A ROBUST DIRECT DATA DOMAIN LEAST SQUARES BEAMFORMING WITH SPARSE CONSTRAINT

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Abstract—A robust direct data domain least squares (D^3LS) beamforming algorithm that is capable of reducing the sidelobe level of the beam pattern is presented. By exploiting the sparsity of the desired beam pattern, the proposed method can enhance the performance with its lower sidelobe level and deeper null for interference while the robustness against steering vector mismatch is increased when a proper regularization parameter is selected. Simulation results demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Adaptive beamforming, a technique for extracting the desired signal while suppressing interference at the output of a sensor array, has been widely used in radar, communications, sonar and many other areas [1]. In past literatures, two macro classes of adaptive beamforming algorithms have been proposed: statistical and deterministic algorithms [2–4].

Statistical algorithms such as Capon beamformer [5] generally require several successive snapshots of data to form an estimate of the covariance matrix of interference to recover the desired signal from the noise contaminated measurements. However, conventionally, the computation of the covariance matrix requires much storage room and heavier calculation compared with deterministic methods. And in some systems such as radar and sonar often work in a nonstationary environment, the statistic property of interference may fluctuate

rapidly over a short distance. Namely, the covariance matrix may be difficult to calculate. Thus, a single snapshot interference cancellation technique will be more suitable for the dynamic scenarios.

In contrast to the statistical methods, deterministic algorithms that can be performed in real time have been proposed and developed mainly for dynamic scenarios where the direction of signal of interest (SOI) is known as a priori information [2–4]. The approach adaptively analyzes the data from a particular snapshot and then solves the weights which are used for the cancellation of the interference without having to estimate its covariance matrix. Consequently, deterministic algorithms are ideally suited to suppress the blinking jammers and rapidly changing clutter in a highly transient environment.

In the past two decades, ever since the Direct Data Domain (D^3) algorithm was first proposed to overcome the drawbacks of statistical techniques by Sarkar and coauthors [6, 7], many literatures have made further research on D^3 algorithm. In these papers, some refined algorithms are proposed to make the original D^3 algorithm more suitable for different scenarios. Mutual coupling between the elements of an array severely undermines the interference suppression capabilities of D^3 algorithms. To eliminate the effects of mutual coupling, the method of moments (MoM) with multiple basis functions per element is exploited in [8]. A direct data domain least squares (D^3LS) space-time adaptive processing (STAP) approach is present in [9] for adaptively enhancing signals. And its performance is compared with statistical-based STAP in [2]. For the purpose of enhancing the use of D^3 approaches, [10] introduces the Pre-Doppler concept. Moreover, in this study, a vector space-based theory which enabled us to form rules for constructing a set of linear independent data vectors has been established. Normally, D^3LS algorithm is performed by changing the complex weights, i.e., magnitudes and phases. However, for large array systems, the computation complexity becomes quite large. To solve this problem, Tapan K. Sarkar and his coauthors have proposed phase-only weight control [11] and amplitude-only weights adaptive algorithms [12]. To efficiently calculate the real weight coefficients in [12], a new two-step convex optimization framework derived by using Centro-Hermitian matrix manipulations has been presented in [13]. When a priori direction of arrival (DOA) of SOI is not known exactly, the performance of D^3LS approach may deteriorate seriously. Their work [14] presents a minimum norm property for the sum of the adaptive weights that can be used to refine the estimate of DOA of SOI in the D^3LS algorithm. Besides, D. Cristallini and W. Burger [15] propose a robust approach to take into account the mismatch between the nominal and the actual target

parameters to overcome the main drawback of deterministic STAP, which is the need of exact knowledge of target parameters.

Although the entire D^3LS algorithms mentioned above can suppress interferences and noise effectively, it suffers from several drawbacks such as reduced degrees of freedom (DOF), and relatively high sidelobe level [2, 8]. Some improved algorithms that are capable of dealing with these defects have been put forward. For the disadvantage of the reduced DOF, Madurasinghe [16] presents a new flexible direct data domain (FD^3) approach to increase the DOF by inverting a smaller data matrix. Another flexible algorithm that allows us to control the DOF is a multiple snapshots based approach [17] which mitigates the degradation of performance caused by the reduced DOF. A modified D^3LS approach making use of the contribution of each element to increase the DOF is presented in [18]. For D^3 STAP approaches, [19] proposes a new D^3 approach using sparse representation of clutter spectrum to maintain the full system DOF and achieve better performance. To mitigate the disadvantage of the high sidelobe level, Xing and Cai [20] set a sidelobe constraint to restrain the sidelobe level within a preset threshold, and the resulting problem is a convex optimization problem, which can be solved by CVX [21]. Recently, sparse constraint on beam pattern has been used in beamforming algorithms [22–24] to gain performance improvement. Some modified sparse constraints are used in [25, 26] to enhance the performance of sidelobe suppression. In this paper, the sparsity of the beam pattern is used in the D^3LS algorithm to reduce the high sidelobe level while sustain its capability to suppress interference and noise within a snapshot. By adding this sparse constraint, we can not only get a lower sidelobe level, but also increase robustness against the mismatch problem caused by imprecise knowledge of the desired signal.

The rest of this paper is organized as follows. In the next section, we describe the original D^3LS algorithm. In Section 3, we present our modified D^3LS with sparse constraint (SC-D^3LS). Some simulation experiments are showed in Section 4. Finally, the conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

Consider a uniform linear array (ULA) with N omnidirectional sensors and inter-sensor space d. Suppose there are L ($L \leq N$) narrowband signals with wavelength λ_c impinging on the array from the far field. We assume that the desired signal comes from θ_s and our objective is to estimate its complex amplitude. The output signal of the nth sensor

at time t can be expressed as [12]

$$X_n(t) = \alpha e^{j2\pi(\frac{nd}{\lambda_c})\sin\theta_s} + \sum_{p=1}^{P} m_p e^{j2\pi(\frac{nd}{\lambda_c})\sin\theta_p} + r(t) \tag{1}$$

where α is the complex amplitude of the SOI, and m_p and θ_p are the amplitude and direction of arrival of the pth interference signal. P is the number of interference, and $r(t)$ denotes the contribution of thermal noise at antenna elements.

Then the received data for a given snapshot can be arranged as a $M \times M$ ($M = \frac{N+1}{2}$) dimensional matrix

$$[\mathbf{X}] = \left[\begin{array}{cccc} X_1 & X_2 & \dots & X_M \\ X_2 & X_3 & \dots & X_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_M & X_{M+1} & \dots & X_N \end{array}\right]_{M\times M} \tag{2}$$

For a particular row of (2), the row-to-row phase difference is

$$Z = \exp\left[j2\pi\frac{d}{\lambda_c}\sin\theta_s\right] \tag{3}$$

Then it can be inferred that the item $X_{n-1} - X_n Z^{-1}$ contains the contribution due to signal multipath, clutters and jammers except the desired source signal. Thus, we can construct a $(M-1)\times M$ dimensional whole noise and interference matrix

$$\mathbf{T} = \left[\begin{array}{cccc} X_1 - X_2 Z^{-1} & X_2 - X_3 Z^{-1} & \dots & X_M - X_{M+1} Z^{-1} \\ X_2 - X_3 Z^{-1} & X_3 - X_4 Z^{-1} & \dots & X_{M+1} - X_{M+2} Z^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{M-1} - X_M Z^{-1} & X_M - X_{M+1} Z^{-1} & \dots & X_{N-1} - X_N Z^{-1} \end{array}\right] \tag{4}$$

Then the DOF descends to $\frac{N+1}{2}$. According to the principle of adaptive beamforming techniques, one can minimize the contribution due to the noise and interference by choosing an appropriate weighting vector $\mathbf{w}$. In order to restore the signal component from the array output, we fix the gain on the expected signal direction as a constant C, which provides an additional equation that can be used to construct the following matrix equation,

$$\left[\begin{array}{cccc} 1 & Z & \dots & Z^{M-1} \\ X_1 - X_2 Z^{-1} & X_2 - X_3 Z^{-1} & \dots & X_M - X_{M+1} Z^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{M-1} - X_M Z^{-1} & X_M - X_{M+1} Z^{-1} & \dots & X_{N-1} - X_N Z^{-1} \end{array}\right]$$

$$\cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_M \end{pmatrix} = \begin{pmatrix} C \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{5}$$

A clear way to represent the above equation is to rewrite (5) in a vector form [11],

$$\mathbf{Fw} = \mathbf{y} \tag{6}$$

Therefore, the problem of eliminating interference and noise boils down to solving the above equation with respect to $\mathbf{w}$. Once the weights are solved, α may be estimated from (7) [6]

$$\hat{\alpha} = \frac{1}{C} \sum_{i=1}^{M} w_i x_i \tag{7}$$

All above mentioned is the forward method of D^{3}LS. The extensions of forward method to backward procedure and forward-backward method are straightforward, and the difference among them is only the coefficient matrix. Coefficient matrix of backward procedure is

$$\mathbf{B} = \begin{bmatrix} 1 & Z & \dots & Z^{M-1} \\ X_N^* - X_{N-1}^* Z^{-1} & X_{N-1}^* - X_{N-2}^* Z^{-1} & \dots & X_M^* - X_{M-1}^* Z^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{M+1}^* - X_M^* Z^{-1} & X_M^* - X_{M-1}^* Z^{-1} & \dots & X_2^* - X_1^* Z^{-1} \end{bmatrix} \tag{8}$$

where $*$ denotes complex conjugate, $M = (N + 1)/2$. In the forward-backward model, the forward and the backward method are combined to double the given data and thereby increase the degrees of freedom. Its coefficient matrix is represented by

$$\mathbf{FB} = \begin{bmatrix} 1 & Z & \dots & Z^{M-1} \\ X_1 - X_2 Z^{-1} & X_2 - X_3 Z^{-1} & \dots & X_M - X_{M+1} Z^{-1} \\ \vdots & \vdots & \vdots & \vdots \\ X_{M-1} - X_M Z^{-1} & X_M - X_{M+1} Z^{-1} & \dots & X_{N-1} - X_N Z^{-1} \\ X_N^* - X_{N-1}^* Z^{-1} & X_{N-1}^* - X_{N-2}^* Z^{-1} & \dots & X_M^* - X_{M-1}^* Z^{-1} \\ \vdots & \vdots & \vdots & \vdots \\ X_{M+1}^* - X_M^* Z^{-1} & X_M^* - X_{M-1}^* Z^{-1} & \dots & X_2^* - X_1^* Z^{-1} \end{bmatrix} \tag{9}$$

3. THE PROPOSED D³LS WITH SPARSE CONSTRAINT (SC-D³LS)

D³LS adaptive algorithms are able to estimate signal in the highly nonstationary environment by processing the data snapshot by snapshot [7]. However, the sidelobe level of the original D³LS beamforming is high and is comparable to that of the mainlobe in the experiments of [2, 8]. Thus, the output signal-to-noise ratio (SNR) is reduced by the energy of interferences and noise which enter from the high level sidelobe.

Based on the above considerations, in order to suppress the high sidelobe level, a regularization item, which exploits the sparsity of the whole beam pattern, is added to the D³LS cost function. By adding the sparse constraint, (6) may not hold true. In fact, only in the ideal environment in which the noise and clutter are absent, the equality (6) is absolutely valid. In this paper, without loss of generality, we assume that the noise r is an additive complex Gaussian random vector. Therefore we can minimize the L_2 norm of the noise energy instead.

Accordingly, the proposed method can be formulated in the following constrained form

$$\min J(\mathbf{w}) = \|\mathbf{F}\mathbf{w} - \mathbf{y}\|_2^2 + \lambda \left\|\bar{\mathbf{A}}^H \mathbf{w}\right\|_p^p \tag{10}$$

where λ is a non-negative regularization parameter and $(\cdot)^H$ denotes conjugate transpose. $\bar{\mathbf{A}}$ consists of the steering matrix in the DOA range with a uniform sample, namely,

$$\bar{\mathbf{A}} = [\mathbf{a}(-90°), \mathbf{a}(-89°), \ldots, \mathbf{a}(90°)]_{N \times 181} \tag{11}$$

where $\mathbf{a}(\theta) = [1, e^{j2\pi(\frac{d}{\lambda_c})\sin\theta}, \ldots, e^{j2\pi(M-1)(\frac{d}{\lambda_c})\sin\theta}]^T$ is the steering vector for a signal from angle θ broadside, wherein $(\cdot)^T$ denotes transpose. $\| \mathbf{x} \|_p^p = \sum_i |x_i|^p$ is the L_p-norm of $\mathbf{x}$. When $p \leq 1$, the L_p-norm can be regarded as the diversity measure and lead to a sparse solution [22]. The smaller the $\| \mathbf{x} \|_p^p$, the sparser the $\mathbf{x}$, which means that the number of trivial entries in $\mathbf{x}$ is larger. This is always true in the whole beam pattern whose main lobe is only a small part compared with sidelobe pattern. Consequently, the beam pattern $\bar{\mathbf{A}}^H \mathbf{w}$ has the sparsity property. λ is an important parameter balancing the equation allowable error and sparsity on the beam pattern that needs to be chosen properly. If λ is set to zero, the modified method degrades to the original D³LS. A large λ means that more DOF are used to suppress the sidelobe, which will lead to performance degeneration dramatically for lack of DOF to suppress the interferences.

To get the solution of Equation (10), we take the partial derivative of J with respect to $\mathbf{w}^*$, where $(\cdot)^*$ denotes conjugate, and using the factored gradient approach [27], we get

$$\nabla_{\mathbf{w}^*} J = \mathbf{F}^H \mathbf{F} \mathbf{w} - \mathbf{F}^H \mathbf{y} - \lambda \bar{\mathbf{A}} \mathbf{\Pi} \bar{\mathbf{A}}^H \mathbf{w} \qquad (12)$$

where

$$\mathbf{\Pi} = \text{diag} \left[\left| \left(\bar{\mathbf{A}}^H \mathbf{w} \right)_1 \right|^{p-2}, \ldots, \left| \left(\bar{\mathbf{A}}^H \mathbf{w} \right)_{181} \right|^{p-2} \right] \qquad (13)$$

wherein $(\bar{\mathbf{A}}^H \mathbf{w})_i$ is the ith element of $\bar{\mathbf{A}}^H \mathbf{w}$. Setting (12) to zero, we get

$$\mathbf{w}(n+1) = \text{inv} \left(\mathbf{F}^H \mathbf{F} + \lambda \bar{\mathbf{A}} \mathbf{\Pi} \bar{\mathbf{A}}^H \right) \mathbf{F}^H \mathbf{y} \qquad (14)$$

where $\text{inv}(\cdot)$ denotes the inverse operation.

For $p \leq 1$, the proposed algorithm can be achieved by iterative steps based on the focal underdetermined system solver (FOCUSS) algorithm [28], the iterative procedure can be summarized as follows

1) Set a proper p and λ, and initialize $\mathbf{w}(0)$.

2) Construct the weighting matrix $\mathbf{\Pi}(n)$. The weighting matrix is a diagonal matrix which is constructed by taking the previous estimation as its diagonal elements

$$\mathbf{\Pi}(n) = \text{diag} \left[\left| \left(\bar{\mathbf{A}}^H \mathbf{w}(n) \right)_1 \right|^{p-2}, \ldots, \left| \left(\bar{\mathbf{A}}^H \mathbf{w}(n) \right)_{181} \right|^{p-2} \right] \quad (15)$$

3) Update changes to $\mathbf{w}(n+1)$. The basic FOCUSS algorithm uses the affine scaling transformation (AST), which scales the entries of the current solution by those of the solutions of previous iterations to construct the weighted minimum norm constraint. Then the updated $\mathbf{w}(n+1)$ is presented as

$$\mathbf{w}(n+1) = \text{inv} \left(\mathbf{F}^H \mathbf{F} + \lambda \bar{\mathbf{A}} \mathbf{\Pi}(n) \bar{\mathbf{A}}^H \right) \mathbf{F}^H \mathbf{y} \qquad (16)$$

4) Set $n = n + 1$, and repeat the steps starting from (2) until the solution $\mathbf{w}(n+1)$ approximately no longer changes, i.e.,

$$|\mathbf{w}(n+1) - \mathbf{w}(n)| \leq \delta \qquad (17)$$

where δ is a predefined small positive number. Finally, the solution $\mathbf{w}$ is obtained. Note that, when $p = 1$, since the objective function of (10) is convex, the optimal $\mathbf{w}$ can also be solved out by CVX [21] and SeDuMi [29].

In this section, we only present the forward D³LS method with sparse constraint and the other methods (such as backward method and forward-backward method) can be constructed straightly.

4. SIMULATIONS

In this section, three examples are presented to demonstrate the effectiveness of the above modified procedure which is capable to adaptively extract SOI from a known direction in the presence of interferences and thermal noise and can keep a lower sidelobe level than the conventional method. Without loss of generality, we consider the situation of $p = 1$, $p = 0.8$ with $\delta = 0.0001$ in the three examples and the parameter λ is chosen experimentally to specify a well output signal-to-interference-plus-noise ratio (SINR), which is defined as

$$\text{SINR} = \frac{\sigma_x^2 \left| \mathbf{w}^H \mathbf{a}(\theta_0) \right|^2}{\mathbf{w}^H \left(\sum_{i=1}^{q-1} \sigma_i^2 \mathbf{a}(\theta_i) \mathbf{a}^H(\theta_i) + \sigma_n^2 \mathbf{I} \right) \mathbf{w}} \tag{18}$$

where σ_x^2 and σ_i^2 are the variance of the SOI from θ_0 and the interference from θ_i, respectively. $\sigma_n^2 \mathbf{I}$ is the noise covariance matrix. Note that we take the forward-backward method of FD^3 (FD^3-FB) [16] as a baseline in the first two examples as it has a lower sidelobe level than that of other original D^{3}LS [7] algorithms. For the third example, the baselines become D^{3}LS and sample matrix inversion (SMI) method which serves as an example of statistical algorithms.

4.1. Example 1: Beam Patterns in No Mismatch Case

In the first example, consider a signal of unit amplitude arriving from $\theta_s = 0°$. We consider a 25-element uniform linear array (ULA) with an element spacing of $\lambda_c/2$. Spatially white Gaussian noise is assumed with unity variance. We assume that SNR is 10 dB, and all interference signals have the same power with Interference to Noise Ratio (INR) of 80 dB. The parameter λ is set to 0.02 for $p = 1$ and $p = 0.8$. All the signal intensities and directions of arrival are summarized in Table 1.

Table 1. Parameters of the incident signals.

	Magnitude	Phase	DOA
Signal	1 V/m	0	0°
Interference #1	10000 V/m	0	−65°
Interference #2	10000 V/m	0	−40°
Interference #3	10000 V/m	0	−20°
Interference #4	10000 V/m	0	35°
Interference #5	10000 V/m	0	70°

In this simulation, we first compare the beam pattern of the proposed method, i.e., the forward-backward method of FD^3 with sparse constraint (SC-FD^3-FB), with that of FD^3-FB and original forward-backward method of D^3LS (D^3LS-FB) algorithm without mismatch in the DOA of SOI. As for the FD^3 method, we choose $M = 21$ in (9). The derived beam patterns are shown in Fig. 1. It is observed that the interference nulls of the SC-FD^3-FB beam pattern are much deeper than that of both FD^3-FB and D^3LS-FB and occur along the correct directions.

4.2. Example 2: Beam Patterns in the Presence of DOA Mismatch

In the second example, we assume that there is a $3°$ mismatch in the DOA of SOI, i.e., the true incident angle is $0°$, and the assumed known DOA of SOI is $3°$. λ for $p = 1$ and $p = 0.8$ are set to be 0.5. The other experimental parameters are set the same as that of the first example. It is observed from Fig. 2 that the proposed method with $p = 0.8$ and $p = 1$ has a better performance than the others. The new method is not sensitive to the mismatch and can provide a high array response in the direction of SOI. In contrast, the FD^3 method has a null in $0°$. It implies that ignoring the property of beam pattern, i.e., sparsity, may cause potential performance loss when there exists steering vector mismatch. This is because in the proposed method, some of the DOF are spared to suppress the high sidelobe level, which helps to lower the algorithm's sensitivity to the steering vector mismatch.
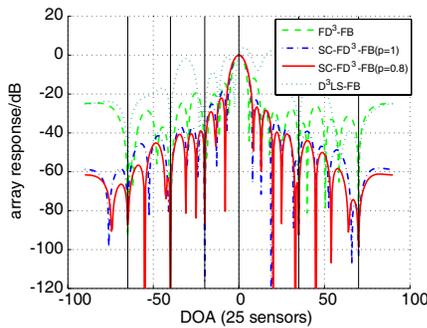

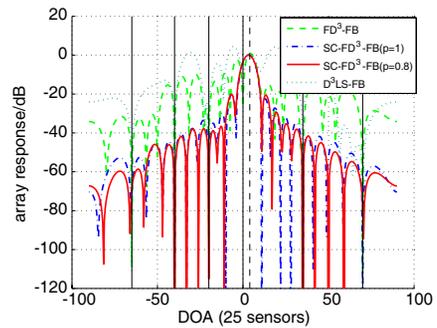**Figure 1.** Beam pattern for SNR = 10 dB in no mismatch case.

Figure 2. Beam pattern for SNR = 10 dB in $3°$ mismatch case.

4.3. Example 3: Comparison of Reconstructed Signals

In this example, we will compare the modified algorithm's and the conventional algorithm's output waveforms instead of the beam pattern [16]. The illustration of Fig. 3 is based on a 21-channel ULA with half wavelength spacing where the three interference signals' DOA are $25°$, $-10°$, $45°$ and $10°$ corresponds to the DOA of SOI. The SNR and INR are set to $10\,\text{dB}$, $30\,\text{dB}$ respectively and all 300 samples are used to compute weights for the sample matrix inversion (SMI-300) solution while the D^3LS algorithm including the forward method of D^3LS with sparse constraint (SC-D^3LS-F) and original forward method of D^3LS (D^3LS-F) only require one snapshot. We set $\lambda = 0.02$ in the SC-D^3LS-F algorithm. The amplitude of SOI is $s_1(t) = \sqrt{2}\sin(16\pi * \frac{k}{320}) * \sqrt{p_s}$ $(k = 1, \ldots, 300)$, p_s is the power of SOI. Fig. 3 shows that the performance of SC-D^3LS-F is superior to that of D^3LS-F and the SMI-300. Thus, it can be inferred that the SC-D^3LS-F algorithm can maintain a high performance when the situation comes to limited snapshots available and interference existing.
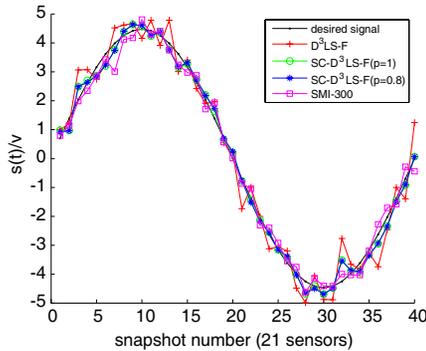

Figure 3. First 40 snaps of estimated signal.

It should be mentioned that the selection of the regularization parameter λ for the performance of the proposed method is important, because λ represents a tradeoff between the equation allowable error and sparsity on the beam pattern. From the three examples above, we can see that when the steering vector mismatch problem exists, a larger λ should be chosen to alleviate the algorithm's sensitivity to the mismatch. Instead, in the presence of a large number of interference signals, a small λ should be selected to restrain the interference signals. When an appropriate λ is selected, we can not only get a better array output performance (such as output SINR), but also can get a lower sidelobe level compared with conventional D^3LS beamformer.

5. CONCLUSION

In this paper, a robust direct data domain least squares adaptive processing approach exploiting the sparsity of the desired beam pattern is presented. The sparse constraint can be viewed as a priori information of the desired beam pattern, which can be utilized for D^3LS beamforming to reduce the high sidelobe level and increase the robustness against mismatch problem with a proper regularization parameter. The experiment results show that the high sidelobe for conventional D^3LS methods is much alleviated and the robustness against steering mismatch is improved simultaneously.

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