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A DUALITY BETWEEN METAMATERIALS AND CONVENTIONAL MATERIALS IN MULTILAYERED ANISOTROPIC PLANAR STRUCTURES

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Abstract—Consider a plane wave incident on a multilayered anisotropic planar structure composed of conventional materials and metamaterials and surrounded by two half- spaces. In this paper, we aim to prove three theorems which indicate a kind of duality in these structures. *Theorem 1:* Assume that an arbitrarily polarized plane wave is obliquely incident on the structure. Now each layer is filled with by dual media according to the interchanges DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG. Then, the reflection (R) and transmission (T) coefficients of the structure become the complex conjugates of their counterparts. Consequently, the reflected power and transmitted power from the structure are the same for the two dual cases of anisotropic media. *Theorem 2:* If the interchanges DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG are made in all the layers except in the half spaces on the two sides of the multilayer structure (which is more realizable), then the reflection coefficients become complex conjugates and the reflected power remains the same. *Theorem 3:* If the structure is backed by a perfect electric conductor and the media interchanges DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG are made in the layers, then the reflection coefficients of the two dual structures become complex conjugates of each other, and the reflected powers are equal. Independent of wave frequency, the number of layers, their thickness, and the type of polarization, these theorems hold true in case of any change in any of these conditions. In the last section, some examples are provided to verify the validity of the proposed theorems.

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1. INTRODUCTION

Metamaterials are materials in which the real parts of the components of permittivity or/and permeability tensors can be negative. They may be classified as epsilon negative (ENG), mu negative (MNG) and double negative (DNG). Conventional materials are called double positive (DPS).

The structure, taken into consideration here, is a multilayered anisotropic planar structure composed of conventional materials and metamaterials and situated between two half spaces. As shown in Figure 1, a plane wave is obliquely incident on the structure. In recent years, numerous methods have been employed for analyzing this structure [1–12].

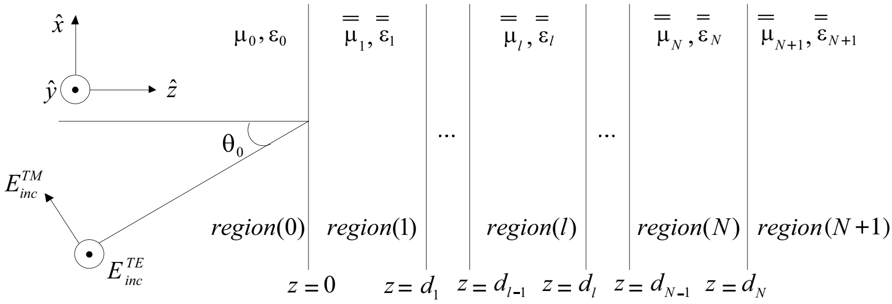

Figure 1. Geometry of the problem.

Initial works on the subject were based on 4×4 characteristic matrix of a single anisotropic slab [1, 2]. Later efforts include generalization of the problem for stratified structures by different methods [3–5]. Morgan et al. paid attention to a numerical solution, and introduced an efficient and simple algorithm for this case [6]. Others proposed various techniques based on eigenvalue computation, Ricatti differential equation, and transmission line method [7–11], which are more complex. The characteristic matrix algorithm [1, 2] had a serious drawback and showed instability for thick layers compared to the wavelength. To avoid this instability which was due to the numerical finite difference algorithm, the use of hybrid matrix of the structure is suggested [12].

In this paper, we employ a full-wave matrix method, whereby the domain is decomposed into forward and backward travelling waves [13, 14]. We prove three theorems for the reflection and transmission coefficients of incident plane waves onto Figure 1.

In [18] these duality theorems are proved in the special case of isotropic structures, limiting application since most metamaterials are anisotropic. In this paper we expand the application scope of these theorems to include anisotropic media as well as isotropic ones.

The purposed theorems are given a mathematical proof and indicate a duality for the propagation and reflection of electromagnetic waves in dual media of DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG. Independent of wave frequency, the number of layers, their thickness, and the type of polarization hold true in case of any change in any of these conditions.

At last, the purposed theorems are validated through some examples solved using state space method.

2. PROBLEM CONFIGURATION

Consider a multilayered anisotropic planar structure in which layer l is a layer with thickness d_l and the anisotropic constitutive parameters below:

$$\bar{\bar{\varepsilon}}_l = \varepsilon_0 \begin{bmatrix} \varepsilon_{lx} & 0 & 0 \\ 0 & \varepsilon_{ly} & 0 \\ 0 & 0 & \varepsilon_{lz} \end{bmatrix} \tag{1}$$

$$\bar{\bar{\mu}}_l = \mu_0 \begin{bmatrix} \mu_{lx} & 0 & 0 \\ 0 & \mu_{ly} & 0 \\ 0 & 0 & \mu_{lz} \end{bmatrix} \tag{2}$$

Supposing $e^{+j\omega t}$, the solution of the wave equation in layer l, which would be some of two waves with orthogonal TE and TM polarizations, is thus [13, 14]:

$$\begin{aligned} H_{xl} &= -\frac{k_{zl}^{I}}{\omega\mu_0\mu_{lx}}\left(A_l e^{-jk_{zl}^{I}z} - C_l e^{+jk_{zl}^{I}z}\right)e^{-jk_x x} \\ E_{xl} &= \frac{k_{zl}^{II}}{\omega\varepsilon_0\varepsilon_{lx}}\left(B_l e^{-jk_{zl}^{II}z} - D_l e^{+jk_{zl}^{II}z}\right)e^{-jk_x x} \\ E_{yl} &= \left(A_l e^{-jk_{zl}^{I}z} + C_l e^{+jk_{zl}^{I}z}\right)e^{-jk_x x} \\ H_{yl} &= \left(B_l e^{-jk_{zl}^{II}z} + D_l e^{+jk_{zl}^{II}z}\right)e^{-jk_x x} \end{aligned} \tag{3}$$

where $l = 0, 1, \ldots N+1$, and superscripts I and II relate to TE and TM polarizations, respectively. A_l and C_l are the amplitudes of forward and backward traveling TE waves; B_l and D_l are the amplitudes of forward and backward traveling TM waves; in the above relations, $e^{-jk_x x}$ is obtained from phases matching and is identical for all layers.

From the dispersion relation [13] for each layer, we have:

$$\begin{cases} k_{zl}^{I^2}+\frac{\mu_{lx}}{\mu_{lz}}k_x^2=k_0^2\varepsilon_{ly}\mu_{lx} \\ \\ k_{zl}^{II^2}+\frac{\varepsilon_{lx}}{\varepsilon_{lz}}k_x^2=k_0^2\varepsilon_{lx}\mu_{ly} \quad , \quad l=0,1,\ldots,N+1 \\ \\ k_x=k_0\sin\theta_0 \end{cases} \tag{4}$$

In the region $l = 0$, we have the following expressions:

$$A_0 = E_0 \tag{5}$$
$$B_0 = H_0 \tag{6}$$
$$C_0 = R_{TE}\cdot E_0 \tag{7}$$
$$D_0 = R_{TM}\cdot H_0 \tag{8}$$

In which R_{TE} and R_{TM} are reflection coefficients for TE and TM polarizations in the left most half space, respectively.

And in the region $l = N + 1$, we have:

$$A_{N+1} = T_{TE}\cdot E_0 \tag{9}$$
$$B_{N+1} = T_{TM}\cdot H_0 \tag{10}$$
$$C_{N+1} = 0 \tag{11}$$
$$D_{N+1} = 0 \tag{12}$$

In which T_{TE} and T_{TM} are transmission coefficients for TE and TM polarizations in the right most half-space, respectively.

The boundary conditions at the boundaries of the layers may be written as a matrix equation:

$$MX=Y \tag{13}$$

In which the M, X and Y are matrix of coefficients, unknowns' vector and sources' vector, respectively and defined as:

$$M=\begin{bmatrix} [A]_{6\times4} & [\phi]_{6\times4} & & \cdots & \cdots & & [\phi]_{6\times4} \\ [\phi]_{8\times4} & \ddots & [\phi]_{8\times4} & & \cdots & & [\phi]_{8\times4} \\ & & \ddots & & & \ddots & \\ \vdots & & \ddots & [B]_{8\times4} & \ddots & & \vdots \\ & \ddots & & & \ddots & & \\ [\phi]_{8\times4} & & \cdots & & [\phi]_{8\times4} & \ddots & [\phi]_{8\times4} \\ [\phi]_{6\times4} & & \cdots & \cdots & & [\phi]_{6\times4} & [C]_{6\times4} \end{bmatrix} \tag{14}$$

$$X = [\, R_{TE}\ R_{TM}\ A_1\ C_1\ B_1\ D_1\ \ldots\ A_l\ C_l\ B_l\ D_l\ A_{l+1}\ C_{l+1}\ B_{l+1}\ D_{l+1}\\ \ldots\ A_N\ C_N\ B_N\ D_N\ T_{TE}\ T_{TM}]^T \tag{15}$$

$$Y = [-E_0\ -E_0\ -H_0\ -H_0\ 0\ 0\ \ldots\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \ldots\ 0\ 0\ 0\ 0\ 0\ 0]^T \tag{16}$$

where $[\phi]$ is the null matrix and:

$$A=\begin{bmatrix} E_0 & 0 & -1 & -1 & 0 & 0 \\ -E_0 & 0 & -p_0^I & p_0^I & 0 & 0 \\ 0 & H_0 & 0 & 0 & -1 & -1 \\ 0 & -H_0 & 0 & 0 & -p_0^{II} & p_0^{II} \end{bmatrix} \tag{17}$$

$$B=\begin{bmatrix} r_l^I & (r_l^I)^{-1} & 0 & 0 & s_l^I & (s_l^I)^{-1} & 0 & 0 \\ r_l^I & -(r_l^I)^{-1} & 0 & 0 & p_l^I s_l^I & -p_l^I (s_l^I)^{-1} & 0 & 0 \\ 0 & 0 & r_l^{II} & (r_l^{II})^{-1} & 0 & 0 & s_l^{II} & (s_l^{II})^{-1} \\ 0 & 0 & r_l^{II} & -(r_l^{II})^{-1} & 0 & 0 & p_l^{II} s_l^{II} & -p_l^{II}(s_l^{II})^{-1} \end{bmatrix} \tag{18}$$

$$C=\begin{bmatrix} r_N^I & (r_N^I)^{-1} & 0 & 0 & E_0 s_N^I & 0 \\ r_N^I & -(r_N^I)^{-1} & 0 & 0 & p_N^I E_0 (s_N^I)^{-1} & 0 \\ 0 & 0 & r_N^{II} & (r_N^{II})^{-1} & 0 & H_0 s_N^{II} \\ 0 & 0 & r_N^{II} & -(r_N^{II})^{-1} & 0 & -p_N^{II} H_0 (s_N^{II})^{-1} \end{bmatrix} \tag{19}$$

in which:

$$\begin{cases} p_l^I = \frac{\mu_{lx}}{k_{zl}^I} \cdot \frac{k_{z(l+1)}^I}{\mu_{(l+1)x}} \\ p_l^{II} = \frac{\varepsilon_{lx}}{k_{zl}^{II}} \cdot \frac{k_{z(l+1)}^{II}}{\varepsilon_{(l+1)x}} \end{cases}, \quad l = 0, 1, \ldots N \tag{20}$$

$$\begin{cases} r_l^i = e^{-jk_{zl}^i d_l} \\ s_l^i = e^{-jk_{z(l+1)}^i d_l} \end{cases}, \quad i = III, \quad l = 0, 1, \ldots N \tag{21}$$

3. PROOF OF THE THEOREMS

3.1. Theorem 1

Consider a multilayered anisotropic planar structure made of a combination of common materials and metamaterials situated between two half spaces composed of lossless media. Now all layers including two half spaces are filled by their dual media according to the interchanges DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG. Then, the reflection (R) and transmission (T) coefficients from the structure become the complex conjugates of their counterparts. Consequently, the reflected power and transmitted power from the structure are the same for the two dual cases.

3.2. Proof of Theorem 1

If we apply the interchanges DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG in all layers including two half spaces, we will have:

$$\begin{aligned} \overline{\overline{\varepsilon}}_{lNew} &= -\overline{\overline{\varepsilon}}_l^* &(22)\\ \overline{\overline{\mu}}_{lNew} &= -\overline{\overline{\mu}}_l^* &(23)\end{aligned}$$

And according to relation (4) and the rules mentioned in [14–17] for choosing the correct sign of wave number, we will have:

$$k_{zlNew}^{I}{}^2 = k_0^2(-\varepsilon_{ly}^*)(-\mu_{lx}^*) - \frac{(-\mu_{lx}^*)}{(-\mu_{lz}^*)}k_x^2 = (-1)(-1)\left(k_0^2\varepsilon_{ly}^*\mu_{lx}^* + \frac{\mu_{lx}^*}{(-\mu_{lz}^*)}\right)k_x^2 \tag{24}$$

$$k_{zlNew}^{II}{}^2 = k_0^2(-\varepsilon_{lx}^*)(-\mu_{ly}^*) - \frac{(-\varepsilon_{lx}^*)}{(-\varepsilon_{lz}^*)}k_x^2 = (-1)(-1)\left(k_0^2\varepsilon_{lx}^*\mu_{ly}^* + \frac{\varepsilon_{lx}^*}{(-\varepsilon_{lz}^*)}\right)k_x^2 \tag{25}$$

$$k_{zlNew}^{I} = -\left(k_0^2\varepsilon_{ly}^*\mu_{lx}^* - \frac{\mu_{lx}^*}{\mu_{lz}^*}k_x^2\right)^{\frac{1}{2}} = -k_{zl}^{I}{}^* \tag{26}$$

$$k_{zlNew}^{II} = -\left(k_0^2\varepsilon_{lx}^*\mu_{ly}^* - \frac{\varepsilon_{lx}^*}{\varepsilon_{lz}^*}k_x^2\right)^{\frac{1}{2}} = -k_{zl}^{II*} \tag{27}$$

Consequently, these changes will be made in the components of the matrix of coefficients:

$$\begin{aligned} p_{lNew}^{I} &= \frac{(-\mu_{lx}^*)}{(-k_{zl}^{I}{}^*)} \cdot \frac{(-k_{z(l+1)}^{I}{}^*)}{(-\mu(l+1)x^*)} = p_l^{I*} &(28)\\ p_{lNew}^{II} &= \frac{(-\varepsilon_{lx}^*)}{(-k_{zl}^{II*})} \cdot \frac{(-k_{z(l+1)}^{I}I^*)}{(-\varepsilon(l+1)x^*)} = p_l^{II*} &(29)\\ r_{lNew}^{i} &= e^{-jk_{zlNew}^{i}d_l} = e^{jk_{zl}^{i}{}^*}d_l = (r_l^i)^* &(30)\\ s_{lNew}^{i} &= e^{-jk_{z(l+1)New}^{i}d_l} = e^{jk_{z(l+1)}^{i}{}^*d_l} = (s_l^i)^* &(31)\end{aligned}$$

It is observed that with the applied interchanges in constitutive parameters of the structure, the entire components of the matrix of coefficients become complex conjugates. As E_0 and H_0 are real, the components of vector of sources are equal with their complex conjugates. Consequently, the reflection and transmission coefficients of the new structure become complex conjugates of the old ones.

$$\begin{cases} [M]\,[X]_{old} = [Y] \rightarrow [X]_{old} = [M]^{-1}\ [Y] \\ [M]^{*}\,[X]_{New} = [Y]^{*} \rightarrow [X]_{New} = ([M]^{*})^{-1}\,[Y]^{*} \end{cases} \Rightarrow [X]_{New} = [X]^{*}_{old} \tag{32}$$

Therefore:

$$\begin{cases} (R_{TE})_{new} = (R_{TE})^{*}_{old},\ (\mathrm{T}_{TE})_{new} = (T_{TE})^{*}_{old} \\ (T_{TE})_{new} = (T_{TE})^{*}_{old},\ (\mathrm{T}_{TM})_{new} = (T_{TM})^{*}_{old} \end{cases} \tag{33}$$

As the reflected and transmitted powers are equal to $P_r = RR^{*} = |R|^2$ and $P_t = TT^{*} = |T|^2$ respectively, they are identical for a multilayer anisotropic planar structure and its dual structure.

3.3. Theorem 2

If the interchanges DPS $\leftrightarrow$ DNG and ENG $\leftrightarrow$ MNG are made in all the layers except in the half spaces on the two sides of the multilayer structure (which is more realizable), then the reflection coefficients become complex conjugates and the reflected power remains the same.

3.4. Proof of Theorem 2

This theorem may be proved by consider the transmission line equivalent circuits of the multilayered anisotropic planar structure corresponding to TE and TM waves shown in Figure 2. Right and left half spaces are replaced by impedances equal to their characteristic impedances.

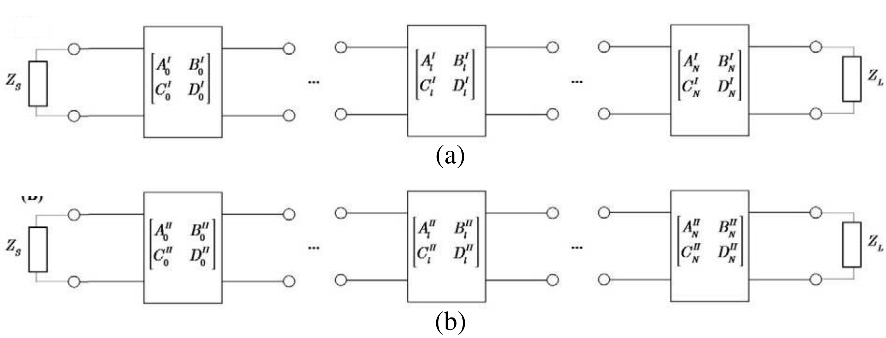

Figure 2. Transmission line model of multilayered anisotropic planar structure for (a) TE wave and (b) TM wave.

The ABCD matrixes of lth layer corresponding to TE and TM waves are given by:

$$[T_l^I] = \begin{bmatrix} A_l^I & B_l^I \\ C_l^I & D_l^I \end{bmatrix} = \begin{bmatrix} \cos[k_l^I(d_l - d_{l-1})] & j\eta_l^I \sin[k_l^I(d_l - d_{l-1})] \\ \frac{j}{\eta_l^I} \sin[k_l^I(d_l - d_{l-1})] & \cos[k_l^I(d_l - d_{l-1})] \end{bmatrix} \tag{34}$$

$$[T_l^{II}] = \begin{bmatrix} A_l^{II} & B_l^{II} \\ C_l^{II} & D_l^{II} \end{bmatrix} = \begin{bmatrix} \cos[k_l^{II}(d_l - d_{l-1})] & j\eta_l^{II} \sin[k_l^{II}(d_l - d_{l-1})] \\ \frac{j}{\eta_l^{II}} \sin[k_l^{II}(d_l - d_{l-1})] & \cos[k_l^{II}(d_l - d_{l-1})] \end{bmatrix} \tag{35}$$

where k_l^I and k_l^{II} may be obtained from (4). η_l^I and η_l^{II} are characteristic impedances of lth layer corresponding to TE and TM waves and are given by [19]:

$$\eta_l^I = \sqrt{\frac{\mu_{lx}}{\varepsilon_{lx}\varepsilon_{lz}/(\varepsilon_{lz}\cos^2\theta_0 + \varepsilon_{lx}\sin^2\theta_0)}} \tag{36}$$

$$\eta_l^{II} = \sqrt{\frac{\mu_{lx}\mu_{lz}/(\mu_{lz}\cos^2\theta_0 + \mu_{lx}\sin^2\theta_0)}{\varepsilon_{lx}}} \tag{37}$$

According to relations (22)–(27) and (34)–(37), if layer l is filled by its dual layer, its dual ABCD matrixes corresponding to TE and TM waves will become equal to the complex conjugates of the former ABCD matrixes.

Consequently, the overall ABCD matrixes of the dual multilayer planar structure (as the product of the individual line sections) corresponding to TE and TM waves will become equal to the complex conjugates of those of the original structure.

The input impedance and the reflection coefficient at the input port of TE and TM transmission lines are:

$$Z_{in}^I = \frac{A^I Z_L + B^I}{C^I Z_L + D^I}, \quad R_{TE} = \frac{Z_{in}^I - \eta_0}{Z_{in}^I + \eta_0} \tag{38}$$

$$Z_{in}^{II} = \frac{A^{II} Z_L + B^{II}}{C^{II} Z_L + D^{II}}, \quad R_{TM} = \frac{Z_{in}^{II} - \eta_0}{Z_{in}^{II} + \eta_0} \tag{39}$$

where η_0 is the intrinsic impedance of the left half space. Considering the fact that the elements of ABCD matrixes of the two dual structures are complex conjugates of each other and that η_0 is real, it can be deduced:

$$(R_{TE})_{new} = (R_{TE})^*_{old}, \quad (R_{TM})_{new} = (R_{TM})^*_{old}, \tag{40}$$

3.5. Theorem 3

If the structure is backed by a perfect electric conductor and the media interchanges DPS $\leftrightarrow$ DNG, and ENG $\leftrightarrow$ MNG are made in the layers, then the reflection coefficients of the two dual structures become complex conjugates of each other, and the reflected powers are equal.

3.6. Proof of Theorem 3

This theorem may also be deduced in a similar way with theorem 2. In this case, $Z_L = 0$. Thus (38) and (39) reduce to:

$$Z_{in}^{I} = \frac{B^I}{D^I}, \; R_{TE} = \frac{Z_{in}^{I} - \eta_0}{Z_{in}^{I} + \eta_0} \tag{41}$$

$$Z_{in}^{II} = \frac{B^{II}}{D^{II}}, \; R_{TM} = \frac{Z_{in}^{II} - \eta_0}{Z_{in}^{II} + \eta_0} \tag{42}$$

and again with the same explanations (40) is deduced.

4. NUMERICAL EXAMPLES

4.1. Example 1

We consider a slab with the parameters shown in the Table 1. The reflection and transmission coefficients of this slab and its dual medium are presented in Figures 3 and 4 versus frequency for a normal incidence. These curves are drawn using Matlab code based on the state space method [20, 21].

Table 1. Parameters of the slab under consideration.

Electric permittivity	Magnetic permeability	Layer thickness (m)
$\overline{\overline{\varepsilon}} = \begin{bmatrix} 10-5i & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -2-3i \end{bmatrix}$	$\overline{\overline{\mu}} = \begin{bmatrix} 1-4i & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5i \end{bmatrix}$	t=0.001

4.2. Example 2

Now consider a bilayer structure with the parameters shown in Table 2 backed by a PEC. The reflection coefficients of the structure and its dual are represented in Figure 5 versus frequency for a normal incidence.

Excellent agreement is observed between the curves, which confirm the validity of our theorems.

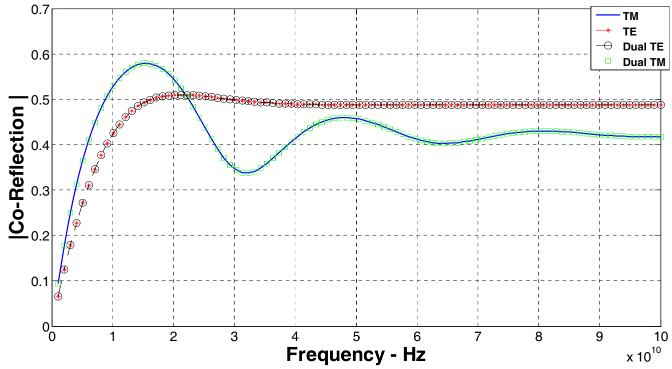

Figure 3. Reflection from the slab with the parameters shown in the Table 1 and its dual medium versus frequency.

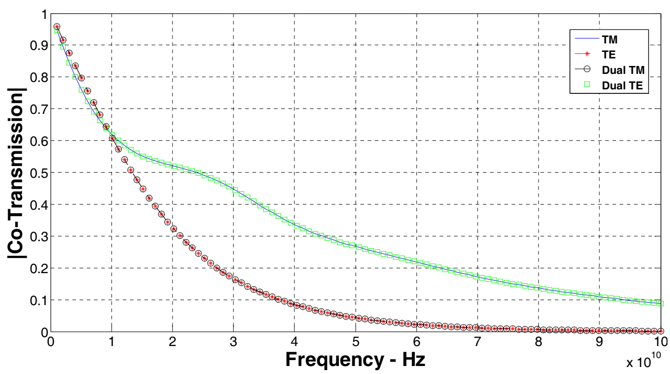

Figure 4. Transmissions from the slab with the parameters shown in the Table 1 and its dual medium versus frequency.

Table 2. Parameters of the structure under consideration.

Layer	Electric permittivity	Magnetic permeability	Layer thickness (m)
1	$\overline{\overline{\varepsilon}}_1=\begin{bmatrix} 1-3i & 0 & 0 \\ 0 & 2\ \ 5i & 0 \\ 0 & 0 & 4 \end{bmatrix}$	$\overline{\overline{\mu}}_1=\begin{bmatrix} 2-3i & 0 & 0 \\ 0 & -5-3i & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$t_1 = 0.001$
2	$\overline{\overline{\varepsilon}}_2=\begin{bmatrix} 1-i & 0 & 0 \\ 0 & 2-5i & 0 \\ 0 & 0 & -3-2i \end{bmatrix}$	$\overline{\overline{\mu}}_2=\begin{bmatrix} 2-5i & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 3 \end{bmatrix}$	$t_2 = 0.001$

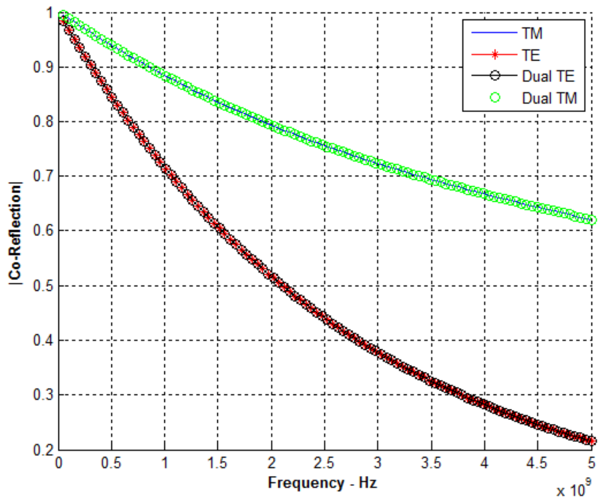

Figure 5. Reflection from the structure with the parameters shown in the table (2) and its dual medium versus frequency.

5. CONCLUSIONS

Several theorems have been proved in this paper, for an incidence of plane waves on a multilayered anisotropic planar structure. These theorems indicate a kind of duality for the reflection and transmission between two dual multilayered anisotropic structures with interchanges DPS $\leftrightarrow$ DNG or ENG $\leftrightarrow$ MNG. Moreover, independent of wave frequency, the number of layers, their thickness, and the type of polarization the theorems hold true in case of any change in any of these conditions.

Each theorem was given a mathematical proof and at last, the validity of the theorems has been verified by some examples solved using the state space method.

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