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GUIDED-MODE RESONANCE FILTER COMPENSATED TO OPERATE ON A CURVED SURFACE

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Abstract—Guided-mode resonance filters (GMRFs) are highly compact structures that can produce a strong frequency response from a single thin layer of dielectric. When a GMRF is formed onto a curved surface, the local angle of incidence varies over the aperture of the device and the overall performance significantly degrades. In the present work, we spatially varied the grating period of a curved GMRF to perfectly compensate for the local angle of incidence. The performance of our curved device actually surpassed that of our flat device because it also compensated for the spherical wave front from the source. This paper summarizes our design process and experimental results obtained around 25 GHz.

1. INTRODUCTION

A guided-mode resonance filter (GMRF) is a simple device composed of just a grating and a slab waveguide that operate in close proximity [1, 2]. Under a precise phase matching condition, the grating partially couples externally propagating waves into guided modes within the slab. The guided modes are "leaky" modes due to the grating so they slowly escape from the waveguide. An overall filtering response is produced from the interference between the applied wave and the leaked wave. Guided-mode resonance is a highly sensitive phenomenon that enables filters to be constructed with extremely narrow pass bands or stop bands. In fact, the bandwidth can be made arbitrarily narrow by reducing the contrast of the grating.

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GMRFs are most commonly used in photonics [3–5], but some devices at radio frequencies can also be found [6–11]. They provide a highly compact means of filtering and do not require using metals. The filter response can be symmetric with virtually no ripple outside of the pass band or stop band. Efficiency can approach 100% and they can be tuned over a very large range of frequencies. Multi-band filters have also been reported [12]. GMRFs tend to be very sensitive to angle of incidence, polarization, material properties, and structural deviations due to the high sensitivity of the phase matching condition. Filters can be made less sensitive to polarization using crossed gratings or even special cases of one-dimensional gratings [13, 14]. The material properties and structural deviations can be more precisely controlled by better manufacturing methods as well as compensated for by tuning the device during fabrication [5]. The angle of incidence remains a problem, especially when the GMRF is to be formed onto a curved surface. In this case, the angle of incidence changes over the aperture of the device due to the slope of the surface changing relative to the direction of the applied wave. The slope of the surface detunes the phase matching condition so portions of the aperture no longer contribute to the resonance. This not only weakens the filter response, but introduces aberrations in the wave front that are difficult to correct.

This paper describes a simple technique to realize all-dielectric filters formed onto arbitrarily curved surfaces where the period of the grating is adjusted to compensate for the curvature. Cannistra et al. fabricated a biologically inspired GMRF onto a curved surface using conformal soft lithography, but the grating was not modified in any way to compensate [15, 16]. Lu et al. formed a GMRF onto a curved surface for the purpose of focusing light [17], but did not adjust the period. This paper presents the first known effort to adjust the grating period of a GMRF in order to obtain the same performance as that of a flat GMRF.

2. DESIGN OF BASELINE GMRF

To best demonstrate our concept, we designed a baseline GMRF that was simple to build and provided a narrow resonance that was sufficiently sensitive to the angle of incidence. The device was manufactured in high density polyethylene (HDPE) which was specified by the vendor to have a dielectric constant of $2.35 \pm 10\%$. First, transmission through two homogeneous layers in air was designed using the transfer matrix method [18, 19] to provide a low background reflection. This optimization dictated that the first layer had $\varepsilon_{r1} = 1.1$ and $d_1 = 1.19\,\mathrm{mm}$, while the second layer had $\varepsilon_{r2} = 2.35$ and

$d_2 = 3.35\,\text{mm}$. Second, to realize the low dielectric constant in the first layer using only the HDPE, the duty cycle of a subwavelength grating was set to $f = 0.36$ so that the effective dielectric constant was 1.1 for the electric field aligned parallel to the grating grooves. Third, the period Λ of the grating was determined using rigorous coupled-wave analysis (RCWA) [20, 21] to place the first-order guided-mode resonance at 25 GHz. This required the period to be $\Lambda = 9.19\,\text{mm}$. The baseline design and its simulated transmission for several angles of incidence are shown in Figure 1.

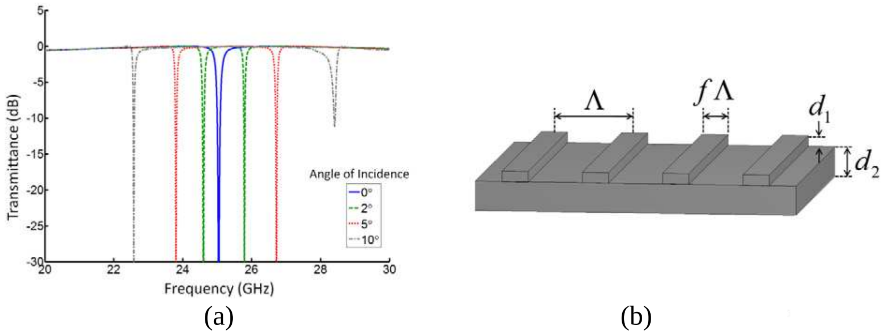

Figure 1. Baseline GMRF in HDPE ($\varepsilon_r = 2.35$) which has $\Lambda = 9.19\,\text{mm}$, $f = 0.36$, $d_1 = 1.19\,\text{mm}$, and $d_2 = 3.35\,\text{mm}$. (a) Simulated transmission at several angles of incidence. (b) Geometry of baseline design.

3. COMPENSATION FOR SURFACE CURVATURE

When a wave is incident on a grating, it diffracts into a number of discrete modes called spatial harmonics, or diffraction orders. The concept, and the geometry of this diffraction, is illustrated in Figure 2. The angles of the diffracted modes θ_m are described by the famous grating equation.

$$n \sin\theta_m = n_{\text{inc}} \sin\theta_{\text{inc}} - \frac{m\lambda_0}{\Lambda} \tag{1}$$

In this equation, λ_0 is the free space wavelength, n_{inc} is the refractive index where the incident wave is applied, n is the refractive index of the material where the angles are being calculated, m is the order of diffraction, and Λ is the period of the grating. This equation can be written in a slightly different form using the grating number K defined for a 1D grating as $K = 2\pi/\Lambda$.

$$\beta_m = k_0 n_{\text{inc}} \sin\theta_{\text{inc}} - mK \tag{2}$$

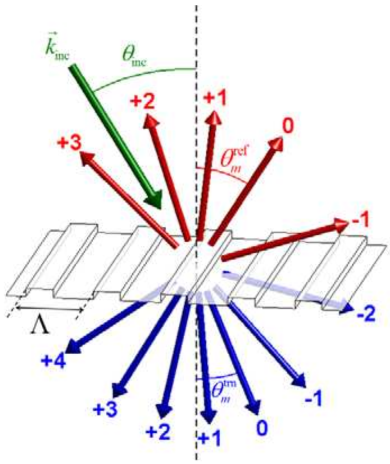

Figure 2. Grating diffraction showing reflected and transmitted spatial harmonics.

The parameter β_m is the propagation constant of the mode guided within the slab. A baseline GMRF was designed to operate at normal incidence and resonate the first order diffracted mode ($m = \pm 1$). The grating number for the baseline design was K_0 and it follows that $K_0 = \beta_m$. When the applied wave is given some angle of incidence, the phase matching condition changes. In order to support resonance across the entire aperture using the same diffraction order, the grating number K must be adjusted so as to maintain proper phase matching. Assuming we wish to keep the -1 order mode phase matched, we set $m = -1$ and solve Eq. (2) for K as a function of θ_{inc}.

$$K\left(\theta_{\text{inc}}\right) = K_0 - k_0 n_{\text{inc}} \sin\theta_{\text{inc}} \tag{3}$$

We now have an equation to adjust the grating period across the device aperture so that the entire device contributes to resonance. To produce a design, the shape and slope of the surface must be known. We chose to wrap our GRMF around 90° of a circle to provide the curvature. The radius of a spherical source beam was also considered in this analysis. The geometry and all of the variables from which we calculated the local angle of incidence across the device are defined and illustrated in Figure 3.

If the device was illuminated by a perfectly flat plane wave, the angle of incidence θ_{inc} would simply equal the angle ϕ. This can be put in terms of the circumferential distance x instead of ϕ as

$$\theta_{\text{inc}}(x) = \phi = \frac{x}{R} \quad \text{Plane wave illumination} \tag{4}$$

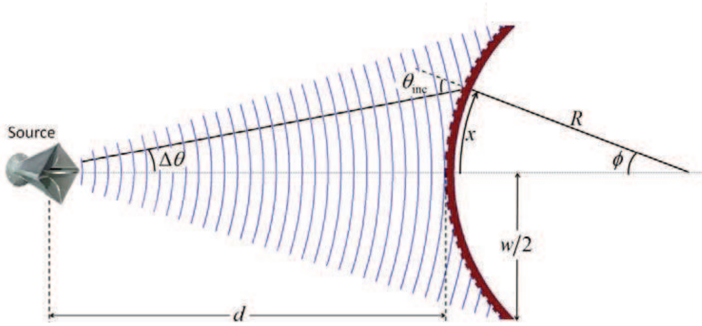

Figure 3. Geometry from which to design a curved GMRF.

In order to compensate for the spherical wave front of a source, the additional angle $\Delta\theta$ must be added to this quantity. The height of the point on the GMRF identified by ϕ above the axis is $R\sin\phi$, where R is the radius of the curved GMRF. The distance along the axis from the source to this point is $d + R(1-\cos\phi)$. Therefore, the angle $\Delta\theta$ is calculated as

$$\tan(\Delta\theta) = \frac{R\sin\phi}{d + R(1-\cos\phi)} = \frac{\sin\phi}{1 + d/R - \cos\phi} \tag{5}$$

Combining these angles, we derive our final equation for the angle of incidence θ_{inc} as a function of the angle ϕ or the circumferential distance x.

$$\begin{aligned}\theta_{\text{inc}}(\phi) &= \phi + \tan^{-1}\left(\frac{\sin\phi}{1 + d/R - \cos\phi}\right)\\ \theta_{\text{inc}}(x) &= \frac{x}{R} + \tan^{-1}\left[\frac{\sin(x/R)}{1 + d/R - \cos(x/R)}\right]\end{aligned} \tag{6}$$

We derived this equation in terms of the circumferential distance x because this parameter translates to the horizontal position when the device is flat. The devices were manufactured as flat panels so this was most convenient. By substituting this expression into Eq. (3), we derive an expression for the grating number K as a function of x. From this, the dielectric constant in the grating region $\varepsilon_r(x)$ is calculated through an intermediate parameter $\Phi(x)$ called the grating phase. Given the desired duty cycle f, the dielectric function is given by Eq. (7). These equations were evaluated numerically in MATLAB and the resulting grating geometry was imported into SolidWorks from which a CAD file was generated.

$$\varepsilon_r(x) = \begin{cases} 1 & \cos[\Phi(x)] < \cos(\pi f) \\ \tilde{\varepsilon}_r & \cos[\Phi(x)] \geq \cos(\pi f) \end{cases} \tag{7}$$

$$\Phi(x) = \int_0^x K(x')\,dx' \tag{8}$$

This simple design methodology spatially varies the grating period to compensate for a local angle of incidence that changes with position. It was discovered through rigorous simulation, however, that the above methodology still produces a small error in the position of the resonance between 0.1% and 2%. It was determined that this error arose because the analytical approach incorrectly assumes the effective refractive index of the grating region is constant with angle of incidence. The error introduced by this assumption is particularly severe for shallow gratings and for large angles of incidence. RCWA was used to determine the exact grating period adjustment as a function of angle of incidence. Figure 4 compares the rigorous data to the data tabulated using the analytical approach outlined above. These curves are nearly identical as the angle approaches 0°, but differ most at the larger angles of incidence.

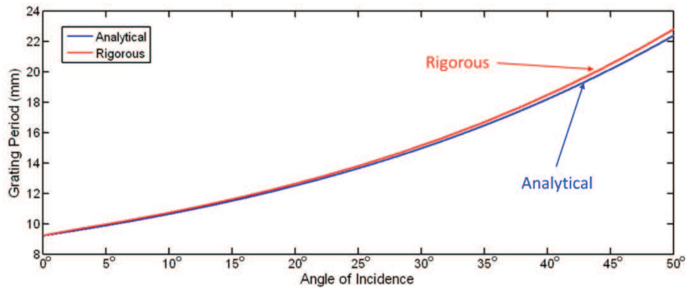

Figure 4. Grating period adjustment as a function of angle of incidence.

Given the baseline design and grating period adjustment data described previously, a design was produced that was 182.88 cm long, 121.92 cm wide, and 4.57 mm thick. The GMRF drawn to scale is shown in Figure 5 along with the simulated field intensity profile on resonance. The simulation of the curved device was performed using finite-difference frequency-domain [22]. While difficult to see in this figure, the grating period near the edges is around 5.49 mm and is 9.19 mm at the center.

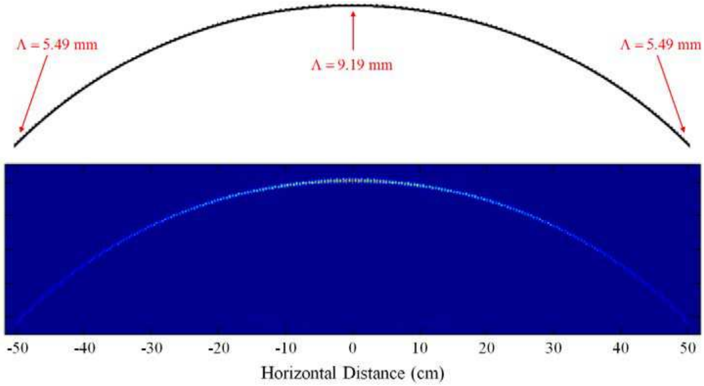

Figure 5. Scale image of the curved GMRF design and the device simulated on resonance.

4. EXPERIMENTAL RESULTS

The design described above was imported into SolidWorks and submitted to a machine shop to be manufactured by computer numerical control (CNC) machining. Two different GMRFs were manufactured. The first was the flat baseline GMRF, while the second was a GMRF to be placed onto a curved surface where the grating period was adjusted to compensate. The parts were mounted onto wooden frames to provide mechanical rigidity and to hold the GMRFs in place with the prescribed curvature. Photographs of the mounted devices are provided in Figure 6. Wooden boards and nylon screws were used instead of metal to minimize electromagnetic scattering and interference during testing.

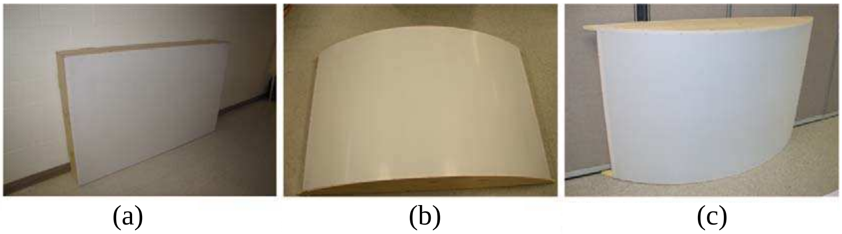

Figure 6. GMRFs installed in flat and curved frames. (a) Flat baseline GMRF. (b) Baseline GMRF in curved frame. (c) Compensated GMRF in curved frame.

The frequency responses of the GMRFs were measured in an anechoic chamber at Virginia Tech using a vector network analyzer (VNA). Three separate tests were performed, corresponding to the configurations shown in Figure 6. Each test entailed measuring the background transmission of the frame without the GMRF installed, measuring the transmission with the GMRF installed, and then dividing the GMRF response by the background response to normalize the data. Anomalous spikes appeared in the final transmittance data at points where the transmission of the frame without the GMRF was very small and we divided by these numbers. The spectral transmittance curves from all of these tests are provided in Figure 7. First, the baseline GMRF was installed in the flat frame and its transmission was measured. A peak suppression of around 15 dB was observed just below 25 GHz. Second, this same uncompensated GMRF was installed in the curved frame to demonstrate dramatically degraded performance. No resonance was observed when the transmission was measured. Third, the compensated GMRF was installed in the curved frame and the transmission measured one final time. In this last case, a strong resonance was observed again just below 25 GHz. In fact, the suppression of the curved GMRF approached 27 dB and exceeded the baseline design by nearly 12 dB. The compensated design outperformed the flat design because both the surface curvature and the divergence of the source were accounted for in the design. The baseline GMRF had a uniform grating so it was designed to filter a perfectly flat and uniform plane wave source. The bandwidth of the curved device remained essentially unchanged. During this work, however, we observed through simulation that the bandwidth of the

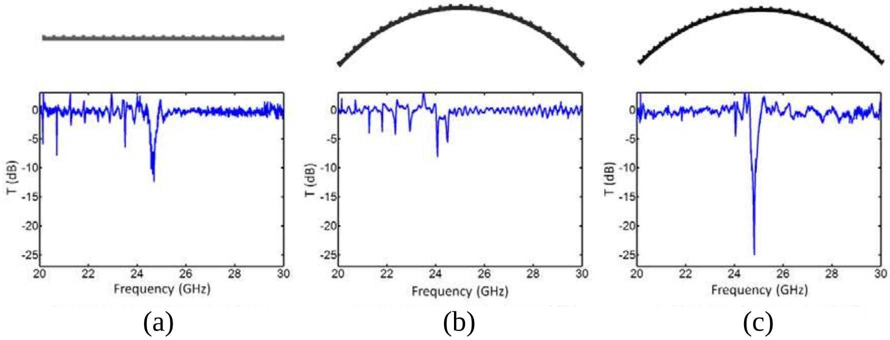

Figure 7. Summary and comparison of test results. (a) Uncompensated flat design. (b) Uncompensated curved GMRF. (c) Compensated curved GMRF.

resonances became broader and somewhat weaker when compensating for large angles of incidence, greater than 70° or so.

5. CONCLUSION

Guided-mode resonance filters are attractive devices for realizing compact and narrowband filters without using metals. This is particularly important at optical frequencies, but can also be important at microwave frequencies when operating at high power or when losses must be absolutely minimized. These devices, however, are highly sensitive to angle of incidence so using them on curved surfaces seriously degrades their performance. In this work, we demonstrated a GMRF on a curved surface by adjusting the local grating period to compensate. By doing this, the entire device contributed to resonance. In fact, the performance of our curved GMRF was better than our baseline flat design because it compensated for divergence of the source beam in addition to surface curvature.

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REFERENCES

1. Magnusson, R. and S. Wang, "New principle for optical filters," *Applied Physics Letters*, Vol. 61, 1022–1024, 1992.
2. Wang, S. and R. Magnusson, "Theory and applications of guided-mode resonance filters," *Applied Optics*, Vol. 32, 2606–2613, 1993.
3. Liu, Z., S. Tibuleac, D. Shin, P. Young, and R. Magnusson, "High-efficiency guided-mode resonance filter," *Optics Letters*, Vol. 23, 1556–1558, 1998.
4. Magnusson, R. and S.-S. Wang, "Optical guided-mode resonance filter," Google Patents, 1993.
5. Rumpf, R. C. and E. G. Johnson, "Modeling fabrication to accurately place GMR resonances," *Optics Express*, Vol. 15, 3452–3464, 2007.
6. Barton, J. H., R. C. Rumpf, R. W. Smith, C. L. Kozikowski, and P. A. Zellner, "All-dielectric frequency selective surfaces with few number of periods," *Progress In Electromagnetics Research B*, Vol. 41, 269–283, 2012.

7. Bertoni, H. L., L.-H. Cheo, and T. Tamir, "Frequency-selective reflection and transmission by a periodic dielectric layer," *IEEE Transactions on Antennas and Propagation*, Vol. 37, 78–83, 1989.
8. Coves, Á., S. Marini, B. Gimeno, and V. Boria, "Full-wave analysis of periodic dielectric frequency-selective surfaces under plane wave excitation," *IEEE Transactions on Antennas and Propagation*, Vol. 60, 2760–2769, 2012.
9. Li, L. and D. H. Werner, "Design of all-dielectric frequency selective surfaces using genetic algorithms combined with the finite element-boundary integral method," *2005 IEEE Antennas and Propagation Society International Symposium*, 376–379, 2005.
10. Tibuleac, S., R. Magnusson, T. A. Maldonado, P. P. Young, and T. R. Holzheimer, "Dielectric frequency-selective structures incorporating waveguide gratings," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 48, 553–561, 2000.
11. Zuffada, C., T. Cwik, and C. Ditchman, "Synthesis of novel all-dielectric grating filters using genetic algorithms," *IEEE Transactions on Antennas and Propagation*, Vol. 46, 657–663, 1998.
12. Boonruang, S., A. Greenwell, and M. Moharam, "Multiline two-dimensional guided-mode resonant filters," *Applied Optics*, Vol. 45, 5740–5747, 2006.
13. Fehrembach, A.-L. and A. Sentenac, "Study of waveguide grating eigenmodes for unpolarized filtering applications," *JOSA A*, Vol. 20, 481–488, 2003.
14. Mizutani, A., H. Kikuta, K. Nakajima, and K. Iwata, "Nonpolarizing guided-mode resonant grating filter for oblique incidence," *JOSA A*, Vol. 18, 1261–1266, 2001.
15. Cannistra, A., M. Poutous, E. Johnson, and T. Suleski, "Fabrication of guided mode resonance filters on conformal surfaces," *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 7927, 23, 2011.
16. Cannistra, A. T., M. K. Poutous, E. G. Johnson, and T. J. Suleski, "Performance of conformal guided mode resonance filters," *Optics Letters*, Vol. 36, 1155–1157, 2011.
17. Lu, M., H. Zhai, and R. Magnusson, "Focusing light with curved guided-mode resonance reflectors," *Micromachines*, Vol. 2, 150–156, 2011.
18. Berreman, D. W., "Optics in stratified and anisotropic media: 4×4-matrix formulation," *JOSA*, Vol. 62, 502–510, 1972.
19. Chen, C.-J., A. Lien, and M. Nathan, "4×4 and 2×2 matrix

formulations for the optics in stratified and biaxial media," *JOSA A*, Vol. 14, 3125–3134, 1997.

20. Moharam, M. and T. Gaylord, "Rigorous coupled-wave analysis of planar-grating diffraction," *JOSA*, Vol. 71, 811–818, 1981.
21. Rumpf, R. C., "Improved formulation of scattering matrices for semi-analytical methods that is consistent with convention," *Progress In Electromagnetics Research B*, Vol. 35, 241–261, 2011.
22. Rumpf, R. C., "Simple implementation of arbitrarily shaped total-field/scattered-field regions in finite-difference frequency-domain," *Progress In Electromagnetics Research B*, Vol. 36, 221–248, 2012.