 1962.
9. Rybak, J. P. and R. J. Churchill, "Progress in reentry

communications," *IEEE Transaction on Aerospace and Electronic Systems*, Vol. AES-7, No. 5, 879–894, 1971.

10. Heald, M. A. and C. B. Wharton, *Plasma Diagnostics with Microwaves,* 156–161, John Wiley and Sons Inc., New York; London; Sydney, 1965.
11. Charles, J. and C. Brown, "Plasma diagnostics," Technical Report, 454, Aug. 15, 1966.
12. Spitzer, L., *Physics of Fully Ionizes Gases,* 2nd Edition, Inter Science, New York, 1967.
13. Buchsbaum, S. J., L. Mower, and S. C. Brown, "Interaction between cold plasmas and guided electromagnetic waves," *Phys. Fluids*, Vol. 3, No. 5, 806–819, 1960.
14. Kornegay, W. M., "Resonant cavity measurements of ionized wakes," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 4, No. 2, 181–186, May 1968.
15. Laroussi, M. and J. R. Roth, "Numerical calculation of the reflection absorption, and transmission of microwaves by a nonuniform plasma slab," *IEEE Transactions on Plasma Science*, Vol. 2, No. 4, 36, 1993.
16. Leich, M., "All optical tree network for interference-free distance multiplexing," *Electronics Letters*, Vol. 31, No. 22, 19–32, Oct. 26, 1995.
17. Wan, K.-W. and J. Austin, "A novel approach to the simultaneous of phase and amplitude noise of oscillators," *IEEE Transactions on Instrumentation and Measurement*, Vol. 40, No. 3, 140–144, 1991.
18. Tuovinen, J., T. M. Hirvonen, and A. V. Raisanen, "Near-field analysis of a thick lens and horn combination: Theory and Measurements," *IEEE Trans. Ant. Prop.*, Vol. 40, No. 6, 613–619, Jun. 1992
19. Hayami, R., K. Kelley, et al., "Open microwave resonators for ionized wake measurement," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 3, No. 2, 339–348, Mar. 1967.
20. Ikeda, M., T. Fukunaga, and T. Miura, "Influence of sample insertion hole on resonant cavity perturbation measurement method," *2003 IEEE MTT-S International Microwave Symposium Digest*, Vol. 2, 1423–1426, 2003.
21. Po, H. T., H. Kawada, and Y. Kohayashi, "Permittivity measurement of dielectric rod samples using a TM_{010} mode circular cavity," *Proc. 2001, Electron. Soc. Conf. of IEICE*, C-2-75, 2001 (in Japanese).