

## **ELECTRIC AND MAGNETIC FIELDS PRODUCED BY A CURRENT FLOWING ALONG A HORIZONTAL CONDUCTOR LOCATED OVER A PERFECTLY CONDUCTING GROUND PLANE — REVISITED**

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**Abstract**—Electromagnetic fields associated with the electric current flowing along a horizontal conductor located over perfectly conducting ground are estimated using electromagnetic fields pertinent to acceleration of electric charges. It is shown that the electric and magnetic fields that exist below a long overhead horizontal conductor are nothing but the radiation fields generated by the acceleration of charge at the point of injection of current into the horizontal conductor.

### **1. INTRODUCTION**

The theory of voltage and current transmission along overhead power lines is a subject that is very well established both in practice and theory. Utilizing these theories one can easily calculate the electric and magnetic fields in the vicinity of energized power lines. A detailed account of the standard procedure to calculate the electric and magnetic fields in the vicinity of power lines with complicated geometry can be found for example in references [1, 2]. These are actually based on the solution of Maxwell's equations pertinent to the geometry under consideration. One interesting aspect of Maxwell's equations is that a given solution can be interpret in many different ways with each interpretation illustrating a certain aspect of the equations which may not be that apparent when another interpretation is sought. The goal of this paper is to interpret the electric and magnetic fields generated by a power line or an overhead conductor in a way to bring into focus certain aspects which were hidden when the problem was solved with the standard procedure. Both the standard procedure

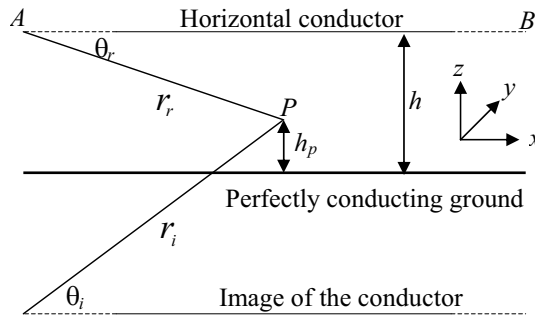
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as illustrated in references [1, 2] and the procedure illustrated in this paper give identical results. Indeed, they are all solutions to the same Maxwell's equations. Actually, mastering any one of these techniques is enough for someone to calculate the electromagnetic fields generated by overhead power lines. However, each technique illustrates the richness of the solutions of Maxwell's equations and one may appreciate being able to apply one technique more than another depending on the problem under consideration and the physical insight that is being sought.

As mentioned above the goal of this paper is to calculate the electromagnetic fields generated by an overhead current carrying conductor using a procedure that had not been described in the literature previously. The new procedure will give results identical to the standard procedure [1, 2], but the reader will get a different insight into the physics of the problem that may not be that apparent when using standard technique.



**Figure 1.** Geometry relevant to the analysis of the electromagnetic fields produced by a single conductor power line.

Consider a long uniform horizontal conductor (or a single conductor power line) of radius  $a$  located at a height  $h$  meters above a perfectly conducting ground. The geometry relevant to the problem under consideration is given in Figure 1. In this figure  $A$  and  $B$  denote the two ends of the conductor. Consider a section of the conductor, which is located far away from both of its ends. The capacitance per unit length,  $C$  (in F/m), and the inductance per unit length,  $L$  (H/m), of this section of the conductor are given for  $h/a \gg 1$  by (i.e., the electrical parameters corresponding to an infinitely long horizontal conductor located over perfectly conducting ground)

$$C = \frac{2\pi\epsilon_0}{\log \left[ \frac{2h}{a} \right]} \quad (1)$$

$$L = \frac{\mu_o}{2\pi} \log \left[ \frac{2h}{a} \right] \quad (2)$$

The impedance per unit length of the conductor,  $Z$ , is given by

$$Z = \frac{1}{2\pi} \sqrt{\frac{\mu_o}{\epsilon_o}} \log \left[ \frac{2h}{a} \right] \quad (3)$$

In the above equations the parameter 'a' is the radius of the conductor.

Assume that a uniform current, fed into the conductor from the end  $A$ , is flowing along the conductor. The standard procedure to calculate the electric and magnetic fields produced by the current flowing along the conductor is the following. Consider a point of observation  $P$  located directly below the current carrying conductor and situated at a height  $h_p$  meters from ground level. The effect of perfectly conducting ground on the electric and magnetic fields is taken into account using image theory. The magnetic field (which is in the azimuthal direction or in the  $y$  direction),  $B$ , at the point of observation can be obtained using Amperes Law and the result is

$$B = \frac{\mu_o I h}{\pi (h^2 - h_p^2)} \mathbf{a}_y \quad (4)$$

This can also be written as

$$B = \frac{I h}{\pi \epsilon_o (h^2 - h_p^2) c^2} \mathbf{a}_y \quad (5)$$

Let us now calculate the electric field at the same point. If the uniform current flowing in the conductor is  $I$ , then the voltage  $V$  of the conductor can be calculated using  $V = IZ$ . That is

$$V = \frac{I}{2\pi} \sqrt{\frac{\mu_o}{\epsilon_o}} \log \left[ \frac{2h}{a} \right] \quad (6)$$

Since the capacitance of the conductor is given by (1) the charge per unit length,  $\rho$ , of the conductor is given by the product of  $V$  and  $C$  which reduces to

$$\rho = I/c \quad (7)$$

where  $c = 1/\sqrt{\epsilon_o \mu_o}$  is the speed of light in vacuum.

Now, the conductor is very long and, therefore, the electric field at the point of observation  $P$  can be calculated by treating the conductor as a uniform line charge. The resulting electric field,  $E$ , which is directed along the  $z$  axis is given by

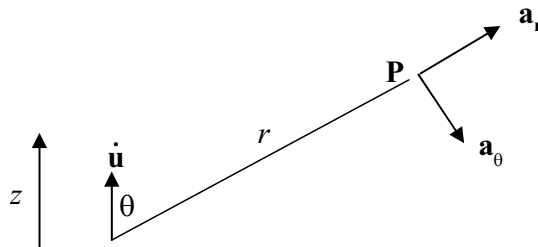
$$E = -\frac{I h}{\pi (h^2 - h_p^2) \epsilon_o c} \mathbf{a}_z \quad (8)$$

These expressions are identical to those presented in references [1,2] when the number of conductors in the power line is reduced to one. First, observe that the magnetic field and the electric fields are perpendicular to each other. Second, note that the ratio of the electric field to the magnetic field is equal to the speed of light in free space. In other words, the field components have all the characteristics of a radiation field moving in free space. This is in agreement with the TEM mode of propagation of waves along overhead horizontal power lines.

Now, even though the current flowing along the conductor is uniform, this current has to be injected into the horizontal conductor at a certain point, i.e., the source point of the current. In our case the source point is assumed to be at  $A$ . During this process of current injection, electric charges undergo acceleration and this acceleration of charges gives rise to a radiation field. The goal of this paper is to show that the field components derived earlier can be re-interpreted as radiation fields generated at the source of the uniform current. This can be done conveniently with the electromagnetic field calculation procedure introduced by Cooray and Cooray [3]. These authors have utilized the electromagnetic fields generated by accelerating charges to evaluate the electromagnetic fields of both lightning return strokes and current pulses propagating along vertical and horizontal conductors. This technique will be used in the present paper.

## 2. ELECTROMAGNETIC FIELDS OF ACCELERATING CHARGES

The theory of electromagnetic fields generated by moving charges is described in any standard text book on electromagnetic theory, and it suffices to quote the results directly [4]. The geometry relevant to



**Figure 2.** Definition of the parameters that appear in Equations (1) and (2).

the problem under consideration is depicted in Figure 2. A charged particle is moving with speed  $\mathbf{u}$  and acceleration  $\dot{\mathbf{u}}$ . We assume that the direction of  $\mathbf{u}$  does not change with time; that is, both  $\mathbf{u}$  and  $\dot{\mathbf{u}}$  are acting in the same direction. The electric field produced by this charge at point  $P$  (with  $\boldsymbol{\beta} = \frac{\mathbf{u}}{c}$  and  $\mathbf{a}_r = \frac{\mathbf{r}}{r}$ ) is given by

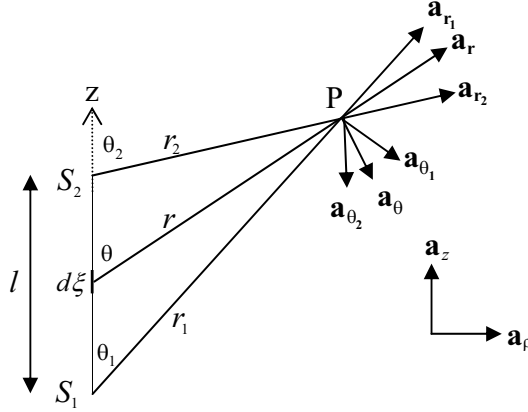
$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{r^2(1 - \boldsymbol{\beta} \cdot \mathbf{a}_r)^3} (\mathbf{a}_r - \boldsymbol{\beta})(1 - \beta^2) \right]_{ret} + \frac{q}{4\pi\epsilon_0} \left[ \frac{1}{cr(1 - \boldsymbol{\beta} \cdot \mathbf{a}_r)^3} [\mathbf{a}_r \times (\mathbf{a}_r \times \dot{\boldsymbol{\beta}})] \right]_{ret} \quad (9)$$

$$\mathbf{B} = \frac{q}{4\pi\epsilon_0 c} \left[ \frac{1}{r^2(1 - \boldsymbol{\beta} \cdot \mathbf{a}_r)^3} (\boldsymbol{\beta} \times \mathbf{a}_r)(1 - \beta^2) \right]_{ret} + \frac{q}{4\pi\epsilon_0 c^2} \left[ \frac{1}{(1 - \boldsymbol{\beta} \cdot \mathbf{a}_r)^3 r} \left\{ \mathbf{a}_r \times [\mathbf{a}_r \times (\mathbf{a}_r \times \dot{\boldsymbol{\beta}})] \right\} \right]_{ret} \quad (10)$$

In the above equations the quantities inside the brackets are the retarded quantities. Note that the expressions for  $E$  and  $B$  both consist of two terms. The second term, which depends on the acceleration of the charge, is the radiation field. The first term is called the velocity field. Note also that the term for the velocity field becomes zero when the speed of propagation of the charge is equal to the speed of light. Cooray and Cooray [3] utilized these expressions to calculate the electric and magnetic fields generated by current pulses moving along conductors. Cooray and Cooray [5] utilized the same field expressions to re-interpret the electric and magnetic fields of a short dipole. The expressions derived by Cooray and Cooray [3, 5] will be utilized here to calculate the electric and magnetic fields of a current carrying horizontal conductor located over perfectly conducting ground.

### 3. ELECTROMAGNETIC FIELDS GENERATED BY A CURRENT PULSE PROPAGATING WITH UNIFORM VELOCITY AND WITHOUT ATTENUATION

The geometry under consideration is shown in Figure 3. A current pulse originates at point  $S_1$  and travels with uniform speed  $u$  without attenuation or dispersion towards  $S_2$ . At  $S_2$ , the current is terminated. As shown by Cooray and Cooray [3, 5] the total electric field at point  $P$ , generated by this process has five components. They are as follows: (i) the radiation field generated from  $S_1$  during the acceleration of charge when the current is initiated, (ii) the radiation field generated from  $S_2$  during the charge deceleration as the current is terminated, (iii) the electrostatic field generated by the negative



**Figure 3.** Geometry used in deriving the electromagnetic fields of a current channel.

charge accumulated at  $S_1$  when the positive charge travels towards  $S_2$ , (iv) the electrostatic field generated by the accumulation of positive charge at  $S_2$ , and (v) the velocity field generated as the current pulse moves along the element. The magnetic field generated by the current flow consists of three terms, namely, two radiation fields generated at  $S_1$  and  $S_2$ , and the velocity field generated as the current propagates along the path. Let us now write down the expressions obtained by Cooray and Cooray [3, 5] for these field components.

### 3.1. The Electric Radiation Field Generated from $S_1$

Let us assume that the current pulse leaving  $S_1$  can be represented by  $i(t)$ . The radiation field generated due to the acceleration of charges at point  $P$  is given by Cooray and Cooray [3, 5]

$$\mathbf{e}_{rad,S_1} = \frac{i(t - r_1/c)u \sin \theta_1}{4\pi\epsilon_0 c^2 r_1} \frac{1}{\left[1 - \frac{u \cos \theta_1}{c}\right]} \mathbf{a}_{\theta_1} \quad (11)$$

### 3.2. The Electric Radiation Field Generated from $S_2$

The radiation field generated due to the deceleration of charges at  $S_2$  is given by

$$\mathbf{e}_{rad,S_2} = -\frac{i(t - r_2/c)u \sin \theta_2}{4\pi\epsilon_0 c^2 r_2} \frac{1}{\left[1 - \frac{u \cos \theta_2}{c}\right]} \mathbf{a}_{\theta_2} \quad (12)$$

### 3.3. The Static Field Generated by the Accumulation of Charge at $S_1$

The charge accumulation at  $S_1$  is equal to the integral of the current, and the field component generated by the charges is given by

$$\mathbf{e}_{stat,S_1} = -\frac{\int_0^{t-r_1/c} i(\xi)d\xi}{4\pi\epsilon_0 r_1^2} \mathbf{a}_{r_1} \quad (13)$$

### 3.4. The Static Field Generated by the Accumulation of Positive Charge at $S_2$

The component of the static field generated by the accumulation of positive charge at  $S_2$  is given by

$$\mathbf{e}_{stat,S_1} = -\frac{\int_0^{t-l/u-r_2/c} i(\xi)d\xi}{4\pi\epsilon_0 r_2^2} \mathbf{a}_{r_2} \quad (14)$$

### 3.5. The Velocity Field Generated as the Current Pulse Propagates along the Channel Element

The component attributable to the velocity field generated as the current pulse propagates along the channel element can be written as [3, 5]

$$\mathbf{e}_{vel} = \int_0^l \frac{i(t - \xi/u - r/c) \left\{ 1 - \frac{u^2}{c^2} \right\}}{4\pi\epsilon_0 r^2 \left[ 1 - \frac{u}{c} \cos \theta \right]^2} \left[ \frac{\mathbf{a}_r}{u} - \frac{\mathbf{a}_z}{c} \right] d\xi \quad (15)$$

Since the vector  $\mathbf{a}_r$  varies along the channel the above equation can be decomposed into components along vertical ( $z$ -direction) and horizontal ( $\rho$ -direction) as follows:

$$\mathbf{e}_{vel} = \int_0^l \frac{i(t - \xi/u - r/c) \left\{ 1 - \frac{u^2}{c^2} \right\}}{4\pi\epsilon_0 r^2 \left[ 1 - \frac{u}{c} \cos \theta \right]^2} \left[ \frac{\cos \theta \mathbf{a}_z}{u} + \frac{\sin \theta \mathbf{a}_\rho}{u} - \frac{\mathbf{a}_z}{c} \right] d\xi \quad (16)$$

### 3.6. Magnetic Radiation Field Generated from $S_1$

The magnetic radiation field generated from  $S_1$  is given by

$$\mathbf{b}_{rad,S_1} = \frac{i(t - r_1/c)u \sin \theta_1}{4\pi\epsilon_0 c^3 r_1} \frac{1}{\left[ 1 - \frac{u \cos \theta_1}{c} \right]} \mathbf{a}_\varphi \quad (17)$$

Note that the magnetic field is in the azimuthal direction.

### 3.7. Magnetic Radiation Field Generated from $S_2$

The magnetic radiation field generated from  $S_2$  is given by

$$\mathbf{b}_{rad,S_2} = -\frac{i(t - r_2/c)u \sin \theta_2}{4\pi\epsilon_0 c^3 r_2} \frac{1}{\left[1 - \frac{u \cos \theta_2}{c}\right]} \mathbf{a}_\varphi \quad (18)$$

### 3.8. Magnetic Velocity Field Generated as the Current Pulse Propagate along the Channel Element

The velocity field generated as the current pulse propagate along the channel element is given by

$$\mathbf{b}_{vel} = \int_0^l \frac{i(t - \xi/u - r/c) \left\{1 - \frac{u^2}{c^2}\right\} \sin \theta}{4\pi\epsilon_0 r^2 c^2 \left[1 - \frac{u}{c} \cos \theta\right]^2} \mathbf{a}_\varphi dl \quad (19)$$

The field components given by Equations (11) to (19) provide a complete description of the electric and magnetic fields generated by the current pulse propagating with uniform velocity and without attenuation. Now, let us consider the case of a current pulse moving along the horizontal conductor depicted in Figure 1.

## 4. ELECTROMAGNETIC FIELDS GENERATED BY A CURRENT PULSE PROPAGATING ALONG A HORIZONTAL CONDUCTOR

The geometry relevant to the analysis is shown in Figure 1. At time  $t = 0$  a current pulse (step current pulse in the case of uniform current) is injected into the end  $A$  of the horizontal conductor. We assume that the current pulse propagates with speed of light along the conductor. The goal is to evaluate the electric field at the point of observation marked  $P$  shown in Figure 1. As mentioned in the introduction, this point of observation is located directly below the horizontal conductor and at a height  $h_p$  from ground level. The traditional way to get the electromagnetic fields at point  $P$  is to divide the horizontal conductor into elementary dipoles and summing up the contribution from each dipole taking into account the corresponding delays. However, as one can see in the sections to follow the application of field equations derived in the previous section to the problem will provide a simple and physically intuitive way to write down the equations that describe the electromagnetic field.



Now, the injection of the current into the horizontal conductor causes charges to accelerate and this results in a radiation field. Moreover, in order to satisfy conservation of charge one has to assume that an amount of charge equal to the integral of the current but of opposite polarity will be accumulated at the source of the current (i.e., at the point of injection). This charge will generate an electrostatic field. There will be no velocity fields because the speed of propagation of the current pulse is equal to the speed of light. Let us first write down an expression for the radiation electric field at the point of observation. This can easily be done with the equations presented earlier. For convenience let us resolve this field into two components in the direction  $z$  and  $x$  (the direction of the axes are shown in Figure 1). Let us denote by  $e_{rad,z,r}(t)$ ,  $e_{rad,z,i}(t)$ ,  $e_{rad,x,r}(t)$  and  $e_{rad,x,i}(t)$  the components of the radiation fields produced in the  $z$  direction (subscript  $z$ ) and in the  $x$  direction (subscript  $x$ ) direction by the real (subscript  $r$ ) and the image currents (subscript  $i$ ) respectively. These components are given directly from (11) and can be written as

$$\mathbf{e}_{rad,z,r}(t) = -\frac{i(t - r_r/c) \sin \theta_r \cos \theta_r}{4\pi\epsilon_0 cr_r} \frac{1}{[1 - \cos \theta_r]} \mathbf{a}_z \quad (20)$$

$$\mathbf{e}_{rad,z,i}(t) = -\frac{i(t - r_i/c) \sin \theta_i \cos \theta_i}{4\pi\epsilon_0 cr_i} \frac{1}{[1 - \cos \theta_i]} \mathbf{a}_z \quad (21)$$

$$\mathbf{e}_{rad,x,r}(t) = -\frac{i(t - r_r/c) \sin \theta_r \sin \theta_r}{4\pi\epsilon_0 cr_r} \frac{1}{[1 - \cos \theta_r]} \mathbf{a}_x \quad (22)$$

$$\mathbf{e}_{rad,x,i}(t) = \frac{i(t - r_i/c) \sin \theta_i \sin \theta_i}{4\pi\epsilon_0 cr_i} \frac{1}{[1 - \cos \theta_i]} \mathbf{a}_x \quad (23)$$

where  $\mathbf{a}_x$ ,  $\mathbf{a}_z$  are unit vectors directed in  $x$  and  $z$  directions.

The magnetic radiation field components produced by the initiation of the real and the image currents are in the  $y$  direction and they are given by

$$b_{rad,y,r}(t) = \frac{i(t - r_r/c) \sin \theta_r}{4\pi\epsilon_0 c^2 r_r} \frac{1}{[1 - \cos \theta_r]} \mathbf{a}_y \quad (24)$$

$$b_{rad,y,i}(t) = \frac{i(t - r_i/c) \sin \theta_i}{4\pi\epsilon_0 cr_i} \frac{1}{[1 - \cos \theta_i]} \mathbf{a}_y \quad (25)$$

Note that in the above equations

$$\cos \theta_r = D / \sqrt{(h - h_p)^2 + D^2} \quad (26)$$

$$\sin \theta_r = (h - h_p) / \sqrt{(h - h_p)^2 + D^2} \quad (27)$$

$$\cos \theta_i = D / \sqrt{(h + h_p)^2 + D^2} \quad (28)$$

$$\sin \theta_i = (h + h_p) / \sqrt{(h + h_p)^2 + D^2} \quad (29)$$

where  $D$  is the horizontal distance from the point of injection to the point of observation.

Now let us consider the static electric field generated by the accumulation of charge at the point of current injection. The charge that is being left behind at the point of injection at any given time is

$$Q(t) = - \int_0^t i(\tau) d\tau \quad (30)$$

Once this charge is given the  $z$  and  $x$  components of the static electric field at point  $P$  can easily be written down using Coulombs law. These field components are

$$\mathbf{e}_{sta,z,r}(t) = - \frac{Q(t - r_r/c) \sin \theta_r}{4\pi\epsilon_o c r_r^2} \mathbf{a}_z \quad (31)$$

$$\mathbf{e}_{sta,z,i}(t) = - \frac{Q(t - r_i/c) \sin \theta_i}{4\pi\epsilon_o c r_i^2} \mathbf{a}_z \quad (32)$$

$$\mathbf{e}_{sta,x,r}(t) = \frac{Q(t - r_r/c) \cos \theta_r}{4\pi\epsilon_o c r_r^2} \mathbf{a}_x \quad (33)$$

$$\mathbf{e}_{sta,x,i}(t) = - \frac{Q(t - r_i/c) \cos \theta_i}{4\pi\epsilon_o c r_i^2} \mathbf{a}_x \quad (34)$$

Equations (20) to (34) describe completely the electromagnetic field at the point of observation produced by the propagating current pulse along the horizontal wire. Now, let us consider a point of observation which is located in such a way that  $D \gg h$  where  $h$  is the height of the wire. Under this approximation one can write

$$\cos \theta_r = 1 - \frac{(h - h_p)^2}{2D^2} \quad (35)$$

$$\cos \theta_i = 1 - \frac{(h + h_p)^2}{2D^2} \quad (36)$$

$$\sin \theta_r = \frac{(h - h_p)}{D} \quad (37)$$

$$\sin \theta_i = \frac{(h + h_p)}{D} \quad (38)$$

Substituting these in the equations obtained previously we find

$$\mathbf{e}_{rad,z,r}(t) = - \frac{i(t - D/c)}{2\pi\epsilon_o c} \frac{1}{[h - h_p]} \mathbf{a}_z \quad (39)$$

$$\mathbf{e}_{rad,z,i}(t) = -\frac{i(t - D/c)}{2\pi\epsilon_0 c} \frac{1}{[h + h_p]} \mathbf{a}_z \quad (40)$$

$$\mathbf{e}_{rad,x,r}(t) = -\frac{i(t - D/c)}{2\pi\epsilon_0 c D} \mathbf{a}_x \quad (41)$$

$$\mathbf{e}_{rad,x,i}(t) = \frac{i(t - D/c)}{2\pi\epsilon_0 c D} \mathbf{a}_x \quad (42)$$

$$\mathbf{e}_{sta,z,r}(t) = -\frac{Q(t - D/c)}{4\pi\epsilon_0 D^2} \frac{(h - h_p)}{D} \mathbf{a}_z \quad (43)$$

$$\mathbf{e}_{sta,z,i}(t) = -\frac{Q(t - D/c)}{4\pi\epsilon_0 D^2} \frac{(h + h_p)}{D} \mathbf{a}_z \quad (44)$$

$$\mathbf{e}_{sta,x,r}(t) = \frac{Q(t - D/c)}{4\pi\epsilon_0 D^2} \left\{ 1 - \frac{(h - h_p)^2}{2D^2} \right\} \mathbf{a}_x \quad (45)$$

$$\mathbf{e}_{sta,x,i}(t) = -\frac{Q(t - D/c)}{4\pi\epsilon_0 D^2} \left\{ 1 - \frac{(h + h_p)^2}{2D^2} \right\} \mathbf{a}_x \quad (46)$$

Now summing up the contributions from the image current and the real current we obtain for the total radiation field and the static field in the  $z$  direction as

$$\mathbf{e}_{rad,z}(t) = -\frac{i(t - D/c)}{2\pi\epsilon_0 c} \frac{2h}{[h^2 - h_p^2]} \mathbf{a}_z \quad (47)$$

$$\mathbf{e}_{sta,z}(t) = -\frac{Q(t - D/c)}{2\pi\epsilon_0 D^2} \frac{(h)}{D} \mathbf{a}_z \quad (48)$$

The corresponding components in the  $x$  direction are

$$\mathbf{e}_{sta,x}(t) = -\frac{Q(t - D/c)}{2\pi\epsilon_0 D^4} h h_p \mathbf{a}_x \quad (49)$$

Note that the static field decreases very rapidly with distance compared to the radiation field. If we select the distance in such a way so that the static field can be neglected in comparison to the radiation field, the electric and magnetic field at point  $P$  will become

$$\mathbf{e}_{rad,z}(t) = -\frac{i(t - D/c)}{\pi\epsilon_0 c} \frac{h}{[h^2 - h_p^2]} \mathbf{a}_z \quad (50)$$

$$\mathbf{b}_{rad,y}(t) = +\frac{i(t - D/c)}{\pi\epsilon_0 c^2} \frac{h}{[h^2 - h_p^2]} \mathbf{a}_y \quad (51)$$

Observe that under the above assumptions, the fields do not depend on the distance to the source. Note also that both the electric and magnetic fields are radiation and their directions are perpendicular to each other. This demonstrates the TEM nature of the transmission

line fields and their radiation nature. The Equations (50) and (51) are valid for any current pulse. If the current is uniform with amplitude  $I_0$  the resulting radiation field at the point of observation becomes

$$\mathbf{e}_{rad,z}(t) = -\frac{I_0}{\pi\epsilon_0 c} \frac{h}{[h^2 - h_p^2]} \mathbf{a}_z \quad (52)$$

$$\mathbf{b}_{rad,y}(t) = +\frac{I_0}{\pi\epsilon_0 c^2} \frac{h}{[h^2 - h_p^2]} \mathbf{a}_y \quad (53)$$

Note that the above equations describe the fields produced by the termination  $A$ . That is, we have assumed that the termination  $B$  is located at infinity and that it will not generate any fields at the point of observation, i.e., it is an infinitely long conductor. These radiation fields are identical to the expressions for the field components given by Equations (5) and (8) which are also derived for an infinitely long conductor. This demonstrates the nature of these field components and the reason why they resemble radiation fields.

In demonstrating the above fact we have assumed that the end  $B$  of the line is located at infinity. At this end of the line the current is terminated. This is accompanied by deceleration of electric charges and it will generate a radiation field. Moreover, the accumulation of charge at the line end will give rise to a static electric field too. However, if the point  $B$  is far away from the point of observation neither the static field nor the radiation field will contribute to the field at the point of observation. Note that unlike the radiation field from  $A$  which contained a term of the form  $(1.0 - \cos\theta)$  in the denominator which made it possible for this field term to sustain as the distance to the point of observation is increased, the radiation field from point  $B$  will contain a term of the form  $(1.0 + \cos\theta)$  in the denominator and consequently attenuate as the distance to the point of observation is increased.

## 5. CONCLUSIONS

The electric and magnetic fields associated with a current flowing along a horizontal conductor located over perfectly conducting ground can be interpreted as the radiation fields generated by the acceleration of electrical charges at the point of injection of the current into the horizontal conductor.

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