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FEATURES OF THE GOUY PHASE OF NONDIFFRACTING BEAMS

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Abstract—It is shown how the linear Gouy phase of an ideal nondiffracting beam of $\pm(k - k_z)z$ form, with k_z being the projection of the wavevector of modulus k of the plane wave spectrum onto the propagation axis z, is built from a rigorous treatment based on the successive approximations to the Helmholtz equation. All of different families of nondiffracting beams with a continuum spectrum, as Bessel beams, Mathieu beams and Parabolic ones, as well as nondiffracting beams with a discrete spectrum, as kaleidoscopic beams, have an identical Gouy phase that fully governs the beam propagation dynamics. Hence, a real beam whose Gouy phase is close to that linear Gouy phase in a given range, will have nondiffracting-like properties on such a range. These results are applied to determine the effective regime in which a physically realizable beam can be treated as a nondiffracting one. As a fruitful example, the Gouy phase analysis is applied to fully establish the regime in which a Helmholtz-Gauss beam propagates with nondiffracting-like properties.

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1. INTRODUCTION

Nondiffacting beams (NDBs) unchange their transverse intensity distribution in free space propagation even for those having nonzero transversal energy flux [1, 2]. They are ideal beams because their infinite extent and energy. Theoretically, they are not only exact solutions of the Helmoltz equation (HE) but also separable into transverse and longitudinal parts [3]. Hence, only the cylindrical frames support nondiffracting solutions [4]. For instance, the Bessel beams form a orthogonal and complete set in the circular frame [5], the Mathieu beams form such a set in the elliptic frame [6], and the parabolic beams, in the parabolic frame [7]. Undoubtedly, the more famous family of NDBs are the Bessel beams due to their wide range of applications [8–10]. Zeroth-order mode [11] and higher-order modes of Bessel-like beams were experimentally generated [12] and their propagation properties were studied in free space [2, 3] and in other nonconventional media, for instance, in the so-called q-plates [13]. In addition to Bessel beams, other kinds of quasi-nondiffracting beams were also experimentally realized: Mathieu-like beams [14], parabolic-like beams [15] and others as the Hermite-Bessel-like beams [16].

On the other hand, just as conventional diffracting beams, NDBs also experience an axial phase shift. This is the so-called Gouy phase shift, playing an important role in several optical phenomena [17]. This phase was interpreted as another manifestation of the geometric phase [18] and the source of the Gouy shift for paraxial beams was recognized to be the beam transverse confinement [19]. Moreover, it was showed that all paraxial shape-invariant beams have a universal Gouy phase of arctan form [20]. The Gouy phase role on the propagation dynamics for several kinds of beams continues being, at the present time, subject of numerous investigations [21, 22]. With respect to the Gouy phase of NDBs, no analysis, neither theoretical nor experimental, was done until recently when the Gouy phase shift has been measured for the 3th-order Bessel beam modes of the first kind [23]. In that work, a linear form for the Gouy phase of all Bessel beam modes was proposed but a theoretical analysis that points out the features on the structure of the Gouy phase covering all kinds of nondiffracting beams was not yet performed. This is necessary to elucidate the role of this phase on a nondiffracting-like behavior of a physically realizable beam and, besides, it will determine the propagation range and experimental conditions in which a beam behaves close to an ideal NDB.

In this paper, a robust analysis on the Gouy phase structure for all kinds of nondiffracting beams is presented. We apply the

propagation operator method [24–26] assuming the separability of the nondiffracting field into longitudinal and transverse parts. Hence, the Gouy phase expression for the exact nondiffracting field solution arises from the Gouy phase expressions for the successive approximations to the exact HE solution. This result will be thus used to determine the role of the Gouy phase on the propagation dynamics of a NDB and, thereby, the conditions in which a real physically realizable beam will have nondiffracting properties are determined. As an example of this, the propagation behavior of an experimentally feasible Helmholtz-Gauss beam (HzGB) is fully determined from the single analysis of the Gouy phase structure. The propagation range and regime related to nondiffracting-like properties of such a beam is also derived.

2. STRUCTURE OF THE GOUY PHASE OF NONDIFFRACTING BEAMS

In free space, any nondiffracting optical field with $e^{-i\omega t}$ time dependence propagating along $+z$ direction can be expressed as

$$u(\mathbf{r}_t, z) = u_t(\mathbf{r}_t)e^{ik_z z}, \tag{1}$$

where u_t is the field propagation-invariant transverse pattern, $\mathbf{r}_t$ the transverse spatial coordinate, and k_z the projection of the wavenumber of modulus k of the plane wave spectrum onto the propagation axis z. The three-dimensional HE:

$$\left(\Delta + k^2\right) u(\mathbf{r}_t, z) = 0 \tag{2}$$

leads to the bi-dimensional HE for the transverse field u_t:

$$\left(\Delta_t + k_t^2\right) u_t(\mathbf{r}_t) = 0, \tag{3}$$

where $k_t^2 = k^2 - k_z^2$ represents the projection of k onto the transverse plane to the propagation direction z and Δ, and Δ_t are the full and transverse Laplacian operators, respectively. It is well known that the separable and translationally invariant solution (1) is only possible in a cylindrical frame [4]. It is straightforward that the field intensity is proportional to $|u_t|^2$, independent of z. Our aim is to derive the structure of the Gouy phase for any arbitrary field represented by (1) by using the propagation operator method [24–26]. First, we define the exact propagation operator as

$$\hat{\mathbb{P}} \equiv \exp\left[ikz\left(1 + \Delta_t/k^2\right)^{1/2}\right]. \tag{4}$$

A HE solution can be expressed in terms of $\hat{\mathbb{P}}$ if the field distribution at a given plane $z = z_0$ is known. Hence, we can write

$$u(\mathbf{r}_t, z) = \hat{\mathbb{P}} u_t(\mathbf{r}_t), \tag{5}$$

since $e^{ik_z z_0}$ is a constant phase factor. From the expansion of $\hat{\mathbb{P}}$ in a power series, one is able to define the approximated propagation operators as

$$\hat{\mathbb{P}}_1 \equiv \exp\left\{ikz\left[1+\Delta_t/\left(2k^2\right)\right]\right\}, \tag{6}$$

and, for $n \geq 2$ as

$$\hat{\mathbb{P}}_n \equiv \exp\left[\frac{-ikz(2n-3)!!(-\Delta_t)^n}{(2n)!!k^{2n}}\right]\prod_{m=1}^{n-1}\hat{\mathbb{P}}_m, \tag{7}$$

since the commutation rule, $[\hat{\mathbb{P}}_\alpha, \hat{\mathbb{P}}_\beta] = 0$, is satisfied and in the limit $n \to \infty$, $\hat{\mathbb{P}}_n$ converges to $\hat{\mathbb{P}}$. Hence, $\hat{\mathbb{P}}_1, \hat{\mathbb{P}}_2, \ldots, \hat{\mathbb{P}}_n, \ldots$, applied to u_t, get the field that are successive approximations to the HE (2). The paraxial field is then

$$u_1(\mathbf{r}_t, z) = \hat{\mathbb{P}}_1 u_t(\mathbf{r}_t), \tag{8}$$

and for $n \geq 2$, the successive nonparaxial fields are

$$u_n(\mathbf{r}_t, z) = \hat{\mathbb{P}}_n u_t(\mathbf{r}_t). \tag{9}$$

Then, by calculating $\Delta_t u_t, \ldots, \Delta_t^n u_t, \ldots$, one can to know the explicit form of $u_1, \ldots, u_n, \ldots$. But the transverse structure for the several nondiffracting fields is, in general, a nontrivial distribution in the $\mathbf{r}_t$-subspace. For example, u_t is represented by complicated Mathieu and parabolic functions in elliptic and parabolic-cylindrical coordinates, respectively [6, 7]. Also, a random angular spectrum yields a nontrivial irregular u_t [27] and a regular discrete plane wave spectrum produces nondiffracting kaleidoscopic patterns with complex u_t-structure [3]. The main question arising is whether one can obtain an explicit expression for $\Delta_t^m u_t$. To do this, some calculation difficulties have to be circumvented. In fact, the key point is that the separability condition of a nondiffracting field leads to an eigenvalue equation that is Eq. (3). Hence, the successive applications of Δ_t to this equation produce the following eigenvalue equation:

$$\left(\Delta_t\right)^n u_t(\mathbf{r}) = \left(-k_t^2\right)^n u_t(\mathbf{r}), \tag{10}$$

where $n = 1, \ldots, \infty$. From (10), one finally obtains that any nondiffracting transverse pattern u_t is an eigenfunction of the complete set of approximated propagation operators $\{\hat{\mathbb{P}}_1, \hat{\mathbb{P}}_2, \ldots, \hat{\mathbb{P}}_n, \ldots\}$ such that

$$u_1(\mathbf{r}_t, z) = e^{ikz}e^{i\phi_1(z)}u_t(\mathbf{r}_t), \tag{11a}$$

$$u_2(\mathbf{r}_t, z) = e^{ikz}e^{i\phi_2(z)}u_t(\mathbf{r}_t), \tag{11b}$$

$$\cdots$$

$$u_n(\mathbf{r}_t, z) = e^{ikz}e^{i\phi_n(z)}u_t(\mathbf{r}_t), \tag{11c}$$

where the axial phase terms are given by

$$\phi_1(z) = z\left[-k_t^2/(2k)\right] \tag{12a}$$

$$\phi_2(z) = z\left[-k_t^2/(2k) - k_t^4/(8k^3)\right] \tag{12b}$$

$$\cdots$$

$$\phi_n(z) = z\left[-\frac{k_t^2}{2k} - \sum_{m=2}^{n} \frac{(2m-3)!!\, k_t^{2m}}{(2m)!!\, k^{2m-1}}\right], \tag{12c}$$

which turn out to be the Gouy phases of all the successive approaches $u_1, u_2, \ldots, u_n, \ldots$. All ϕ_n are linear functions on z with slopes given by the polynomials in brackets. Thus, when the Gouy phase of a real nondiffracting-like beam can be measured as a function of z, the slope will determine the regime in which such a beam propagates and the departure from the linear dependence on z will determine the nondiffracting quality of such a beam. For an ideal NDB:

$$u(\mathbf{r}_t, z) = \lim_{n\to\infty} u_n(\mathbf{r}_t, z) = e^{ikz} e^{i\phi(z)} u_t(\mathbf{r}_t), \tag{13}$$

its Gouy phase is explicitly given by

$$\phi(z) = z\left[-\frac{k_t^2}{2k} - \sum_{m=2}^{\infty} \frac{(2m-3)!!k_t^{2m}}{(2m)!!k^{2m-1}}\right]. \tag{14}$$

The correctness of (14) provides a reduction of (13) to (1). As a verification, from (13)–(14), one has the total phase for the exact nondiffracting field

$$kz + \phi = kz\left[1 - \frac{k_t^2}{2k^2} - \sum_{m=2}^{\infty} \frac{(2m-3)!!}{(2m)!!}\left(\frac{k_t^2}{k^2}\right)^m\right].$$

Provided that $k_t/k < 1$, the series in brackets converges to $\sqrt{1-(k_t/k)^2}$ such that the full phase of an ideal NDB is $kz\sqrt{1-(k_t/k)^2} = \sqrt{k^2-k_t^2}\; z = k_z z$. This ensures the correctness of (14). The Gouy phase for any nondiffracting field u is finally expressed as

$$\phi(z) = -(k - k_z)z, \tag{15}$$

that coincides with the result given in [23] unlike the sign. This is because we choose $+z$ as the propagation direction and Ref. [23] chooses the opposite one. Our analysis in terms of the successive approximations to HE extends for any arbitrary nondiffracting fields and shows how the slope of the Gouy phase determines the regime in which the beam propagates.

The above results can also be obtained by working in the transverse spatial frequency space, $\mathbf{q}_t$, where the nondiffracting field is now characterized by the plane wave angular spectrum. In this representation, the Fourier transform of $u(\mathbf{r}_t, z)$ is $\mathcal{F}\{u(\mathbf{r}_t, z)\} = A(\mathbf{q}_t, z) = \exp(ikz\sqrt{1 - q_t^2/k^2})A_t(\mathbf{q}_t)$ [28], where $A_t(\mathbf{q}_t) = \mathcal{F}\{u_t(\mathbf{r}_t)\}$ is the angular spectrum of the nondiffracting transverse pattern. When $\exp(ikz\sqrt{1 - q_t^2/k^2})$ is expanded in a power series in the expression for $A_t(\mathbf{q}_t)$, the successive polynomials define the successive Fourier transform of the HE approximations, i.e., $A_1 = \mathcal{F}\{u_1\}, \ldots, A_n = \mathcal{F}\{u_n\}, \ldots$. Then, we must apply the inverse transform to obtain the HE approximations u_n. Doing this, the direct way to get Eqs. (11)–(15) is to assume that $A_t(\mathbf{q}_t)$ is defined on a ring of radius k_t in frequency space, i.e., $A_t(\mathbf{q}_t) = A(\varphi)\delta(q_t - k_t)$ for $\mathbf{q}_t$ placed in polar coordinates (q_t, φ), with δ being the Dirac delta distribution. Physically, this ring is interpreted as the superposition of all the plane waves in the McCutchen sphere whose wavevectors of modulus k lie on a conical surface of angle $\theta_0 = \arctan(k_t/k_z)$ with respect to z axis and satisfying $k^2 = k_z^2 + k_t^2$ [29]. Finally, note that a single plane wave propagating along z can be viewed as the particular case of a nondiffracting beam with null Gouy phase. In fact, its aperture angle is $\theta_0 = 0$ leading to $k_t = 0$ and $\phi_1 = \ldots \phi_n = \ldots = \phi = 0$ so that all the HE approximations collapse into the exact solution: $u_1 = \ldots = u_n = \ldots = u$.

3. CHARACTERISTICS OF THE GOUY PHASE OF NONDIFFRACTING BEAMS

The above analysis covers ideal nondiffracting beams that are physically unrealizable due to their infinite energy and spatial extension. However, such an analysis is of great importance to rigorously determine the propagation regime and conditions in which a real beam, possessing finite energy and spatial extension, can be a good representation of an ideal NDB. The nondiffracting-like properties of a real beam can only manifest when its propagation dynamic is exclusively governed by its Gouy phase, as in the ideal case. By this fact, it is important to highlight the features on this phase from the results obtained in Section 2. This will help to a better understanding about the nondiffracting-like properties of a physically realizable beam.

3.1. Role of the Gouy Phase: Nondiffracting vs. Diffracting Beams

The linear dependence for the Gouy phase on propagation coordinate is the hallmark of a nondiffracting field that implies a constant rate

in its variation according to the beam propagates. As the rate of the Gouy phase shift is related to the spatial confinement of the beam [19], the constant rate characterizing the nondiffracting fields is a direct consequence of its profile spatial invariance. The Gouy phase of an ideal nondiffracting beam only depends on the cone aperture angle of its angular spectrum no matter the structure of the transverse profile pattern of such a field, contrary to what happens for diffracting beams, as for example shape-invariant beams [20], where the transverse profile plays an important role in the Gouy phase structure. This is a direct consequence of that the Gouy phase fully dominates the propagation dynamics as Eqs. (11) and (13) point out. The phase of an ideal nondiffracting beam has no radial dependence contrary to what happens to real diffracting beams. For small cone aperture angles, $k_t \ll k$, as it happens in the most usual experimental situations, the nondiffracting beam is completely represented by the paraxial Gouy phase $-k_t^2 z/2k$ rather than (15). If greater aperture angles are used leaving the validity of the paraxial approximation [30–32], a greater number of terms in Eq. (12) contributes to the Gouy phase. As this last fully governs the beam dynamic, the only modification in the nondiffracting beam will be in its effective Gouy phase. Thereby, the mandatory condition for a real beam to behave as a nondiffracting one is that its effective Gouy phase must be equivalent to that of an ideal NDB. Otherwise, the nondiffracting properties of the real beam will necessarily be lost.

3.2. Dynamics of a Nondiffracting-like Beam Based on the Gouy Phase Analysis

NDBs are not physically realizable because of their infinite range and infinite energy content. A physically consistent representation of such a beam may be provided by a HzGB carrying finite energy content [33], since, under certain conditions, this can propagate over a given distance without significant spreading [34]. The functional form of a HzGB incorporates the product of a Gaussian envelope with waist size w_0 and the transverse shape of an ideal NDB. HzGBs were experimentally generated [34]. It was pointed out in Ref. [33] that the propagation dynamic of a HzGB is basically governed by a parameter, γ, that is the ratio between the half-aperture angle of the conical surface traced by the Gaussian beam propagation axes that superimposes to forming the HzGB [35] and the diffraction angle of such Gaussian constituents. It was established [33] that γ determines when a HzGB behaves near to an ideal NDB and when it behaves as a diffracting Gaussian-like beam. Besides, Ref. [33] has noted that a HzGB remains quasi-nondiffracting on a propagation distance z_R/γ, where z_R is the Rayleigh distance,

in the picture in that the NDB beam is properly represented by a HzGB. In this section, based on the analytical expressions for the Gouy phase of a NDB derived here and for the Gouy phase of a HzGB extracted from Ref. [33], we perform a detailed analysis on as the Gouy phase fully dominates the beam propagation dynamics of beams having nondiffracting-like properties. Besides, extending the results presented in [33], we show that the Gouy phase of a beam possessing nondiffracting characteristics exclusively depends on the parameter γ. As a proper example, the conditions for a HzGB to behave as an ideal NDB are presented in the following.

From the analytical expression for a HzGB [33], one can derive its Gouy phase, ψ, as a function of z, which is given by

$$\psi = -\frac{k_t^2 z}{2k\left[(1+(z/z_R)^2\right]} - \arctan\left(z/z_R\right). \tag{16}$$

Performing a series expansion of Eq. (16) and having into account that the parameter γ can be expressed as $\gamma^2 = z_R k_t^2/2k$ from its definition {Eq. (13) in [33]}, one finally arrives to

$$\psi = -\left(1+\gamma^2\right)\left(\frac{z}{z_R}\right) + \left(\frac{1}{3}+\gamma^2\right)\left(\frac{z}{z_R}\right)^3 - \left(\frac{1}{5}+\gamma^2\right)\left(\frac{z}{z_R}\right)^5 + \ldots. \tag{17}$$

Notice that ψ exclusively depends on the parameter γ for a given propagation range z/z_R. Hence, the Gouy phase plays a central role in the main propagation features of a HzGB. Only if ψ might be represented by the Gouy phase of an ideal NDB, namely ϕ, derived in the above section, one can ensure the nondiffracting-like properties of a HzGB. Let us see under what conditions this fact occurs. To this aim, ϕ is rewrote in terms of γ and z/z_R from Eq. (15) to give:

$$\phi = -z_R k\left[1-\sqrt{1+\left(\frac{k_t}{k}\right)^2}\right]\left(\frac{z}{z_R}\right). \tag{18}$$

Once the term in brackets is expanded in a power series, the Gouy phase of an ideal NDB is finally expressed as

$$\phi = -\gamma^2\left[1+\frac{1}{4}\left(\frac{k_t}{k}\right)^2 + \frac{1}{8}\left(\frac{k_t}{k}\right)^4 + \ldots\right]\left(\frac{z}{z_R}\right). \tag{19}$$

At this point, we investigate under what propagation regime the Gouy phases of both kinds of beams can be treated as equivalents, i.e., $\phi \equiv \psi$. Only in such a case, a HzG beam will behave as a nondiffracting-like beam. By comparing Eqs. (17) and (19), the phase equivalence is only

possible if

$$\phi \equiv \psi \approx -\gamma^2 \left(\frac{z}{z_R} \right). \tag{20}$$

To (20) be fulfilled, ψ must retain only the linear term in (17) what happens in the propagation range $z \ll z_R$ where the higher-order terms can be neglected. Furthermore, the propagation regime must also be paraxial: $k_t/k \ll 1$, such that only the first term in brackets in Eq. (19) survives, or equivalently, the Gouy phase of an ideal NDB must be necessarily given by expression (12a). With respect to the parameter γ, Eq. (20) holds true if $\gamma^2 + 1$ can be approximated by γ^2 in (17). This latter leads to the regime $\gamma \gg 1$, in agreement with the result obtained in Ref. [33] where nondiffracting-like properties of a HzGB were univocally related to this range of γ. Therefore, one can see that the simples comparison of both Gouy phases fully establishes the representation of an ideal NDB by a real HzGB.

Figure 1 illustrates the Gouy phase shift of a NDB and a HzGB simultaneously as a function of the normalized propagation distance z/z_R for a paraxial regime $k_t/k = 0.1$ and for several values of γ. Important features are emphasized from this figure. The phase ψ holds linear for low values of z/z_R since the higher-order terms are negligible with respect to the linear term. However, the linearity of ψ is not a sufficient condition for guaranteeing the equivalence between both phases. In fact, for values of γ around the unit or lesser, the phases ψ

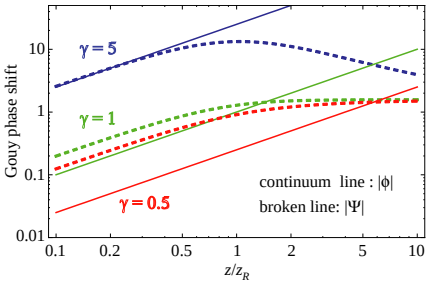

Figure 1. Gouy phase shift of an ideal nondiffracting beam, $|\phi|$ (continuum line), and of a Helmholtz-Gauss beam, $|\psi|$ (broken line), as a function of the normalized propagation distance z/z_R for several values of the parameter γ and $k_t/k = 0.1$.

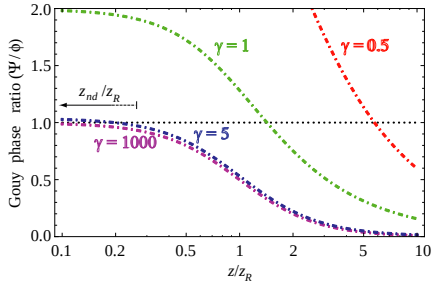

Figure 2. Gouy phase ratio ψ/ϕ as a function of the normalized propagation distance z/z_R for several values of γ and $k_t/k = 0.1$. Notice that the relative nondiffracting propagation range z_{nd}/z_R slightly varies even if γ increases considerably from $\gamma = 5$.

and ϕ have different shift rate as Figure 1 shows because $(1+\gamma^2)$ in Eq. (17) cannot be approximated by γ^2 in Eq. (19). Thereby, the HzGB does not behave as a nondiffracting one for low values of the parameter γ. As this latter increases, the slope of both curves approximates and ψ and ϕ overlap from values of $\gamma \approx 5$ (or still less) in the range $z/z_R \lessapprox 0.3$. There exists a transition zone ($\gamma \sim 1$) between a Gaussian-like behavior ($\gamma \ll 1$) and a nondiffracting-like behavior ($\gamma \gg 1$). These results, based exclusively on the Gouy phase analysis, agree with those obtained in Ref. [33]. On the other hand, when the higher-order terms in Eq. (17) begin to be significative to the Gouy phase shift as z/z_R increases, ψ departures from the linear behavior. In this case, the beam loses its nondiffracting properties according as it propagates out of the range $z/z_R \ll 1$. Figure 1 illustrates this fact. Notice how ϕ and ψ disjoin according as z/z_R increases from $z/z_R \gtrapprox 0.3$ for $\gamma = 5$. This also happens for larger values of γ. At this point, a central question arises to know how the parameter γ affects the propagation distance, namely z_{nd}, wherein a HzGB is equivalent to an ideal NDB. To this aim, the Gouy phase ratio ψ/ϕ was depict in Figure 2 as a function of z/z_R for several values of γ. Notice the strong variation of such a ratio for $\gamma = 1$ and for $\gamma = 0.5$ in the overall interval z/z_R, pointing out a full nonequivalence between both Gouy phases. The single point in which $\psi/\phi = 1$ for both curves in Figure 2 represents the intersection point between ψ and ϕ as viewed from Figure 1. Of course, this isolated point does not represent the phase equivalence as it was already remarked in the previous discussion around Figure 1. Our main interest is then to analyze the regime $\gamma \gg 1$ where the HzGB may possess nondiffracting-like properties. Figure 2 clearly shows that the curves for $\gamma = 5$ and $\gamma = 10^3$ practically overlap on the overall z/z_R-range with ψ and ϕ being equivalents for $z/z_R \lesssim 0.3$. This emphasizes that z_{nd}/z_R does not significantly vary even if γ increases considerably. Inasmuch as z_{nd}/z_R just diminishes slightly, then z_{nd} does not overcome $\sim 0.3 z_R$ when $\gamma \to \infty$. Of course, for longer γ, one has longer z_R so that the absolute propagation range of nondivergence increases.

4. CONCLUSIONS

In summary, it was shown how the linear Gouy phase for any arbitrary nondiffracting beam is building from a rigorous analysis based on the successive approximations to the Helmholtz equation. The variation rate of such a phase only depends on the cone aperture angle, that determines the effective Gouy phase of an ideal nondiffracting beam. Thus, the results on the structure of its Gouy phase are useful to elucidate the nondiffracting quality of a real beam having potential nondiffracting properties. Hence, a physically realizable beam whose

Gouy phase is closely equivalent to the effective linear Gouy phase of an ideal nondiffracting beam in a given propagation range, will posses nondiffracting-like properties on such a range. As an fruitful example, the Gouy phase structure analysis here presented was applied to fully establish the regime in which an experimentally feasible Helmholtz-Gauss beam propagates with nondiffracting-like properties.

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