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CIRCULAR ANTENNA ARRAY DESIGN BY USING EVOLUTIONARY SEARCH ALGORITHMS

Pinar Civicioglu*

Department of Aircraft Electrics and Electronics, College of Aviation, Erciyes University, Kayseri 38039, Turkey

Abstract—Evolutionary Search Algorithms (EA) have been intensively used in solving numerical optimization problems. Since design of antenna arrays is a numerical optimization problem, EAs have been intensively used in solving antenna arrays design problems. Although EAs are widely used in antenna array design problems, a performance comparison study of the intensively used EAs for circular antenna array design problem has been scarcely studied. In this paper, 3 different circular antenna array design problems have been solved by using 15 different evolutionary search algorithms (i.e., ABC, ACS, BSA, CK, CLPSO, CMAES, DE, E2-DSA, EPSDE, GSA, JADE, JDE, PSO, SADE, S-DSA). The objective function designed for solution of the relevant circular antenna array design problems ensures minimization of side lobe levels, acquisition of maximum directivity, and null control of the non-uniform, planar circular antenna array. Obtained statistical analysis results show that S-DSA solves the relevant circular antenna array design problems statistically better than the other evolutionary algorithms used in this paper.

1. INTRODUCTION

Since antenna arrays are intensively used in mobile and wireless communication systems, optimum design of array patterns is of vital importance for increasing channel capacities of these systems, broadening their coverage areas, and ensuring the efficient spectrum utilization. When a single element based antenna is used, it gets difficult to meet the gain or highly directive radiation pattern conditions required in the long distance communication. Antenna arrays use many individual antennas by means of a geometric and

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* Corresponding author: Pinar Civicioglu (civici@erciyes.edu.tr).

electrical configuration, a variety of features of which have been optimized previously. The antenna arrays are widely used in many applications such as radar [1], GPS [2], sonar [3], radios [4], and third generation wireless communication systems [5].

The objective function to be used for obtaining the optimum parameters pertaining to antenna array geometry aims at determining the positions of array elements [6–12]. Optimization of the positions of array elements is very important for producing a radiation pattern that is as similar as possible to the radiation pattern required to be produced via the relevant antenna array [13–20]. Many researchers studying in the field of electromagnetic optimization problems have conducted many studies on the subject of non-uniformly spaced linear antenna arrays [21–24].

Most of the classical optimization algorithms need very good initial-solution values pertaining to the relevant problem in order to solve an antenna array design problem. In addition, classical optimization algorithms usually need the derivative of the relevant problem. Since antenna array design problems are generally in a multimodal form, it is a high possibility that most of the classical optimization algorithms are caught up in a local-solution. Evolutionary search algorithms can sub-optimally solve a multimodal numerical-optimization problem defined by using an objective function without needing the derivative of the relevant problem. Thus, the use of evolutionary search algorithms has become widespread in solution of antenna array design problems [10, 13–16, 18–22, 25]. Evolutionary algorithms have been commonly used for optimizing Side Lobe Level (SLL) [24–26] values of antenna arrays, and ensuring an optimized null control from the designed arrays. Many studies were carried out in order to develop antenna arrays having different geometric characteristics [10–20]. Circular shaped antenna arrays [7, 9, 11, 12] are intensively used in applications such as sonars, radars, mobile and commercial satellite communication systems [27–32]. Evolutionary algorithms, which are widely used for solution of the circular array design problem, are Particle Swarm Optimization (PSO) [14–16, 18, 20, 21], Differential Evolution Algorithm (DE) [19, 33–35], Biogeography-Based Optimization algorithm [10], Invasive Weed Optimization algorithm [13, 22, 26], Genetic Algorithm [16] and the derivatives of these algorithms [36–41].

This paper includes 15 different evolutionary search algorithms (Artificial Bee Colony Algorithm (ABC) [42], Artificial Cooperative Search Algorithm (ACS) [43], Backtracking Search Optimization Algorithm (BSA) [44], Cuckoo Search Algorithm (CS) [45], Comprehensive Learning Particle Swarm Optimizer (CLPSO) [46], Covariance Matrix Adaptation Evolution Strategy (CMAES) [47], Differential Evolution Algorithm (DERND; DE/rand/1/bin) [48], Differential Evolution Algorithm with Ensemble of Parameters (EPSDE) [49], Gravitational Search Algorithm (GSA) [50], Adaptive Differential Evolution Algorithm (JADE) [51], Self-Adaptive Differential Evolution Algorithm (JDE) [52, 53], Particle Swarm Optimization Algorithm (PSO) [45, 54], Strategy Adaptation Based Differential Evolution Algorithm (SADE) [55] and Differential Search Algorithm (DSA; E2-DSA and S-DSA) [53, 56]) in order to solve 3 different circular antenna array design problems.

This paper is organized as follows: Sections 2 and 3 include the *Evolutionary Search Algorithms* and *Problem Formulation*, respectively. Section 4 describes the *Experiments* and Section 5 presents the *Conclusions*.

2. EVOLUTIONARY SEARCH ALGORITHMS

The general structures and basic evolutionary computing philosophy foundation of the evolutionary search algorithms used in this paper are briefly explained below.

ABC is a population-based evolutionary search algorithm that analogically simulates nectar source search behavior of honey-bees. ABC has an elitist, and two-phased search strategy. Because ABC's search strategy is elitist, its problem solving success of multimodal problems is limited. ABC has only two control parameters and its mutation strategy is rather similar to DE. ABC has no crossover operator.

ACS is a swarm intelligence algorithm developed for solving real valued numerical optimization problems. ACS is a bijective search algorithm, that means each random solution evolves towards a random solution of the related problem. The non-elitist structure of ACS gives it ability to solve multimodal problems more successfully. The swarm intelligence philosophy behind ACS is based on the migration of two artificial superorganisms as they biologically interact to achieve the global minimum value pertaining to the problem. In ACS, a superorganism consisting of random solutions of the related problem corresponds to an artificial superorganism migrating to more productive feeding areas.

BSA has a simple structure that is effective, fast and capable of solving multimodal problems and that enables it to easily adapt to numerical optimization problems. BSA can be considered as a modernized PSO. BSA has new strategies for crossover and mutation operations and it has quite powerful local and global search abilities.

In particular, BSA possesses a short-range memory in which it stores a population from a randomly chosen previous generation for use in generating the search-direction matrix. Thus, BSAs memory allows it to take advantage of experiences gained from previous generations when it generates a new trial preparation.

CS is a population based, elitist stochastic search algorithm. CK has a tendency to evolve each random solution towards to the best solution obtained beforehand. CK is structurally similar to DE and ABC. However, it has a superior problem solving success in comparison to ABC, DE and some DE variants. CK has only two control parameters.

CLPSO is an advanced PSO version. Differently from the standard PSO, CLPSO uses the historical best information of all the other particles simultaneously to update the speed of a particle. Instead of the *pbest* and *gbest* values used in the standard PSO, CLPSO uses the pbest values of all random solutions in the swarm. In contrary to the other numerous PSO versions, CLPSO can also solve the multimodal functions.

CMAES is an evolution-strategy based metaheuristic algorithm. CMAES is based on the update of a covariance matrix used to model the relations among the parameters of the optimization problem. CMAES uses the ranks of the random solutions in the population to update the elements of relevant covariance matrix. In consequence of the *recombination*, *mutation*, *selection*, and *adaptation* stages it structurally has, CMAES tries to evolve the population iteratively. CMAES needs the dimension of the problem, the initial mean and the standard deviation values of the multivariate normal distribution to initialization process.

DE is a non-gradient-based, evolutionary raw-genetic algorithm. Its mathematical structure is very simple and it can be adapted to many problem types easily. For such reasons, DE is perhaps the mostly used optimization algorithm in the literature. The basic difference of DE from the genetic algorithm is its multi-structured mutation operator. The most frequently used mutation operator in the literature of DE is DERND (i.e., DE/rand/1/bin), which is used as DE in this paper.

EPSDE uses the mutation strategies existing in the DE algorithm. In EPSDE, a mutation strategy existing in DE algorithm is assigned to each element of the population. The value of the mutation coefficient to be used for the mutation strategy selected in EPSDE is randomly selected from a pool in such a way that it will remain between 0.4 and 0.9 in each iteration. Despite of its low-speed, EPSDE is a quite successful algorithm in problem solving.

GSA is an evolutionary search algorithm and it has been inspired from the universal gravitational laws. Random solution of the respective problem desired to be solved in GSA has been modeled as artificial-bodies that apply newtonian gravitational force to each other. Mass of an artificial-body is related to the quality of the solution that artificial-body provides for the respective problem. The higher the quality of the solution, the slower the speed that artificial-body abandons that position due to the gravitation force applied to it by other artificial-bodies. Speed of the artificial-bodies with inferior quality of solution is higher in the search-space. This phenomenon allows GSA to search the search space very efficiently to find a solution for a problem.

JADE has a new mutation strategy (i.e., *DE/current-to-pbest*) that has been developed to be used together with DE. The *mutation operator* used in JADE evolves a solution that is randomly selected from the population, towards a random top-best solution that provides the best solution at the moment in the population. JADE algorithm can solve numerical optimization problems with much greater success than *DE/current-to-best/1* and *DE/best/1* strategies that are used in standard DE algorithms.

JDE uses a developed form of *DE/rand/1/bin* of DE. The initial values of the *mutation coefficient* and *crossover coefficient* that are the basic control parameters of DE algorithm are generally determined on the basis of the structure of the problem to be solved. JDE similar to DE operates on the basis of a fixed population size. JDE uses a different *mutation* and *crossover* coefficients for each element of each random solution making up the population. Although the structure of JDE is very simple its success in problem solving is much higher than *DE/rand/1/bin* algorithm.

PSO is a *metaheuristic* algorithm inspired by the joint movements of the members of superorganisms. The artificial particles creating a population in PSO and corresponding to the chromosomes used in the genetic algorithm benefit from their *initiative* and *social* experiences while searching the solution of a problem. The *initiative* and *social* experience notions used in PSO correspond philosophically to the *local search* and *global search* notions in the global search algorithms. PSO has been used to solve numerous numeric optimization problems. As a result of the ongoing researches, numerous relatively new PSO structures that are generally more successful than the standard PSO have been developed (e.g., CLPSO, PSO2007). In this paper, the PSO2011, which is an advanced version of PSO2007 that includes a considerable part of the improvements acquired through the long years of researches carried out on the PSO algorithm has been used.

SADE is an advanced DE version capable of using the mutation strategies used in the standard DE adaptively. In SADE, the mutation strategy to be used in any area is determined using the probability values calculated on the basis of the successes the relevant mutation strategies have had in the past iteration steps. Global search capabilities of the SADE are rather advanced.

DSA is a multi-strategy based, advanced evolutionary swarm-algorithm. DSA analogically simulates a superorganism that migrates between two stopovers. DSA has only unique mutation and crossover operators. The structure of mutation operator of DSA contains just one direction pattern apart from the target pattern. The structure of crossover operator of DSA is very different from the structures of crossover operators used in advanced DE algorithms. DSA has only two control parameters. These parameters pertaining to the crossover process of DSA are used for controlling the degree to which the trial

Table 1. Control parameters of the related algorithms.

Algorithm	Initial Values of Control Parameters
ABC	$limit = N \cdot D$ $Size\ of\ EmployedBee = (Size\ of\ \text{Colony})/2$
ACS	$p = 0.10$
BSA	$mixrate = 1$
CK	$p = 0.25, \quad \beta = 1.5$
CLPSO	$[c_1, c_2] = [1.49445,\ 1.49445], \quad m = 0$ $p_c = 0.5 \cdot \frac{e^t - e^{t(1)}}{e^{t(p_s)} - e^{t(1)}}$ where $t = 0.5 \leq \left(0 : \frac{1}{p_s - 1} : 1\right)$
CMAES	$N = 4 + \lfloor 3 \cdot \log(Dimension\ of\ problem) \rfloor$ $\sigma = 0.25$ $\mu = \left\lfloor \frac{4 + \lfloor 2 \cdot \log(N) \rfloor}{2} \right\rfloor$
DERND	$F = 0.50, \quad CR = 0.90$
E2-DSA	$p_1 = p_2 = 0.30 \cdot \kappa \vert \kappa \sim U[0,1]$
EPSDE	F=[0.4 0.5 0.6 0.7 0.8 0.9] CR=[0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9]
GSA	$R_{norm} = 2, \quad R_{power} = 1 \quad \alpha = 20, \quad G_0 = 100$
JADE	$c = 0.10, \quad p = 0.05 \quad CR_m = 0.50, \quad F_m = 0.50, \quad A_{factor} = 1$
JDE	$F_{init} = 0.50, \quad CR_{init} = 0.90$
PSO	$C_1 = 1.80, \quad C_2 = 1.80 \quad \omega = 0.5 + (1 - \text{rand}) \vert \text{rand} \sim U[0,1]$
SADE	$F \sim N(0.5, 0.3), \quad CR \sim N(CR_m, 0.10) \quad c = 0.10, \quad p = 0.05$
S-DSA	$p_1 = p_2 = 0.30 \cdot \kappa \vert \kappa \sim U[0,\ 1]$

pattern will mutate in comparison to the target pattern. Each trial pattern uses the corresponding target pattern for evolving towards stopovers that provide a better fitness value. Standard DSA has 4 different options for obtaining direction matrix. In *Bijective DSA; B-DSA*, population evolves in each cycle into the randomly permuted form of current population. In *Surjective DSA; S-DSA*, population evolves into artificial organisms in which relatively better solutions are found. In *Elitist DSA; E1-DSA*, population evolves into the randomly selected top-best solutions of the original population. In *Elitist DSA; E2-DSA*, population evolves into the better solution of the original population. In this paper, *S-DSA* and *E2-DSA* have been employed.

Table 1 gives the initial values of the relevant control parameters for the evolutionary algorithms used in this paper.

3. PROBLEM FORMULATION

The antenna elements constitute a circular antenna array, as it is illustrated in the Figure 1 [17]. Array factor value (AF) belonging to a circular antenna array can be calculated by use of Eq. (1) when the wavefront of incident plane wave is perpendicular to the x-y plane.

$$AF(\phi)=\sum_{n=1}^{N} I_n \cdot \exp\left[j\cdot k\cdot r\cdot\left(\cos\left(\phi_0-\phi_{ang}^n\right)-\cos\left(\phi_0-\phi_{ang}^n\right)\right)+\beta_n\right] \quad (1)$$

Here,

- $\phi_{ang}^n = 2\cdot\pi\cdot\frac{n-1}{N}$: Angular position of the nth element on the x-y plane,

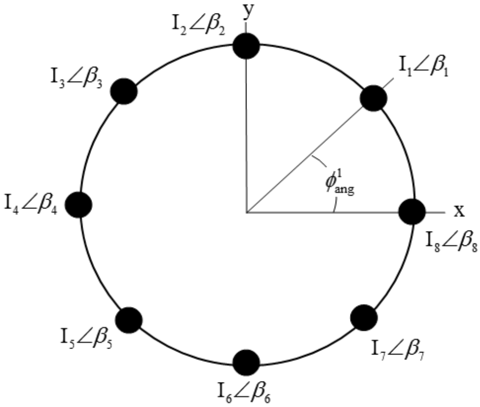

Figure 1. Geometry of circular antenna array.

- k: Wave-number, where $k \cdot r = N \cdot d$ and d denotes angular spacing between elements,
- r: Radius of the circle defined by the circular array,
- φ_0: Direction of maximum radiation,
- φ: Angle of incidence of the plane wave,
- I_n: Current excitation,
- β_n: Phase excitation of the nth element.

Constraints between elements of the designed circular antenna array were defined through Eq. (2).

$$\begin{aligned} I_{\frac{n}{2}+1}\angle\beta_{\frac{n}{2}+1} &= \operatorname{conj}\left(I_1\angle\beta_1\right), \\ I_{\frac{n}{2}+2}\angle\beta_{\frac{n}{2}+2} &= \operatorname{conj}\left(I_2\angle\beta_2\right), \ldots, \\ I_n\angle\beta_n &= \operatorname{conj}\left(I_{\frac{n}{2}}\angle\beta_{\frac{n}{2}}\right) \end{aligned} \tag{2}$$

In this paper, the objective function defined in Eq. (3) has been used for solution of the circular antenna array design problem [17].

$$\begin{aligned} F = {} & w_1 \cdot \left|AR\left(\varphi_{sll}, \vec{I}, \vec{\beta}, \varphi_0\right)\right| \Big/ \left|AR\left(\varphi_{\max}, \vec{I}, \vec{\beta}, \varphi_0\right)\right| \\ & + w_2 \cdot \frac{1}{DIR\left(\varphi_0, \vec{I}, \vec{\beta}\right)} + w_3 \cdot \left|\varphi_0 - \varphi_{des}\right| \\ & + w_4 \cdot \sum_{k=1}^{num} \left|AR\left(\varphi_k, \vec{I}, \vec{\beta}, \varphi_0\right)\right| \end{aligned} \tag{3}$$

where $w_1 = w_2 = w_3 = w_4 = 1$ have been used in this paper.

The purpose of the objective function defined in Eq. (3) is to minimize side-lobe levels, maximize directivity, and obtain null control. To accomplish the purposes of the objective function, phase and amplitude values exciting the antenna elements were investigated. The range of normalized amplitude excitations is [0 1], and the range of phase excitations is [−180 180].

In Eq. (3), the term of $|AR(\phi_{sll}, \vec{I}, \vec{\beta}, \phi_0)|/|AR(\phi_{\max}, \vec{I}, \vec{\beta}, \phi_0)|$ enables minimizing the side lobes. φ_{sll} shows the angle where the maximum side lobe level is obtained.

The term of $\frac{1}{DIR(\phi_0, \vec{I}, \vec{\beta})}$ used in Eq. (3) maximizes the value of the directivity of the array pattern. The directivity is very useful for meeting different array patterns. The term of $|\phi_0 - \phi_{des}|$ strives to drive the maxima of the array pattern close to the desired maxima φ_{des}. The term of $\sum\limits_{k=1}^{num} |AR(\phi_k, \vec{I}, \vec{\beta}, \phi_0)|$ penalizes the objective function if sufficient null control is not achieved. *num* is the number of null control directions and φ_k specifies the kth null control direction.

Table 2 presents the descriptive properties pertaining to the circular antenna design problem used in this paper. The relevant circular antenna array design problems were solved by the relevant algorithms, and current excitation and phase of the antenna elements values enabling to obtain the desired pattern were obtained.

Table 2. The circular antenna array design problems used in tests.

Test #	low	up	dim	ϕ_{des}	null control
Test 1	$[0.2, 0.2, 0.2,$ $0.2, 0.2, 0.2,$ $-180, -180, -180,$ $-180, -180, -180]$	$[1, 1, 1,$ $1, 1, 1,$ $180, 180, 180,$ $180, 180, 180]$	12	$180°$	-
Test 2	$[0.2, 0.2, 0.2,$ $0.2, 0.2, 0.2,$ $-180, -180, -180,$ $-180, -180, -180]$	$[1, 1, 1,$ $1, 1, 1,$ $180, 180, 180,$ $180, 180, 180]$	12	$180°$	$120°$
Test 3	$[0.2, 0.2, 0.2,$ $0.2, 0.2, 0.2,$ $-180, -180, -180,$ $-180, -180, -180]$	$[1, 1, 1,$ $1, 1, 1,$ $180, 180, 180,$ $180, 180, 180]$	24	$180°$	-

4. EXPERIMENTS

This section examines in detail the statistical parameters used in analysis of the test results, algorithmic precision used in the statistical analysis, common control parameters of related algorithms, stopping conditions used by the relevant algorithms, and statistical results.

In the tests conducted, the relevant circular antenna array design problems were solved 30 times, each time using a different initial pattern-matrix [45]. In each test conducted, the relevant evolutionary computing algorithms use the same initial population. Various values acquired in the tests conducted (i.e., *global minimum* and *global minimizer*) are kept for detailed statistical analysis. All tests and statistical analysis performed herein have been carried out in the Matlab platform.

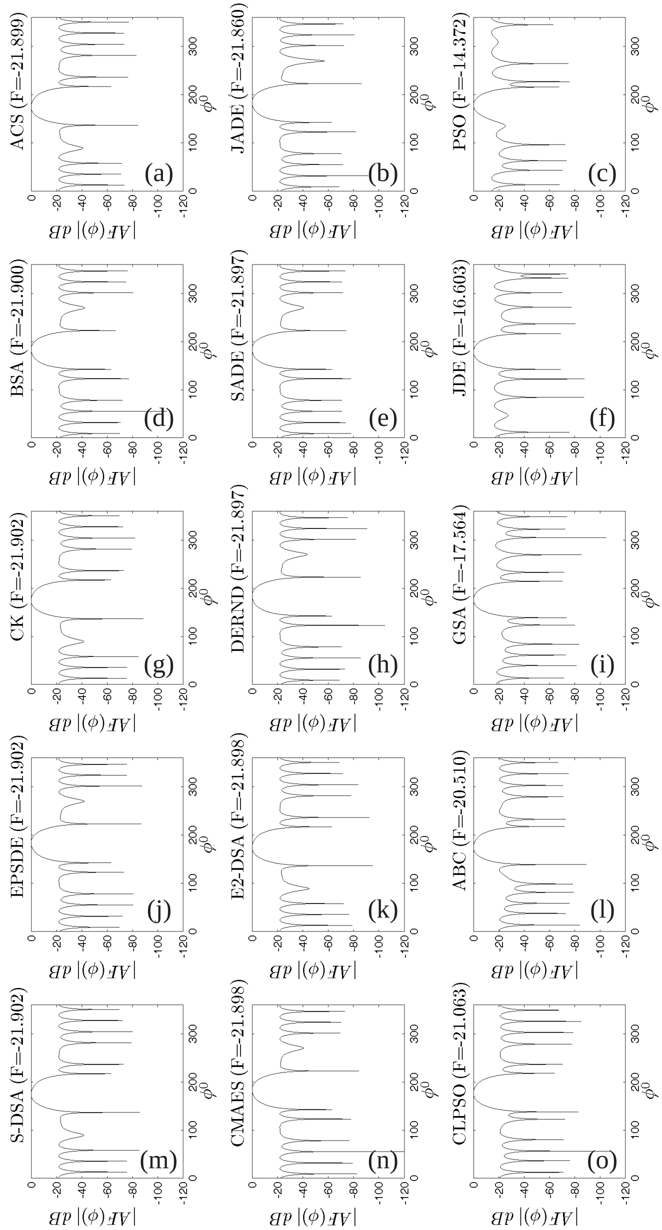

Figure 2. The best array patterns for the Test-2, which is obtained by (a) ACS, (b) JADE, (c) PSO, (d) BSA, (e) SADE, (f) JDE, (g) CK, (h) DERND, (i) GSA, (j) EPSDE, (k) E2-DSA, (l) ABC, (m) S-DSA, (n) CMAES, (o) CLPSO.

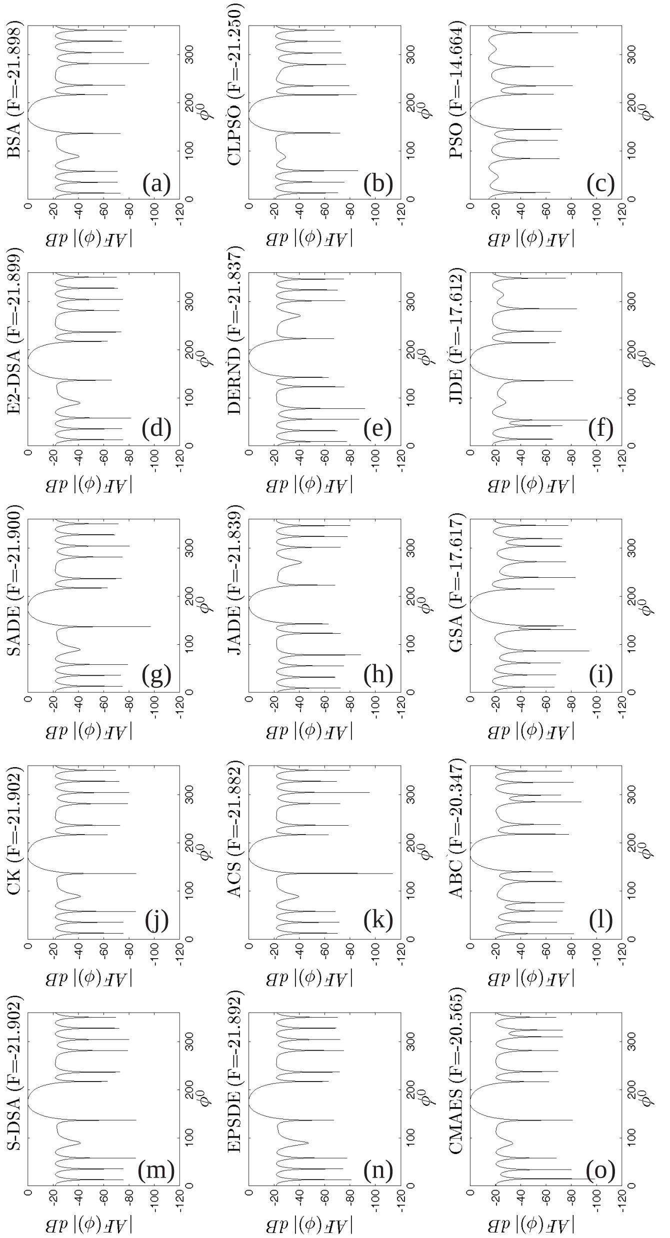

Figure 3. The best array patterns for the Test-2, which is obtained by (a) BSA, (b) CLPSO, (c) PSO, (d) E2-DSA, (e) DERND, (f) JDE, (g) SADE, (h) JADE, (i) GSA, (j) CK, (k) ACS, (l) ABC, (m) S-DSA, (n) EPSDE, (o) CMAES.

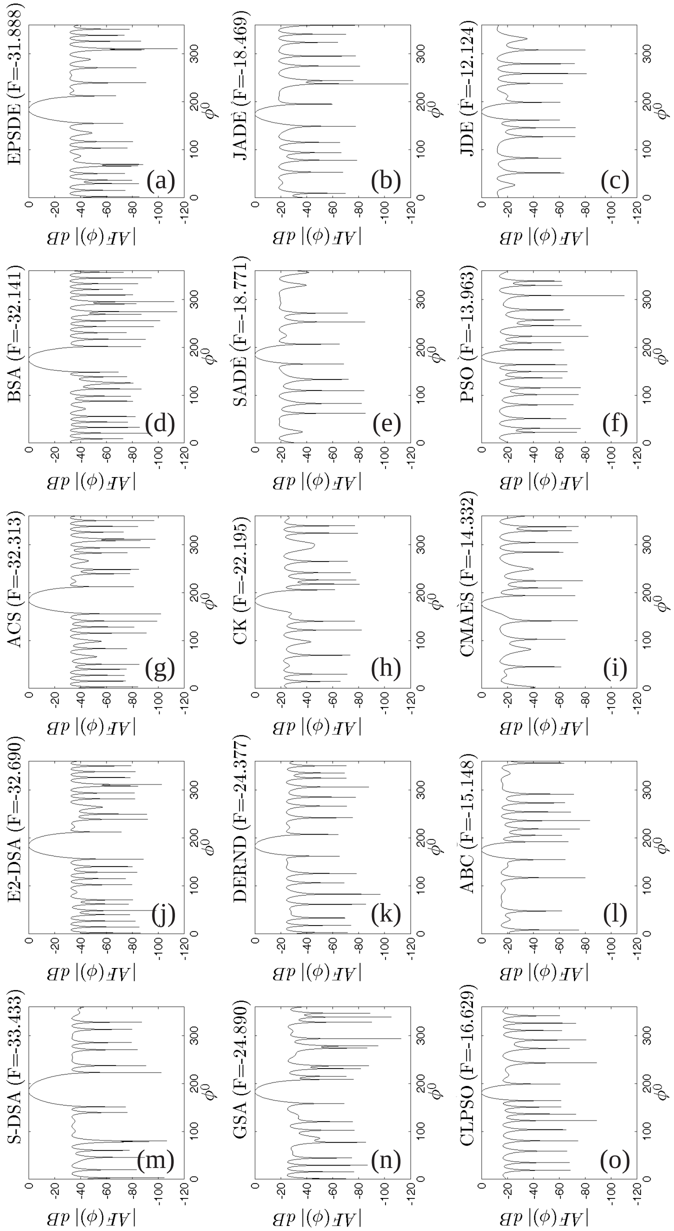

Figure 4. The best array patterns for the Test-2, which is obtained by (a) EPSDE, (b) JADE, (c) JDE, (d) BSA, (e) SADE, (f) PSO, (g) ACS, (h) CK, (i) CMAES, (j) E2-DSA, (k) DERND, (l) ABC, (m) S-DSA, (n) GSA, (o) CLPSO.

4.1. Algorithmic Precision

The *algorithmic precision* of numerous modern software development tools is at the level of 10^{-16} in the *double-precision* mode. If the *arithmetic precision* value is selected higher than necessary, it becomes difficult to compare the *local search* abilities of the algorithms. In the statistical analysis conducted in this paper, the arithmetic precision value has been determined as 10^{-16} so that it can cover the precision level needed in many *practical applications*.

4.2. Common Control Parameters of Algorithms

The common control parameters of the relevant *evolutionary search* algorithms are as follows.

- The maximum generation number value (i.e., *maxcycle*) is 2,000,000.
- Size of the pattern matrix (i.e., size of population) = 30.

Table 3. Statistical values of global minimum values obtained from 30-trials in Test 1; Mean: mean value of global minimum values, Std: standard deviation of the mean global minimum value, Best: the best value of global minimum values and Median: median value of global minimum values.

Algorithm	Statistics			
	Mean	Std	Best	Median
S-DSA	−21.3959	0.955676	−21.9024	−21.7067
EPSDE	−21.581	0.377924	−21.9023	−21.6606
CK	−21.7273	0.150403	−21.9023	−21.7076
BSA	−21.6781	0.198937	−21.9002	−21.7067
ACS	−21.6273	0.174802	−21.899	−21.6389
CMAES	−2.98977	27.6972	−21.8979	−12.1883
E2-DSA	−21.5388	0.449843	−21.8976	−21.6158
DERND	−20.947	1.759608	−21.8975	−21.4957
SADE	−21.3964	0.621627	−21.8967	−21.5322
JADE	−21.353	0.239282	−21.86	−21.3393
CLPSO	−19.5979	1.560356	−21.0628	−20.1316
ABC	−17.534	1.351429	−20.5099	−17.2709
GSA	−16.5437	0.42498	−17.5645	−16.4553
JDE	−9.72518	1.931982	−16.6028	−9.28692
PSO	−13.4075	0.505283	−14.3721	−13.3552

4.3. Stopping Conditions

The predetermined criteria to stop the search processes of the relevant algorithms used in this paper are as follows.

- If absolute value of objective function is less than 10^{-16}, stop.
- If the algorithm has failed to find a better solution than the existing solution even at the end of the last 200,000 function evaluations, stop.
- When the number of function evaluations reaches 2,000,000, stop.

4.4. Statistical Analysis

Tables 3–5 gives basic statistical values (i.e., $Mean$, Std, $Best$, $Median$) of global optimum values obtained in 30 trials by the relevant algorithms for the circular antenna array design problems. Array patterns obtained by the relevant algorithms are showed in the Figures 2–4.

Table 4. Statistical values of global minimum values obtained from 30-trials in Test 2; Mean: mean value of global minimum values, Std: standard deviation of the mean global minimum value, Best: the best value of global minimum values and Median: median value of global minimum values.

Algorithm	Statistics			
	Mean	Std	Best	Median
S-DSA	−21.4399	0.614433	−21.9017	−21.6274
CK	−21.65	0.178681	−21.9017	−21.6677
SADE	−21.277	0.701287	−21.9002	−21.4768
E2-DSA	−21.4905	0.288214	−21.8987	−21.4796
BSA	−21.6608	0.165671	−21.8981	−21.6674
EPSDE	−21.6397	0.183391	−21.8915	−21.6602
ACS	−21.6033	0.173024	−21.8824	−21.506
JADE	−21.4089	0.271234	−21.8393	−21.3857
DERND	−21.3465	0.426849	−21.8368	−21.4212
CLPSO	−20.2716	0.933406	−21.2502	−20.5683
CMAES	18.59502	37.19945	−20.565	4.370296
ABC	−18.0684	1.167902	−20.3474	−18.0741
GSA	−16.6642	0.421773	−17.6169	−16.6748
JDE	−9.61435	2.39578	−17.6116	−8.94924
PSO	−13.2438	0.644399	−14.6641	−13.2706

Table 5. Statistical values of global minimum values obtained from 30-trials in Test 3; Mean: mean value of global minimum values, Std: standard deviation of the mean global minimum value, Best: the best value of global minimum values and Median: median value of global minimum values.

Algorithm	Statistics			
	Mean	Std	Best	Median
S-DSA	−21.5036	6.7538	−33.4334	−19.3358
E2-DSA	−24.6933	4.2943	−32.6903	−25.8875
ACS	−22.2482	4.5013	−32.3133	−21.3762
BSA	−23.1325	4.2970	−32.1408	−24.4438
EPSDE	−19.4702	5.9493	−31.8883	−16.2445
GSA	−22.8905	1.8213	−24.8898	−23.5310
DERND	−17.7545	2.6231	−24.3769	−16.9840
CK	−14.6113	1.7005	−22.1947	−14.4060
SADE	−16.2070	1.2932	−18.7705	−16.4255
JADE	−15.7014	1.0930	−18.4688	−15.5797
CLPSO	−14.3385	0.6390	−16.6294	−14.2788
ABC	−12.7516	0.7458	−15.1482	−12.7759
CMAES	15.3954	21.8090	−14.3321	12.1527
PSO	−12.7375	0.5976	−13.9626	−12.7660
JDE	−8.0792	1.4484	−12.1239	−7.5831

Examining the Figures 2–4, it is seen that S-DSA, in comparison to the other algorithms, achieved better minimized side lobes and the greatest directivity.

5. CONCLUSIONS

This paper includes solutions to 3 different circular antenna array design problems using 15 different evolutionary search algorithms. 30 trials were solved for each problem by using different initial populations in order to avoid negativities of the stochastic natures of evolutionary search algorithms. Results were analyzed by using the well known statistical parameters of *Mean*, *Standard Deviation of Mean*, *Best Solution* and *Median Value of the Solutions*. Simulation results show that results found by S-DSA are statistically better than those produced by using ABC, ACS, BSA, CK, CLPSO, CMAES, DE, E2-DSA, EPSDE, GSA, JADE, JDE, PSO and SADE.

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